A Student's Guide to Fourier Transforms

Fourier transform theory is of central importance in a vast range of applications in physical science, engineering and applied mathematics. Providing a concise introduction to the theory and practice of Fourier transforms, this book is invaluable to students of physics, electrical and electronic engineering and computer science.

After a brief description of the basic ideas and theorems, the power of the technique is illustrated through applications in optics, spectroscopy, electronics and telecommunications. The rarely discussed but important field of multi-dimensional Fourier theory is covered, including a description of Computerized Axial Tomography (CAT) scanning. The book concludes by discussing digital methods, with particular attention to the Fast Fourier Transform and its implementation.

This new edition has been revised to include new and interesting material, such as convolution with a sinusoid, coherence, the Michelson stellar interferometer and the van Cittert–Zernike theorem, Babinet's principle and dipole arrays.

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with Applications in Physics and Engineering

Third Edition

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Preface to the first edition

Showing a Fourier transform to a physics student generally produces the same reaction as showing a crucifix to Count Dracula. This may be because the subject tends to be taught by theorists who themselves use Fourier methods to solve otherwise intractable differential equations. The result is often a heavy load of mathematical analysis.

This need not be so. Engineers and practical physicists use Fourier theory in quite another way: to treat experimental data, to extract information from noisy signals, to design electrical filters, to 'clean' TV pictures and for many similar practical tasks. The transforms are done digitally and there is a minimum of mathematics involved.

The chief tools of the trade are the theorems in Chapter 2, and an easy familiarity with these is the way to mastery of the subject. In spite of the forest of integration signs throughout the book there is in fact very little integration done and most of that is at high-school level. There are one or two excursions in places to show the breadth of power that the method can give. These are not pursued to any length but are intended to whet the appetite of those who want to follow more theoretical paths.

The book is deliberately incomplete. Many topics are missing and there is no attempt to explain everything: but I have left, here and there, what I hope are tempting clues to stimulate the reader into looking further; and of course, there is a bibliography at the end.

Practical scientists sometimes treat mathematics in general, and Fourier theory in particular, in ways quite different from those for which it was invented.[1] The late E. T. Bell, mathematician and writer on mathematics, once described mathematics in a famous book title as 'The Queen and Servant of Science'.

[1] It is a matter of philosophical disputation whether mathematics is invented or discovered. Let us compromise by saying that theorems are discovered; proofs are invented.

The queen appears here in her role as servant and is sometimes treated quite roughly in that role, and furthermore, without apology. We are fairly safe in the knowledge that mathematical functions which describe phenomena in the real world are 'well-behaved' in the mathematical sense. Nature abhors singularities as much as she does a vacuum.

When an equation has several solutions, some are discarded in a most cavalier fashion as 'unphysical'. This is usually quite right.[2] Mathematics is after all only a concise shorthand description of the world and if a position-finding calculation based, say, on trigonometry and stellar observations, gives two results, equally valid, that you are either in Greenland or Barbados, you are entitled to discard one of the solutions if it is snowing outside. So we use Fourier transforms as a guide to what is happening or what to do next, but we remember that for solving practical problems the blackboard-and-chalk diagram, the computer screen and the simple theorems described here are to be preferred to the precise tedious calculations of integrals.

Manchester, January 1994 J. F. James

[2] But Dirac's equation, with its positive and negative roots, predicted the positron.

Preface to the second edition

This edition follows much advice and constructive criticism which the author has received from all quarters of the globe, in consequence of which various typos and misprints have been corrected and some ambiguous statements and anfractuosities have been replaced by more clear and direct derivations. Chapter 7 has been largely rewritten to demonstrate the way in which Fourier transforms are used in CAT scanning, an application of more than usual ingenuity and importance: but overall this edition represents a renewed effort to rescue Fourier transforms from the clutches of the pure mathematicians and present them as a working tool to the horny-handed toilers who strive in the fields of electronic engineering and experimental physics.

Glasgow, January 2001 J. F. James

Preface to the third edition

Fourier transforms are eternal. They have not changed their nature since the last edition ten years ago: but the intervening time has allowed the author to correct errors in the text and to expand it slightly to cover some other interesting applications. The van Cittert–Zernike theorem makes a belated appearance, for example, and there are hints of some aspects of radio aerial design as interesting applications.

I also take the opportunity to thank many people who have offered criticism, often anonymously and therefore frankly, which has (usually) been acted upon and which, I hope, has improved the appeal both of the writing and of the contents.

Kilcreggan, August 2010 J. F. James