

ELEMENTS OF STRING COSMOLOGY

The standard cosmological picture of our Universe emerging from a “big bang” leaves open many fundamental questions: Is the big bang a true physical singularity? What happens to the Universe at ultra-high energy densities when even gravity should be quantized? Has our cosmological history a finite or infinite past extension? Do we live in more than four space-time dimensions? String theory, a unified theory of all forces of nature, should be able to answer these questions.

This book contains a pedagogical introduction to the basic notions of string theory and cosmology. It describes the new possible scenarios suggested by string theory for the primordial evolution of our Universe. It discusses the main phenomenological consequences of these scenarios, stresses their differences from each other, and compares them with the more conventional models of inflation.

The first book dedicated to string cosmology, it summarizes over 15 years of research in this field and introduces current advances. The book is self-contained so it can be read by astrophysicists with no knowledge of string theory, and high-energy physicists with little understanding of cosmology. Detailed and explicit derivations of all the results presented provide a deeper appreciation of the subject.

MAURIZIO GASPERINI is Professor of Theoretical Physics in the Physics Department of the University of Bari, Italy. He has authored several publications on gravitational theory, high-energy physics and cosmology, and has twice received one of the Awards for Essays on Gravitation from the Gravity Research Foundation (1996 and 1998).

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*To Patty
with gratitude*

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Preface

The aim of this book is to provide an elementary, but detailed, introduction to the possible impact of string theory on the basic aspects of primordial cosmology. The content of the book includes a discussion of the new models of the Universe obtained by solving the string theory equations, as well as a systematic analysis of their phenomenological consequences, for a close comparison with more conventional inflationary scenarios based on the Einstein equations.

The book is primarily intended for graduate students, not necessarily equipped with a background knowledge of cosmology and string theory; but any reader in possession of the basic notions of general relativity and quantum field theory should be able to benefit from the use of this book (or, at least, of a great part of it). Some chapters (in particular, Chapters 1, 7 and 8) could also be used as a “soft” introduction to modern cosmology for string theorists, while other chapters (in particular, Chapters 2 and 3) as a soft introduction to string theory for astrophysicists; however, all readers are strongly advised to refer to other, more specialized books for a rigorous (independent) study of cosmology and string theory. It should be stressed, also, that this book *is not* aimed as a comprehensive and up-to-date review of all research work available in a string cosmology context: it only provides a pedagogic introduction to the basic ideas and theoretical tools, hopefully useful to the interested reader as a starting point towards more advanced research topics currently in progress in this field.

This book grew out of lectures given in May 2001 at the *First International Ph.D. Course on “Gravitational Physics and Astrophysics”*, jointly organized by the Universities of Berlin, Portsmouth, Potsdam, Salerno and Zurich. The style is that of class lectures: I have tried to be self-contained as much as possible, and I have not hesitated to insert many computational details and explanations, which may even appear to be trivial to the expert reader, but which may result in being of crucial importance for many students, as I have personally verified during the lectures. Besides organizing known material in a form appropriate to

a pedagogic presentation, the book also presents explicit calculations never seen in the literature; in addition, it contains new results obtained through simple generalizations of previous studies. In particular, all topics are discussed (whenever possible) in the general context of a $(d + 1)$ -dimensional space-time manifold: known results in $d = 3$ are thus extended (some of them for the first time) to a generic number d of spatial dimensions.

A possible objection concerning the explicit absence of exercises and problems can be preempted by noting that the main text of the various chapters is literally “filled” with *solved* exercises, in the sense that all computations are displayed in full details, including all the explicit passages required for a reader’s easy understanding. In view of such a large “equation density” in all sections and appendices, the inclusion of additional exercises seemed to be inappropriate.

Another warning concerns the appendices. In contrast with the common use of presenting technical details and computations (and with the exception of Appendix 2A), here the appendices are devoted to a self-contained discussion of specific topics which are closely related to the subject of the chapter, but which are not essential for the understanding of other chapters, and can be skipped in a first reading.

It should be explained, finally, why some chapters are characterized by a list of references much longer than others. The reason is that in some cases (for instance in Chapters 2 and 3) one can conveniently refer to existing books, which provide an excellent discussion of the subject; in other cases (for instance in Chapters 7, 8 and 10), no such book is presently available, and one has to resort to a more detailed bibliography with explicit references to the original papers on the subject.

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But, above all, I am greatly indebted to Gabriele Veneziano for his friendship and invaluable help and advice over all the years, for the generous sharing of his mastery of string theory, and for his crucial role in encouraging me to concentrate on the study of string cosmology scenarios. It is fair to say that this book would not exist without his basic input.

Finally, I would like to thank Simon Capelin (Publishing Director at Cambridge) for his kind advice and encouragement, Lindsay Barnes (Assistant Editor) for her prompt assistance, Margaret Patterson (Copy Editor) for her very careful correction of the original manuscript, and Jeanette Alfoldi (Production Editor) for her kind and efficient help during the final stages of the publishing process.

Notation, units and conventions

Unless otherwise stated, we adopt the following conventions:

- spatial indices: $i, j, k, \dots = 1, \dots, d$;
- space-time indices: $\mu, \nu, \alpha, \dots = 0, 1, \dots, d$;
- metric signature: $g_{\mu\nu} = \text{diag}(+, -, -, -, \dots)$;
- Riemann tensor: $R_{\mu\nu\alpha}{}^{\beta} = \partial_{\mu}\Gamma_{\nu\alpha}{}^{\beta} + \Gamma_{\mu\rho}{}^{\beta}\Gamma_{\nu\alpha}{}^{\rho} - (\mu \leftrightarrow \nu)$;
- Ricci tensor: $R_{\nu\alpha} = R_{\mu\nu\alpha}{}^{\mu}$;
- covariant derivatives: $\nabla_{\mu}V^{\alpha} = \partial_{\mu}V^{\alpha} + \Gamma_{\mu\beta}{}^{\alpha}V^{\beta}$; $\nabla_{\mu}V_{\alpha} = \partial_{\mu}V_{\alpha} - \Gamma_{\mu\alpha}{}^{\beta}V_{\beta}$.

Covariant objects are referred to the symmetric, metric-compatible Christoffel connection,

$$\Gamma_{\alpha\beta}{}^{\mu} = \frac{1}{2}g^{\mu\nu}(\partial_{\alpha}g_{\beta\nu} + \partial_{\beta}g_{\alpha\nu} - \partial_{\nu}g_{\alpha\beta}), \quad (1)$$

satisfying $\nabla_{\alpha}g_{\mu\nu} = 0$. We use natural units $\hbar = c = k_{\text{B}} = 1$, where k_{B} is the Boltzmann constant. The fundamental string mass, M_{s} , and string length, λ_{s} , are thus related to the string tension $T = (2\pi\alpha')^{-1}$ by

$$M_{\text{s}}^2 = \lambda_{\text{s}}^{-2} = (2\pi\alpha')^{-1}. \quad (2)$$

The four-dimensional (reduced) Planck mass M_{P} , and the Planck length λ_{P} , are related to the Newton constant G (in $d = 3$ spatial dimensions) by

$$M_{\text{P}}^2 = \lambda_{\text{P}}^{-2} = (8\pi G)^{-1}. \quad (3)$$

The current experimental value $G \simeq 6.709 \times 10^{-39} \text{ GeV}^{-2}$ [1] then leads to

$$M_{\text{P}} = (8\pi G)^{-1/2} \simeq 2.43 \times 10^{18} \text{ GeV} \quad (4)$$

(note the difference from an alternative – often used – definition, $M_{\text{P}} = G^{-1/2} \simeq 1.22 \times 10^{19} \text{ GeV}$). In a manifold with $D = d + 1$ space-time dimensions Eq. (3) becomes

$$M_{\text{P}}^{d-1} = \lambda_{\text{P}}^{1-d} = (8\pi G_D)^{-1}, \quad (5)$$

where G_D is the D -dimensional gravitational coupling constant, and M_P , λ_P are gravitational scales, possibly different (in principle) from the numerical value (4) determined by four-dimensional phenomenology. If the geometry of the higher-dimensional manifold has a factorized, Kaluza–Klein structure, then G_D is related to the four-dimensional Newton constant G through the proper volume of the internal space $\mathcal{M}_{d-3}$ as follows:

$$(8\pi G_D)^{-1} V_{d-3} = (8\pi G)^{-1}, \quad V_{d-3} = \int_{\mathcal{M}_{d-3}} \sqrt{|g|} \, d^{d-3}x. \tag{6}$$

The relative strength of M_s and M_P is controlled by the scalar dilaton field ϕ , defined in such a way that, at the tree-level, and in d spatial dimensions,

$$\left(\frac{M_s}{M_P}\right)^{d-1} = e^\phi. \tag{7}$$

Masses, energies and temperatures are usually expressed in eV (or multiples of eV), and distances in cm (or eV^{-1}), using the equivalence relations:

$$(1 \text{ eV})^{-1} \simeq 1.97 \times 10^{-5} \text{ cm} \simeq 6.59 \times 10^{-16} \text{ s} \simeq 8.6 \times 10^{-5} \text{ kelvin}^{-1}. \tag{8}$$

The Planck length, defined as in Eq. (3), corresponds to

$$\lambda_P = (8\pi G)^{-1/2} \simeq 8.1 \times 10^{-33} \text{ cm}. \tag{9}$$

The curvature scale of the cosmological manifolds, parametrized by the Hubble parameter H , is often expressed in Planck units, and the energy densities in units of critical density $\rho_c = 3H^2/8\pi G$. For the present Universe, in particular,

$$H_0 = 3.2h \times 10^{-18} \text{ s}^{-1} \simeq 8.7h \times 10^{-61} M_P, \tag{10}$$

where $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. Recent observations suggest

$$h = 0.73^{+0.04}_{-0.03} \tag{11}$$

as the current standard [1]. The corresponding critical density is

$$\rho_c(t_0) = \frac{3H_0^2}{8\pi G} = 3H_0^2 M_P^2 \simeq 1.88h^2 \times 10^{-29} \text{ g cm}^{-3} \simeq 2.25h^2 \times 10^{-120} M_P^4. \tag{12}$$

Reference

[1] Particle Data Group webpage at pdg.lbl.gov/