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Introduction

Sand in stasis or in motion – the image these words conjure up is one of lifelong familiarity and intuitive simplicity. Despite appearances, however, matter in the granular state combines some of the most complex aspects of known physical systems; to date, a detailed understanding of its behaviour remains elusive.

Granular media are neither completely solid-like nor completely liquid-like in their behaviour – they pack like solids, but flow like liquids. They can, like liquids, take the shape of their containing vessel, but unlike liquids, they can also adopt a variety of shapes when they are freestanding. This leads to the everyday phenomenon of the *angle of repose*, which is the angle that a sandpile makes with the horizontal. The angle of repose can take values between θ_r (the angle below which the sandpile is stationary) and θ_m (the angle above which avalanches spontaneously flow down the slope); in the intervening range of angles, the sandpile manifests *bistability*, in that it can either be at rest or have flowing down it. This avalanche flow is such that all the motion occurs in a relatively narrow boundary layer, so that granular flow is strongly non-Newtonian.

Sandpiles are not just disordered in their geometry – the shape and texture of the grains, on which physical parameters like friction and restitution depend, are also sources of disorder. These features, along with their amorphous packings, have important consequences for granular statics and dynamics. It is well known that sand must expand in order to flow or deform, since voids must become available for passing grains to flow through – this is the so-called phenomenon of Reynolds *dilatancy* [1], whose origin lies in the ability of powders to sustain voids. This also results in cooperative phenomena such as *bridge formation*, or its twin avatar, the propagation of *force chains*, both of which will be discussed comprehensively later in the book.

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Since grains are typically massive, so that the ambient thermal energy kT is insufficient to impart to them kinetic energies of any significance, they do not undergo Brownian motion. Consequently, the phenomenon of thermal averaging does not occur, and hence bridges persist, once formed. This is unique to granular materials, since analogous structures would simply be thermally averaged away in gases or liquids. Bridge formation and kinetics are crucial to a proper description of the collective aspects of granular flow.

The *athermal* nature of granular media implies in turn that granular configurations cannot relax spontaneously in the absence of external perturbations. This leads typically to the generation of a large number of *metastable* configurations; it also results in *hysteresis*, since the sandpile carries forward a memory of its initial conditions. Bistability at the angle of repose is yet another consequence, since the manner in which the sandpile was formed determines whether avalanche motion will, or will not, occur at a given angle.

The above taken together, suggest that sandpiles show *complexity*; that is, the occurrence and relative stability of a large number of metastable configurational states govern their behaviour. Analogies between sandpiles and other complex systems, such as spin glasses, Josephson junction arrays, flux creep in superconductors and charge-density waves have been made: for example, de Gennes [2] has drawn analogies between vortex motion in superconductors and in sandpiles.

It should be mentioned at this point that granular matter has been studied extensively by engineers, and that it is beyond the scope of this book to provide a comprehensive review of all such contributions. In particular, there have been significant advances made in the study of the frictional properties of grains, for which the reader is referred to the book by Briscoe and Adams [3]. The regime of rapid flow in powders has also been extensively studied, and some of the relevant developments in an engineering context can be found in review articles by Savage [4].

1.1 Statistical mechanics framework, packing and the role of friction

As mentioned above, true thermal agitation in granular media takes place on an atomic rather than a particulate scale; therefore it is external vibration or shear that initiates and maintains the motion of grains. A characterisation of the relevant dynamic regimes was carried out in the pioneering work of Bagnold [5, 6]; he showed that, depending on the ratio of interparticle collision forces and interstitial viscous forces, a granular system could be in a macroviscous or grain-inertial regime. This ratio, subsequently named the Bagnold number N, was small ($N < 40$) for macroviscous flows (such as flows of slurries or mud where the viscosity of interstitial fluid predominates over grain inertia) and large ($N > 40$) for

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grain-inertial flows (such as granular flows in air, where fluid viscosity can be neglected in comparison with the effects of interparticle collisions).

Since this book is largely concerned with the flows of dry grains in air, it suffices to limit the discussion that follows to the grain-inertial regime; however, the nature of the externally applied shear needs to be specified. In the regime of rapid shear, a loosely packed granular system can be treated like a 'gas' of randomly colliding grains; 'kinetic theories' of grains based on a 'granular temperature' given by the root-mean-square of the fluctuating component of grain velocities [7] can be written down. These have been extensively studied via fluid-mechanical approaches [8]. However, such techniques are clearly inappropriate for situations when the applied shear is weak, and when the system under study consists of densely packed grains in slow, or no, motion with respect to one another. This regime of quasistatic flow needs new physical concepts, and it was to answer this need that Edwards [9] put forward a pathbreaking thermodynamics of granular media in the late 1980s. This was based on the observation that the volume occupied by a granular system (as measured by its packing fraction) is bounded, analogously to the energy in a thermal system.

Assemblies of grains normally pack in a disordered way; and the rigidity as well as the geometrical disorder of the packing are important determinants of granular flow. Although it has long been assumed without proof that the densest possible packing in three dimensions is the regular hexagonally close-packed structure with a volume fraction $\phi_{\mathrm{hcp}} = 0.74$, the highest *available* packing for a disordered assembly such as a powder is closer to the random close-packed limit $\phi_{\mathrm{rcp}} = 0.64$ in three dimensions [10]. The opposite limit of random loose packing, i.e. the least dense limit at which the powder is mechanically stable, is less clearly defined, but some experiments on sphere suspensions [11, 12] suggest values around $\phi_{\mathrm{rlp}} = 0.52$.

Given the existence of these limits, Edwards [9] assumed that an analogy could be drawn between the volume V occupied by a powder and the energy of a thermal system. In addition, he put forward the concept of a new equivalent temperature for a powder; he called this the *compactivity* X, and defined it in terms of the configurational entropy S as $X = \mathrm{d}V/\mathrm{d}S$. The significance of the compactivity is that it is a measure of the disorder: when $X = 0$, the powder is constrained to be at its most compact, whereas the reverse holds for $X = \infty$. The importance of Edwards' formulation lies in the definition of this effective temperature, which is valid for powders at rest or in slow flow, unlike the previously defined granular temperature.

While the reader is referred elsewhere for further details of the statistical mechanics framework [9, 13, 14] and for a deeper explanation of the significance of the compactivity [15], it is pertinent here to mention Edwards' recent formulation of a pressure-related temperature, named by him as the 'angoricity'. Although still largely conceptual, this fills an important void in a theory of seminal importance in the physics of granular media.

The statistical mechanics framework of Edwards has been remarkably successful in various applications. It was used in its earliest form to examine the problem of segregation when a mixture of grains of two different sizes was shaken [16, 17]. An equivalent granular 'Hamiltonian' was written down and solved to increasing levels of sophistication. At the simplest level, the prediction of this model was total miscibility for large compactivities, and phase separation for lower compactivities. At a higher-order level of solution corresponding to the eight-vertex model of spins [18], the prediction for the ordered phase was more subtle: below a critical compactivity, segregation coexists with 'stacking', where some of the smaller grains nestle in the pores created by the larger ones. While it has so far not proved possible to carry out reliable three-dimensional investigations of granular packings at the particulate scale, experiments on concentrated suspensions for high Peclet number (where Brownian motion is greatly diminished) [19] support these predictions.

In our discussions so far, we have said little about the frictional forces that hold dry cohesionless powders together; the first attempt to formulate a macroscopic friction coefficient is attributed to Coulomb [20], who equated it to the tangent of the angle of repose, by defining it to be the ratio of shear and normal stresses on an inclined pile of sand. While the work of Bagnold [5, 6] made it clear that frictional force varied as the square of the shear rate for grain-inertial flow in the regime of rapid shear, it has long been recognised that the nature of the frictional forces in the quasistatic regime is complex; the frictional force between individual grains in a powder can take any value up to some threshold for motion to be initiated [21], so that considerations of global stability reveal little about the nature of microscopic stick–slip mechanisms [22, 23]. The proper microscopic formulation of intergrain friction remains an outstanding theoretical challenge.

1.2 Granular flow through wedges, channels and apertures

The flow of sand through hoppers [21] or through an hourglass [24] has been well studied, in particular to do with the dependence of the flow rate on the radius of the aperture, on the angle of the exit cone and on the grain size. Interest in this subject was rekindled by the experiments of Baxter *et al.* [25], who examined the flow of sand through a wedge-shaped hopper using X-ray subtraction techniques. They demonstrated that for large wedge angles, dilatancy waves formed and propagated upwards to the surface; their explanation was that these propagating regions were due to progressive bridge collapse. Thus, regions of low density trapped under bridges 'travel upwards' when they collapse due to the weight of oncoming material from the top of the hopper. This phenomenon is reversed for the case of small wedge angles, when waves propagate downwards and disappear altogether for totally smooth grains. Evesque [26] has also reported a related phenomenon in his observations of vibrated hourglasses; for large amplitudes of vibration, he observed that flow at the orifice was stopped. Naive reasoning would suggest that an increased flow might result as a consequence of the greater fluidisation of sand in the large-amplitude regime – the observation to the contrary confirms the well-known phenomenon of jamming [27, 28].

Theoretical approaches to this subject have been greatly restricted by their inability so far to deal with the fundamentally discrete and discontinuous aspects of granular flow through narrow channels. While existing kinetic theory approaches (see Chapter 12) can be adequate to cope with regions of the wedge where flow exists, they are inadequate for the regions where flow, if it exists, is quasistatic; an added complication from the theoretical point of view is that the transition between these two phases occurs discontinuously. Also, for narrow channels and orifices, the discreteness of the grains is very important and continuum approaches based on fluid mechanics are not really appropriate: despite this limitation, the continuum calculations of Hui and Haff [29] were able to reproduce experimentally observed features of granular flow in narrow channels, such as the formation of plugs. They predicted that for small inelastic grains, plug flow develops in the centre of the channel, with mobile grains restricted to boundary layers; for large elastic grains, on the other hand, plug flow does not occur at all, although the flow rate decreases near the centre. Caram and Hong [30] have carried out two-dimensional simulations of biased random walks on a triangular lattice based on the notion that the flow of grains through an orifice can be modelled as an upward random walk of voids; this yields a flavour of plug flow and bridge formation. Finally, Baxter and Behringer [31] have demonstrated the effects of particle orientation (see also Behringer and Baxter [32] for a fuller description); their cellular automaton (CA) model includes orientational interactions, whose results are in good agreement with their experiments on elongated grains. The results of both simulation and experiment indicate that elongated grains align themselves in the direction of flow, with the upper free surface exhibiting a series of complex shapes. More recent work on bridges [33] as well as on grain shapes [34, 35], will be discussed in detail in subsequent chapters.

1.3 Instabilities, convection and pattern formation in vibrated granular beds

The occurrence of convective instabilities in vibrated powders is among a class of familiar phenomena (see, for example [36]) that have been reexamined by several groups [37, 38]. When an initially flat pile of sand is vibrated vertically with an applied acceleration Γ such that $\Gamma > g$, the acceleration due to gravity, a spontaneous slope appears, which approaches the angle of repose θ; this is termed a convective instability, since it is then maintained by the flow down the slope, and convective feedback to the top. However, there is still considerable doubt about the mechanisms responsible for the spontaneous symmetry breaking associated with the sign of the slope. On the one hand, it seems very plausible that the presence of rogue horizontal vibrations (which are very difficult to eradicate totally) could be responsible for transients pushing up one side of the pile; the symmetry breaking thus achieved would lead to the resultant slope being maintained by convection in the steady state. Equally, a mechanism due to Faraday [39] has been invoked [38] to explain this, which relies on the notion that air flow in the vibrated pile is responsible for the initial perturbation of the grains and the consequent appearance of the 'spontaneous' slope. Finally, it is possible to draw analogies with the work of Batchelor [40] on fluidised beds, which suggests that one of the key quantities leading to instabilities in those systems is the gradient diffusivity of the grains, related to differences in their spatial concentration; however, for powders well below the fluidisation threshold, where interstitial fluid is expected to play a more minor role than in conventional fluid-mechanical systems, such analogies should be pursued with caution.

An associated problem is the extent to which the vibrated bed can indeed be regarded as fluidised in the sense required for the Faraday mechanism. While kinetic theory approaches suggest that a vibrated sandpile is more fluidised at the bottom than at the top [4], experiments [41] suggest the opposite; this scenario, i.e. that the free surface of a pile is more loosely packed than its base, is one that makes much more intuitive sense.

It is possible that the resolution of this controversy lies in the interpolation of granular temperatures discussed in [42]. In the regime of large vibration or when piles are loosely packed, grains can undergo a kind of Brownian motion in response to the driving force, so that the use of kinetic theories based on the concept of a conventional granular temperature is not inappropriate; it is then also conceivable that the extent of fluidisation is greatest at the base where the driving force is applied. On the other hand, for denser piles as used in the experiments of Evesque [41], providing the amplitude of vibration is not too large, the use of kinetic theory is limited, and the effective temperature is more likely to be the compactivity [15]; in such regimes, one would expect to see denser packings at the base which would then move like a plug in response to vibration, allowing for the greatest agitation to be felt at the free surface. The experiments of Zik and Stavans [43], where the authors measured the friction felt by a sphere immersed in a vibrated granular bed as a function of height from the base and applied acceleration, lend support to this scenario. They show that in a boundary layer at the bottom of the cell, the friction decreases rapidly with height, whereas it is nearly constant in the bulk; however, the size of this boundary layer decreases sharply with increasing acceleration, ranging from the system size at $\Gamma = 1$ to the sphere size at higher accelerations. They conclude that for large accelerations, grains are in a fluidised state, and respond as nearly Brownian particles; while for small accelerations and a denser packing, the presence of a systemwide boundary layer indicates strongly collective behaviour, with free particle motion restricted to the surface.

The phenomenon of convective instability has also been explored by computer simulations. Both Taguchi [44] and Gallas *et al*. [45] have employed granular dynamics schemes to simulate the formation of convective cells in two-dimensional vibrated granular beds containing a few hundred particles. These simulations are based on the molecular dynamics approach but they include parametrised interparticle interactions which model the effects of friction and the dissipation of energy during inelastic collisions. The form of this interaction, which allows a limited number of particle overlaps, precludes a direct quantitative comparison between the simulations and the behaviour of real granular materials.

However, it is clear that convection in a two-dimensional granular bed can be driven by a cyclic sinusoidal displacement imposed on the (hard) base of the simulation cell. In the steady state, a map of the mean particle velocity against position (in the frame of the container) shows two rolls which flow downwards next to the container walls and upwards in the centre. Although experiments have concentrated on the link between convection and heap formation, these simulations show the two phenomena as separate; a causal link between these two effects, if one exists, must be pursued in more realistic three-dimensional simulations. It is also clear that better models of the forces transmitted from the vibration source through grain contacts to the pile surface are necessary for the understanding of extended flow patterns in disordered granular systems. These issues will be further discussed later in this book.

For two-dimensional simulations containing a few hundred particles, the details of the driving force are paramount in determining the strength and the quality of the convective motion. Gallas *et al*. [45] show that there is a special (resonant) driving frequency for which convection is strongest and that the cellular pattern disappears if the vibration displacement amplitude is small. Taguchi [44] has shown that, for small vibration amplitude or large bed depth, convection is limited to an upper, fluidised layer while lower particles respond to the excitation, in large part, as a rigid body. The depth of the fluidised region increases with the vibration strength. Taguchi has identified the release of vertical stress during the vibrational part of the shake cycle as the origin of the convective motion. This occurs for acceleration amplitudes that are above a critical value ($\Gamma \approx 1$).

For larger accelerations yet, experiments report more and more complicated instabilities; Douady *et al*. [38] have reported period-doubling instabilities leading to the formation of spatially stationary patterns. Pak and Behringer [46] also observe these standing waves, and find in addition higher-order instabilities corresponding to travelling waves moving upward to the free surface. In some cases a bubbling effect is observed, where voids created at the bottom propagate upwards and burst at the free surface, indicating that the bed is fluidised. One of the most striking experimental observations is the oscillon, reported by Umbanhowar, Melo and Swinney [47–49]. While there is as yet an insufficient theoretical understanding of these difficult problems, it is clear [50] that the applied acceleration Γ, which has been used canonically as a control parameter for vibrated beds, is inadequate for their complete characterisation. This is corroborated by the experiments of Pak and Behringer [46], who point out that the higher-order instabilities they observe occur only for large amplitudes of vibration at a given value of the acceleration Γ. The previous use of Γ on its own was related to hypotheses [37] that a granular bed behaved like a single entity, e.g. an inelastic bouncing ball, in its response to vibration; while Γ is indeed the only control parameter for this system [51], the many-body aspects of a sandpile and its complicated response to different shear and vibratory regimes defy such oversimplification [52, 53]. We suggest, therefore, that competing regimes of amplitude and frequency should be explored for the proper investigation of pattern formation and instabilities in vibrated granular beds.

1.4 Size segregation in vibrated powders

Still keeping the convection connection, but in the context of segregation, we mention the work of Knight *et al.* [54] which identified convection processes as a cause of size segregation in vibrated powders. Size segregation phenomena, in which loosely packed aggregates of solid particles separate according to particle size when they are subjected to shaking, have widespread industrial and technological importance. For example, the food, pharmaceutical and ceramic industries include many processes such as the preparation of homogeneous particulate mixtures, for which shaking-induced size segregation is a concern. An assessment of the particulate mechanisms that underlie a segregation effect and of the qualities of the vibrations which constitute the driving forces is thus essential in these situations [55].

The convection-driven segregation proposed by Knight *et al.* [54] is clearly distinct from previously proposed segregation mechanisms (see below) which rely substantially on relative particle reorganisations. In a convection flow pattern all the particles, large and small, are carried upwards along the centre of each roll, but only particles which have sizes smaller than the width of the downward moving zone at the roll edges will continue in the flow and complete a convection cycle. Those particles that are larger than this critical size remain trapped on the top of a vibrating bed, and therefore segregation is observed. In the simplest case, such convection-driven segregation leads to a packing that is separated into two distinct fractions, respectively containing particles with sizes above and below critical. In the fully segregated state, there is a gradation of such phase separation: separate convection cells exist for each size fraction, with only a small amount of interference at their internal interface. The experiments of Knight *et al.* [54] show that such convective motion is driven by frictional interactions between the particles and the walls and disappears in its absence; they conclude also that convection is overwhelmingly responsible for size segregation in the regime of low-amplitude and high-frequency vibrations.

Size segregation is, however, frequently observed in vibrated particulate systems even when there is no apparent convective motion (see e.g. [56]). In the most significant process of this kind, collective particle motions cause large particles to rise, relative to smaller particles, through a vibrated bed. In a complementary process, that of interparticle percolation, vibrations assist the fall of small particles through a close-packed bed of larger particles. A large size discrepancy is not essential for these processes to proceed [57], and in many practical examples, it is the segregation of similarly sized particles that is most important. For these processes, it is often the excitation amplitude which is the appropriate control parameter.

Computer simulations have been instrumental in developing an understanding of these processes. The two-dimensional simulations of Rosato *et al.* [58] were designed to explain why Brazil nuts rose to the top, via a model that included sequential as well as nonsequential (cooperative) particle dynamics. They showed that, during a shaking process, the downward motion of large particles is impeded, since it is statistically unlikely that small particles will reorganise below them to create suitable voids. The large particles therefore rise with respect to the small ones, i.e. size segregation is observed.

In general, for a shaken bed containing a continuous distribution of particle sizes, a measure of the segregation is the weighted particle height,

$$s = \Sigma(R_i - R_o)z_i/(\langle z\rangle \Sigma(R_i - R_o)) - 1, \tag{1.1}$$

where R_i is the size of the ith sphere at height z_i, R_o is the minimum sphere size and $\langle z\rangle$ is the mean height. This initially increases linearly with time [59] and, in the fully segregated state, fluctuates around a constant value; in this state there is a continuous gradation of particle sizes in the height profile.

Other simulations [58] follow the progress of a single impurity (tracer) particle that is initially located near the centre of a vibrated packing. For fixed vibration intensity, the mean vertical component, $\langle v\rangle$, of the tracer displacement per shake cycle varies continuously with the relative size, R, of the impurity such that $\langle v\rangle > 0$ when $R > 1$ and $\langle v\rangle < 0$ when $R < 1$. In three dimensions, there is a percolation

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discontinuity at small impurity sizes and $\langle v\rangle$ increases sublinearly for large impurity sizes. For $R \sim 1$, segregation is very slow and long simulation runs are necessary in order to measure accurately the segregation velocity of an isolated impurity. In this regime, the segregation takes place intermittently; that is, the impurity particle jumps sporadically, in between periods of inactivity. The process becomes continuous for larger relative sizes R. Another result from these simulations is that size segregation is retarded for shaking amplitudes which are smaller than some critical value [58].

The segregation results above must be considered carefully because they arise from nonequilibrium Monte Carlo simulations, for which dynamic results may depend on parameters such as the maximum step length and the termination criterion [60]. However, shaking simulations combining Monte Carlo deposition with nonsequential stabilisation which deploy a homogeneous introduction of free volume [61] as a response to shaking, lead to configurations of particles that are virtually independent of the simulation parameters [62]. Able to reproduce the qualitative features of segregation described above [63], their results [64] indicate that the competition between fast and slow dynamical modes determines the statistical geometry of the packing and therefore has a crucial influence on the mode of size segregation. Further details of this can be found in a subsequent chapter.

Convection and particle reorganisation mechanisms are clearly distinct, but they have some features in common which are essential in driving realistic segregation processes. Firstly, they both rely on nonsequential particle dynamics, so that the extent of the segregation (which depends, qualitatively speaking, on the competition between individual and collective dynamics) is dependent on the amplitude of the driving force. Secondly, both mechanisms rely on the complex coupling between a vibration source and a disordered granular structure, i.e. the fact that the driving forces are not transmitted to individual particles independently, but in a way that relies on many-body effects involving friction and restitution. The minimal ingredients for any convincing model of segregation thus must include nonsequential dynamics and complex force–grain couplings, to avoid unphysical results [63].

The above underlines the need for a precise specification of the driving forces if one is to build reliable models of shaking and any associated segregation behaviour. Thus, although the acceleration amplitude of the base is frequently chosen as the control parameter for a vibrated bed, in practice, details such as the extent and the location of free volume that is introduced into a packing at each dilation, and the contact forces at particle–wall collisions, may be required for an accurate analysis of segregation phenomena [50].

It has in fact been suggested [50] that convection-driven segregation dominates in the quasistatic regime of low-amplitude and high-frequency vibrations which