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*Part I*

Single production

## 1 Principles

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It has been economists' long-standing conviction that, in a competitive state, there is a long-term tendency towards uniformity of rates of profit. This is why, after having studied the formation of short-term equilibria, Walras attempted to incorporate this law into his grand construction. The nature of his project should not be eclipsed by its failure in the treatment of time. But it is in the works of the classical economists, who elaborated the concept of the long run, that the most profound developments of the idea are found. In a circular conception of production, the outputs become the inputs of a new cycle. Since the rate of profit links the price of inputs to that of output, the uniform profitability hypothesis implies a consistency within the set of prices. When the number of operated processes and commodities are equal ('square' economic systems) the prices are determined by distribution and the technical coefficients: they are called *prices of production*. The present book is mainly devoted to the study of these prices and the behaviour of long-run equilibria. For economists accustomed to think in terms of the equilibrium of demand and supply, the surprising fact is that demand is apparently missing. However, demand matters for the determination of activity levels and affects prices in a twofold fashion: either because distribution is involved (as illustrated by the treatment of land in classical economics), or because the operated processes are those which comply with demand (in this respect, the distinction between single and joint production is significant). After the marginalist revolution, less attention has been devoted to these aspects but the reference to the rate of profit and long-term equilibria survives under different headings (discount rate, turnpike) in the modern formulations of the neoclassical tradition.

This study refers to such great economists as Ricardo, Marx, Jevons, Böhm-Bawerk, Wicksell, von Neumann, Hicks, Leontief, Arrow, Debreu, Malinvaud, Samuelson, Solow and Morishima. Above all, it is inspired by Sraffa's work. The publication of *Production of Commodities by Means of Commodities* (hereafter, *PCMC*) marks the renewal of that field of research in modern economics. The interesting ideas of the present book,

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when not directly borrowed from Sraffa, derive from an elaboration of his model, sometimes in a critical way. However, we should like to be explicit from the very beginning regarding a principle. Simply put, the reader is invited not to interpret our work as being faithful to Sraffa's ideas before it has been scrutinized and compared with Sraffa's own conception. In fact, we never hesitate to plunder every tree in the garden of economic knowledge: our analytical approach is flexible and looks at connections among ideas from diverse horizons. On the contrary, the chapters devoted to economic thought or methodology draw particular attention to the specificity of an approach. They presuppose that a 'school' is not a gentlemen's club and is characterized by adhesion to a global conception and basic principles.

If the greatest economists are those who have defined a project that illuminates their choice and their methodology, then Sraffa is one of them. First, as an historian: while many marginalists considered that it suffices to extend Ricardo's analysis of rent to all factors and forget his absurd theory of value to make him comprehensible, i.e. to make him a precursor of their own approach, Sraffa rendered justice to Ricardo's own conception and restored the working of his logic by destroying the puppet in marginalist clothes. Secondly, as an economist: Sraffa's strong convictions are reflected in *PCMC*'s sub-title 'Prelude to a Critique of Economic Theory'. *PCMC* intends to be a radical critique of marginalism. It points towards its logical inconsistencies and, simultaneously, constitutes a first step towards a reconstruction of economic theory on classical principles. No pages are more significant for this project than the foreword. Remember that Sraffa spent thirty years elaborating the one hundred pages that he considered worth publishing (compare that with my own shameless participation in the deforestation process). The foreword of *PCMC* stresses that no assumptions on returns are made because no change, either finite or infinitesimal, is contemplated. Prior to any judgement on its validity, the very existence of this position must be recognized. It would be paradoxical to treat Sraffa himself in the very way he critiqued others for giving Ricardo short shrift.

Many analyses in this book derive from Sraffa's model but do not necessarily claim to be Sraffian. Some readers may find the dichotomy somewhat schizophrenic. This choice is dictated by our desire for methodological caution. Opinions and feelings are less important than the endeavour to grasp ideas and evaluate their originality, consistency and validity.

Constant returns are assumed.

## 2 The corn model

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### 1 A simple economy

Let us consider an extremely simple society which produces wheat by means of wheat and labour.<sup>1</sup> Our aim is to introduce some concepts and obtain results which will later be extended to more complex economies. The production process (or method) of  $b$  units of wheat by means of  $a$  units of wheat (seeds) and  $l$  units of labour ( $a > 0, l > 0, b > 0$ ) is written

$$a \text{ wheat} + l \text{ labour} \rightarrow b \text{ wheat.} \quad (1)$$

The notation  $(a, l) \rightarrow b$  will also be used. A certain time period (a ‘year’) separates the inputs (or advances) from the product. Constant returns prevail. In sections 2 and 3, only one method is deemed available. For more complex technologies, the question of the selection of the operated production method will be addressed (section 4).

### 2 Duality

The economy is *viable* in a strict sense if its net product is positive ( $b > a$ ). The economy is viable in a broad sense if  $b \geq a$ . When the surplus is completely consumed, the economy reproduces itself at the same level (simple reproduction). Otherwise, a part of the surplus is invested in order to enhance production. Let  $g$  be the rate of investment and  $c$  the surplus per labourer for final consumption. The division between consumption and investment is written  $b - a = cl + ga$ , hence

$$c = (b - (1 + g)a)/l. \quad (2)$$

Relation (2) shows that:

<sup>1</sup> The organization of the very first chapters is close to that of *Theory of Production* (Kurz and Salvadori 1995). This similarity is explained by the pursuit of a common project with Neri Salvadori in the 1980s, which did not come to fruition.

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- Consumption per head  $c = c(g)$  is a continuous and decreasing function of the rate of investment: there is a trade-off between consumption and the rate of growth.
- There exists a maximum rate of growth  $G = (b - a)/a$ , which is attained in investing the totality of the product. Consumption is then nil.
- The economy is strictly viable if  $G > 0$ .

The net product is the source of incomes. Let there be two classes in society: workers who receive a real wage  $w$  per labour unit, and capitalists who advance the seeds required for production, employ and pay the workers, then sell the final product. Let  $r$  be the rate of profit on these operations. If the workers are paid at the end of the period, the division of the net product between profits and wages is written  $b - a = ra + wl$ , hence

$$w = (b - (1 + r)a)/l. \quad (3)$$

The economy is said to be *profitable* in a strict sense if it is possible to obtain a positive rate of profit (resp. profitable in a broad sense if the maximum rate of profit is positive or zero). According to (3):

- The real wage  $w = w(r)$  is a continuous and decreasing function of the rate of profit: there is a trade-off between the wage and the rate of profit
- There exists a maximum rate of profit  $R = (b - a)/a$  to which corresponds a zero wage
- The economy is profitable if  $R > 0$ .

The parallel with the previous conclusions is striking. In particular:

- The maximum rates of growth and profit are equal:  $G = R$
- The properties of viability and profitability are both equivalent to  $G = R > 0$ .

These results are explained by the formal similarity of (2) and (3): the curves  $c = c(g)$  and  $w = w(r)$  coincide. The reason is that, when the capitalists reinvest all the profits and the workers consume all of their wages (the ‘golden regime’), profit and investment on the one hand and wage and consumption on the other are identified ( $r = g$  and  $w = c$ ). Outside the ‘golden regime’, the  $c$  variable given by (2) no longer represents consumption per worker since the capitalists consume as well.

The identity of curves  $c = c(g)$  and  $w = w(r)$  disappears when ‘the classical idea of a wage “advanced” from capital’ (PCMC, § 9) replaces the hypothesis of wages paid at the end of the period. If  $w^M$  denotes the real wage, the total advances made by capitalists rise to  $a + w^M l$  and the sharing of the net product between profits and wages leads to a rate of profit  $r$  such as  $b = (1 + r)(a + w^M l)$ , hence

$$w^M = (b - (1 + r)a)/l(1 + r). \quad (4)$$

Even though (4) differs from the relation obtained for a wage paid *post factum*, the main conclusions remain unchanged. In particular, the maximum rate of profit is not modified, since it does not matter whether a zero wage is advanced or paid *post factum*. We generally follow the Sraffian tradition of a wage paid *post factum* but we shall assure ourselves that the principal conclusions do not depend on this choice.

### 3 Expressions of price

The wage expresses the exchange ratio between labour and wheat. Its inverse  $p$  defines the price of wheat in wage units. With a wage paid *post factum*, (3) shows that the price of wheat amounts to  $p(r) = l/(b - (1 + r)a)$ . This price is positive, rises with the rate of profit and tends towards infinity when  $r$  tends to its maximum level  $R$ . These conclusions constitute an alternative reading of previous results: it is equivalent to say that the wage decreases with the rate of profit and vanishes at the limit or that the wage price of wheat rises indefinitely. When the choice of the numéraire is left open, the fundamental equation is written

$$(1 + r)ap + wl = bp \quad (5)$$

and the relative prices of wheat and labour are

$$p(r) = wl/(b - (1 + r)a). \quad (6)$$

Let us choose the gross product as the measurement unit of wheat ( $b = 1$ ). Relation (6) is developed as:

$$p(r) = wl + (1 + r)wal + (1 + r)^2wa^2l + \dots + (1 + r)^t wa^t l + R_t \quad (7)$$

with

$$R_t = (1 + r)^{t+1} a^{t+1} p(r). \quad (8)$$

Equality (7) admits an economic interpretation: in order to produce one unit of wheat today (date 0), the capitalists pay  $wl$  to the workers and have invested  $a$  units of seeds at date  $-1$ . These seeds have themselves been obtained as products of the preceding period for which the capitalists have paid  $wal$  to the workers and advanced  $a^2$  units, etc. In pursuing the reduction over  $t$  periods, the capitalists have successively paid the wages  $wa^t l, \dots, wal, wl$  on dates  $-t, \dots, -1, 0$ , whose total present value is the second member of (7). They have also made a 'primitive advance' of  $a^{t+1}$  units of wheat, whose present value  $R_t$  is given by (8). Despite the compounded interests, the present value of the primitive advance is

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negligible when  $t$  is great (because  $(1+r)a < (1+R)a = 1$ ). The formula of infinite reduction to dated wages

$$p(r) = \sum_{t=0}^{\infty} w(1+r)^t a^t l \quad (9)$$

holds for all admissible values of  $r$  ( $r < R$ ). If the wage is advanced, (5) and the reduction formula are written

$$(1+r)(ap + wl) = bp. \quad (10)$$

$$p(r) = \sum_{t=0}^{\infty} w(1+r)^{t+1} a^t l. \quad (11)$$

In relation (1) describing technology, we have assumed that the coefficients  $a$  and  $l$  are positive. What happens if one of them is nil? Let us exclude a land of plenty where there is production without input. If the production of wheat requires labour but not wheat ( $a = 0, l > 0$ ), a simple case of the Austrian model, then the rate of profit has no upper bound ( $R = +\infty$ ) and the reduction is exact from the very first period onwards, with a nil residue. Inversely, the hypothesis ( $a > 0, l = 0$ ) is that of production without labour. Nature alone renders possible the growth of the product, the ‘physiocratic’ polar conception of the former. Sraffa’s conception of wage (*PCMC*, § 8) is intermediary. A part of it, corresponding to minimum level that is historically and socially determined, is incorporated into the input of production: the socio-technical coefficient ‘ $a$ ’ is the sum of the pure technical coefficient and the guaranteed real wage. Sraffa denotes as  $w$  the part of the real wage that exceeds the minimum level and depends on the force relations between workers and capitalists.

From an economic standpoint, the positivity of profit ( $r > 0$ ) is necessary to the survival of a capitalist economy. Formally, it is the value  $-1$  and not  $0$  which constitutes the lower limit (the *factor* of profit  $1+r$  is positive even though the rate of profit is not) and the algebraic properties mentioned previously hold in the extended range  $]-1, R]$ . Similarly, negative rates of growth cover cases of disaccumulation. The values  $g = 0$  (simple reproduction) or  $r = 0$  (labour value) have no particular property.

#### 4 Choice of techniques

Until now, technology has been reductively thought of as one method. The presence of  $m$  methods  $(a_i, l_i) \rightarrow b_i$  ( $i = 1, \dots, m$ ) raises the question of the choice of the operated method. We presuppose the absence of

*a priori* constraints on the available quantities of wheat and labour, therefore a method is not abandoned because of the scarcity of an input. The choice of a technique depends on the objective of the decision maker. If the rate of profit is set at a long-term level  $r$ , the capitalists will look for innovation rents that yield temporary extra profits. Let  $i$  be the currently employed method at the ruling rate of profit  $r$ . The nominal price  $p_i$  and nominal wage  $w_i$  are such that

$$(1 + r)a_i p_i + w_i l_i = b_i p_i. \quad (12)$$

There is no incentive to change the present method if any alternative method  $j$  is more costly:

$$\forall j (1 + r)a_j p_i + w_i l_j \geq b_j p_i. \quad (13)$$

When (12) and (13) are met, the method  $i$  is said to be *cost-minimizing* or *dominant*. Conversely if, when the prices are  $(p_i, w_i)$ , inequality

$$\exists j (1 + r)a_j p_i + w_i l_j < b_j p_i \quad (14)$$

holds, the first contractor who innovates by using method  $j$  obtains a positive extra profit equal to the difference between the two members of (14). By assumption, this extra profit is temporary: once this entrepreneur is imitated the rate of profit will return to the ruling rate  $r$ , but the substitution of method  $j$  for  $i$  leads to new prices  $(p_j, w_j)$

$$(1 + r)a_j p_j + w_j l_j = b_j p_j. \quad (15)$$

The existence of transitory extra profits is therefore compatible with the stability of the long-term rate of profit. An alternative hypothesis consists in assuming that, instead of the long-term rate of profit, the real wage is fixed. The capitalists will then choose the method yielding the maximum rate of profit.

For a given rate of profit, does a dominant method exist? How is it determined? Relations (12)–(14) suggest an algorithm that can be seen as a stylization of the process of technical change. Let us start from the price and the wage  $(p_i, w_i)$  associated with the current method  $i$  and defined by (12). If this method is not dominant, a certain method  $j$  yields extra profits. Once this is substituted for  $i$  and the temporary extra profits have disappeared, new prices  $(p_j, w_j)$  emerge. The procedure is repeated until a dominant method is obtained, for which (12)–(13) hold.

The simplicity of this ‘market algorithm’ conceals a difficulty: the decision to choose the more profitable method  $j$  is taken on the basis of existing prices  $(p_i, w_i)$ ; but the basis will change and become  $(p_j, w_j)$  after the adoption of method  $j$ . It is *a priori* conceivable that method  $i$  that has just been set aside becomes profitable for the new prices. The algorithm



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would then lead, indefinitely, from  $i$  to  $j$  and then to  $i$ . The consistency property stipulates that such a situation is excluded. Consistency is the noteworthy equivalence whereby method  $j$  is cheaper than  $i$  on the basis of prices  $(p_i, w_i)$ :

$$(1 + r)a_j p_i + w_i l_j < b_j p_i \quad (16)$$

if and only if method  $i$  is more costly than  $j$  on the basis of prices  $(p_j, w_j)$ :

$$(1 + r)a_i p_j + w_j l_i > b_i p_j. \quad (17)$$

Let us prove the equivalence between (16) and (17). Prices  $(p_i, w_i)$  associated with the use of method  $i$  verify (12). By calculating  $p_i/w_i$  in (12), (16) can be written

$$(b_i - (1 + r)a_i)/l_i < (b_j - (1 + r)a_j)/l_j. \quad (18)$$

The same transformation can be carried out in relation (17), with prices  $(p_j, w_j)$  defined by (15). Then (17) is also written as (18). Therefore, (16) and (17) are both equivalent to (18).

Consistency is a 'local' property of the algorithm: if method  $j$  is substituted for  $i$ , then the algorithm does not immediately revert from  $i$  to  $j$  (symbolically:  $i \rightarrow j$  implies  $j \rightarrow i$ ). Since consistency alone does not exclude successive substitutions  $i \rightarrow j \rightarrow k \rightarrow i$  and the algorithm progresses in a cyclical way, a supplementary argument is necessary to conclude to convergence. The argument is that, according to (18), there is technical change  $i \rightarrow j$  if and only if the  $r$ -net product per worker  $(b - (1 + r)a)/l$  is higher for method  $j$  than for  $i$ . Cycles are therefore excluded, the algorithm converges and the dominant technique is the one for which the  $r$ -net product is maximum.

According to (3), the  $r$ -net product per worker represents the real wage associated with method  $i$  when the rate of profit is  $r$ . Therefore method  $j$  is preferred to  $i$  if it pays a higher wage and the dominant technique is the one which maximizes the real wage. An application of the property is the graphic selection of the methods. For any method  $i$ , let us draw in figure 2.1 the associated wage-profit curve  $w = w_i(r)$ , which is a segment ((3) with  $w \geq 0$ ,  $r \geq 0$  or  $r \geq -1$ ). For a given rate of profit the selected technique is located on the upper envelope. When the rate of profit varies the set of dominant techniques is represented by the bold curve in figure 2.1. The rates of profit  $r^*$  and  $r^{**}$  correspond to switch points when  $r$  varies. At these points, two methods are equally profitable and the corresponding price and wage vectors are identical, up to the numéraire.

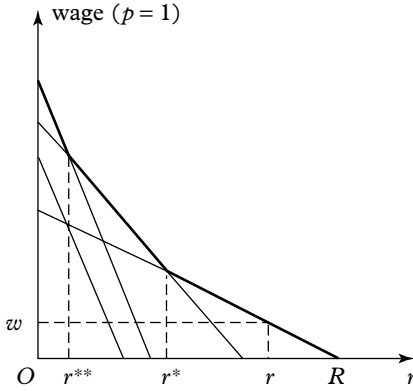


Figure 2.1 Choice of technique: graphical solution

The wage-maximization property results from technical choices. It is not a norm governing the selection of techniques (capitalists do not aim at maximizing wages!) but a manifestation of the good done by the ‘invisible hand’. The change of technique is undertaken by the entrepreneur who comes across an opportunity to obtain extra profits. Because of the *hypothesis* of a profit returning to its initial level in the long run, the surplus is ultimately transferred to the workers as a wage increase.

For a given rate of profit but an advanced wage, the choice of technique is identical to that of the wage paid *post factum*. This results from the equivalence of (12) and (19) on the one hand and (13) and (20) on the other:

$$(1 + r)(a_i p_i^M + w_i^M l_i) = b_i p_i^M \tag{19}$$

$$(1 + r)(a_j p_j^M + w_j^M l_j) \geq b_j p_j^M \tag{20}$$

Alternatively, let us suppose that the choice of technique is made under the hypothesis of a given real wage. The dominant technique maximizes the rate of profit. According to figure 2.1, the introduction of a new method can lead only to an increase in the rate of profit. This result, known as the Okishio Theorem, is famous because it seems to contradict the ‘law of the falling rate of profit’ formulated by Marx. We will examine this law in chapter 8, section 3. Finally, thanks to duality and the golden rule, the market algorithm can also be referred to in a totally different framework, that of a ‘socialist’ economy in which the planner aims at maximizing consumption per head for a given rate of accumulation.