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978-0-521-18649-0 - Neverending Fractions: An Introduction to Continued Fractions

Jonathan Borwein, Alf Van Der Poorten, Jeffrey Shallit and Wadim Zudilin

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