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Bill Pender, David Sadler, Julia Shea and Derek Ward
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YEAR 12

CAMBRIDGE Mathematics 2 Unit

Second Edition

COLOUR VERSION WITH STUDENT CD-ROM
Now in colour with an electronic version of the book on CD

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Contents

Preface

This textbook has been written for students in Years 11 and 12 taking the 2 Unit calculus course 'Mathematics' for the NSW HSC. The book covers all the content of the course at the level required for the HSC examination. There are two volumes — the present volume is roughly intended for Year 12, and the previous volume for Year 11. Schools will, however, differ in their choices of order of topics and in their rates of progress.

Although the Syllabus has not been rewritten for the new HSC, there has been a gradual shift of emphasis in recent examination papers.

- The interdependence of the course content has been emphasised.
- Graphs have been used much more freely in argument.
- Structured problem solving has been expanded.
- There has been more stress on explanation and proof.

This text addresses these new emphases, and the exercises contain a wide variety of different types of questions.

There is an abundance of questions and problems in each exercise — too many for any one student — carefully grouped in three graded sets, so that with proper selection the book can be used at all levels of ability in the 2 Unit course.

This new second edition has been thoroughly rewritten to make it more accessible to all students. The exercises now have more early drill questions to reinforce each new skill, there are more worked exercises on each new algorithm, and some chapters and sections have been split into two so that ideas can be introduced more gradually. We have also added a review exercise to each chapter.

We would like to thank our colleagues at Sydney Grammar School and Newington College for their invaluable help in advising us and commenting on the successive drafts. We would also like to thank the Headmasters of our two schools for their encouragement of this project, and Peter Cribb, Sarah Buerckner and the team at Cambridge University Press, Melbourne, for their support and help in discussions. Finally, our thanks go to our families for encouraging us, despite the distractions that the project has caused to family life.

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How to Use This Book

This book has been written so that it is suitable for the full range of 2 Unit students, whatever their abilities and ambitions.

The Exercises: **No-one should try to do all the questions!** We have written long exercises so that everyone will find enough questions of a suitable standard — each student will need to select from them, and there should be plenty left for revision. The book provides a great variety of questions, and representatives of all types should be attempted.

Each chapter is divided into a number of sections. Each of these sections has its own substantial exercise, subdivided into three groups of questions:

FOUNDATION: These questions are intended to drill the new content of the section at a reasonably straightforward level. There is little point in proceeding without mastery of this group.

DEVELOPMENT: This group is usually the longest. It contains more substantial questions, questions requiring proof or explanation, problems where the new content can be applied, and problems involving content from other sections and chapters to put the new ideas in a wider context.

CHALLENGE: Many questions in recent 2 Unit HSC examinations have been very demanding, and this section is intended to match the standard of those recent examinations. Some questions are algebraically challenging, some require more sophistication in logic, some establish more difficult connections between topics, and some complete proofs or give an alternative approach.

The Theory and the Worked Exercises: All the theory in the course has been properly developed, but students and their teachers should feel free to choose how thoroughly the theory is presented in any particular class. It can often be helpful to learn a method first and then return to the details of the proof and explanation when the point of it all has become clear.

The main formulae, methods, definitions and results have been boxed and numbered consecutively through each chapter. They provide a bare summary only, and students are advised to make their own short summary of each chapter using the numbered boxes as a basis.

The worked examples have been chosen to illustrate the new methods introduced in the section. They should provide sufficient preparation for the questions in the following exercise, but they cannot possibly cover the variety of questions that can be asked.

The Chapter Review Exercises: A Chapter Review Exercise has been added to each chapter of the second edition. These exercises are intended only as a basic review of the chapter — for harder questions, students are advised to work through more of the later questions in the exercises.

The Order of the Topics: We have presented the topics in the order that we have found most satisfactory in our own teaching. There are, however, many effective orderings of the topics, and apart from questions that provide links between topics, the book allows all the flexibility needed in the many different situations that apply in different schools.

The time needed for the Euclidean geometry in Chapter Seven and probability in Chapter Eight will depend on students' experiences in Years 9 and 10.

We have left Euclidean geometry and probability until Year 12 for two reasons. First, we believe that functions and calculus should be developed as early as possible because these are the fundamental ideas in the course. Secondly, the courses in Years 9 and 10 already develop most of the work in Euclidean geometry and probability, at least in an intuitive fashion, so that revisiting them in Year 12, with a greater emphasis now on proof in geometry, seems an ideal arrangement.

The Structure of the Course: Recent examination papers have made the interconnections amongst the various topics much clearer. Calculus is the backbone of the course, and the two processes of differentiation and integration, inverses of each other, are the basis of most of the topics. Both processes are introduced as geometrical ideas — differentiation is defined using tangents and integration using areas — but the subsequent discussions, applications and exercises give many other ways of understanding them.

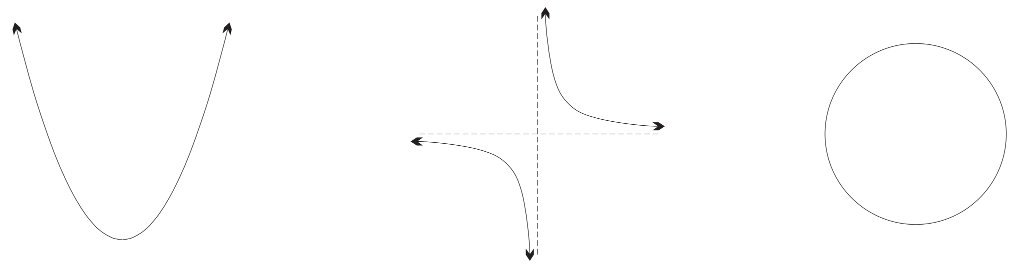
Besides linear functions, three groups of functions dominate the course:

THE QUADRATIC FUNCTIONS: These functions are known from earlier years. They are algebraic representations of the parabola, and arise naturally when areas are being considered or a constant acceleration is being applied. They can be studied without calculus, but calculus provides an alternative and sometimes quicker approach.

THE EXPONENTIAL AND LOGARITHMIC FUNCTIONS: Calculus is essential for the study of these functions. We have begun the topic with the exponential function. This has the great advantage of emphasising the fundamental property that the exponential function with base e is its own derivative — this is the reason why it is essential for the study of natural growth and decay, and therefore occurs in almost every application of mathematics. The logarithmic function, and its relationship with the rectangular hyperbola $y = 1/x$, has been covered in a separate chapter.

THE TRIGONOMETRIC FUNCTIONS: Calculus is also essential for the study of the trigonometric functions. Their definitions, like the associated definition of π, are based on the circle. The graphs of the sine and cosine functions are waves, and they are essential for the study of all periodic phenomena.

Thus the three basic functions in the course, x^2, e^x and $\sin x$, and the related numbers e and π, can all be developed from the three most basic degree-2 curves — the parabola, the rectangular hyperbola and the circle. In this way, everything in the course, whether in calculus, geometry, trigonometry, coordinate geometry or algebra, can easily be related to everything else.



Algebra and Graphs: One of the chief purposes of the course, stressed heavily in recent examinations, is to encourage arguments that relate a curve to its equation. Algebraic arguments are constantly used to investigate graphs of functions. Conversely, graphs are constantly used to solve algebraic problems. We have drawn as many sketches in the book as space allowed, but as a matter of routine, students should draw diagrams for most of the problems they attempt. It is because sketches can so easily be drawn that this type of mathematics is so satisfactory for study at school.

Theory and Applications: Although this course develops calculus in a purely mathematical way using geometry and algebra, its content is fundamental to all the sciences. In particular, the applications of calculus to maximisation, motion, rates of change and finance are all parts of the syllabus. The course thus allows students to experience a double view of mathematics, as a system of pure logic on the one hand, and an essential part of modern technology on the other.

Limits, Continuity and the Real Numbers: This is a first course in calculus, and rigorous arguments about limits, continuity or the real numbers would be quite inappropriate. Any such ideas required in this course are not difficult to understand intuitively. Most arguments about limits need only the limit $\lim_{x \to \infty} 1/x = 0$ and occasionally the sandwich principle. Introducing the tangent as the limit of the secant is a dramatic new idea, clearly marking the beginning of calculus, and is quite accessible. The functions in the course are too well-behaved for continuity to be a real issue. The real numbers are defined geometrically as points on the number line, and any properties that are needed can be justified by appealing to intuitive ideas about lines and curves. Everything in the course apart from these subtle issues of ‘foundations’ can be proven completely.

Technology: There is much discussion about what role technology should play in the mathematics classroom and what calculators or software may be effective. This is a time for experimentation and diversity. We have therefore given only a few specific recommendations about technology, but we encourage such investigation, and to this new colour version we have added some optional technology resources which can be accessed via the student CD in the back of the book. The graphs of functions are at the centre of the course, and the more experience and intuitive understanding students have, the better able they are to interpret the mathematics correctly. A warning here is appropriate — any machine drawing of a curve should be accompanied by a clear understanding of why such a curve arises from the particular equation or situation.

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David Sadler is Second Master in Mathematics at Sydney Grammar School, where he has taught since 1980. He has a BSc from the University of NSW and an MA in Pure Mathematics and a DipEd from Sydney University. In 1979, he taught at Sydney Boys' High School, and he was a Visiting Fellow at the University of NSW in 1991.

Julia Shea is now Director of Curriculum at Newington College, having been appointed Head of Mathematics there in 1999. She has a BSc and DipEd from the University of Tasmania, she taught for six years at Rosny College, a State Senior College in Hobart, and was a member of the Executive Committee of the Mathematics Association of Tasmania for five years. She then taught for five years at Sydney Grammar School before moving to Newington College.

Derek Ward has taught Mathematics at Sydney Grammar School since 1991 and is Master in Charge of Statistics. He has an MSc in Applied Mathematics and a BScDipEd, both from the University of NSW, where he was subsequently Senior Tutor for three years. He has an AMusA in Flute, and is a lay clerk at St James', King Street, where he sings counter-tenor. He also does occasional solo work at various venues.

The Book of Nature is written in the language of Mathematics.

— The seventeenth-century Italian scientist Galileo

It is more important to have beauty in one's equations than to have them fit experiment.

— The twentieth-century English physicist Paul Dirac

Even if there is only one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe? The usual approach of science of constructing a mathematical model cannot answer the questions of why there should be a universe for the model to describe.

— Steven Hawking, *A Brief History of Time*