Cambridge University Press
978-0-521-17495-4 - Talking Mathematics in School: Studies of Teaching and Learning
Edited by Magdalene Lampert and Merrie L. Blunk
Excerpt
More information

1 Introduction

Magdalene Lampert

Consider the collection of regularly shaped flat blocks shown in Figure 1.1. Can two of them be joined to make a hexagon? What is a hexagon anyway? And what does it mean to "join" two of these pieces?

A Sample of Mathematical Talk in School

To introduce the ideas in this volume, I begin with an extended example of mathematical talk from my fifth-grade classroom. This talk will not be analyzed here but will serve to raise the central ideas in the chapters in this book. What does mathematical talk have to do with learning? How do students learn to discuss mathematics? What do the words they use refer to? What does a teacher do in a mathematical discussion? What does classroom discussion have to do with the way mathematical problem solving proceeds outside the classroom?

My fifth-grade students – in groups of four or five – worked with sets of blocks like the ones pictured in Figure 1.1 (called tangrams) to find out if two of them could make a hexagon. And they talked about what they were doing. They disagreed about the "rules of the game": Can "join" mean "overlap"? Can you turn the blocks over? Does "two" mean two of the same shape or two different ones? Does a hexagon have to have equal sides? I did not answer these questions or resolve their disagreements. I encouraged them to ask their classmates for clarification and to talk about why different positions on these questions might make sense and what different assumptions would imply for the solution. As they moved the shapes around, they talked about relationships among the pieces and their attributes: There are two sets of triangles – two small, two larger – that are "the same." If you put two "same" triangles together, you can make a square, or a bigger triangle, depending on how you turn them. The side of the smaller triangle fits exactly on the side of the square, but what would you call the shape it makes? How many sides does it have?

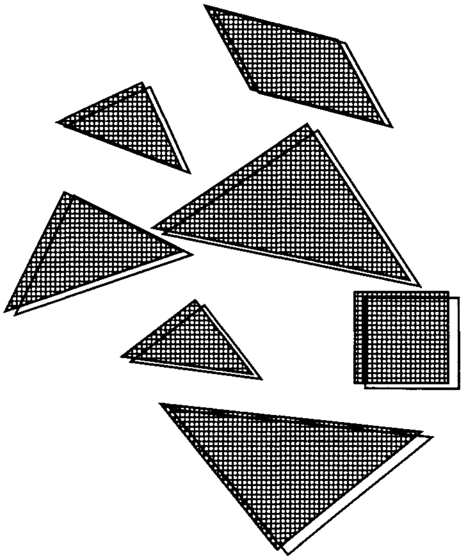

Figure 1.1. Tangrams

After about 20 minutes, I asked the students to stop their small group work and began a discussion intended to involve the whole class in considering whether one could make a hexagon with two tangram pieces. Awad tried to explain why it was impossible:

Lampert: OK. I just heard somebody say "it's impossible." [Some children whisper "yes" or "it is."] How could you prove that it's impossible to make a hexagon with two pieces? It's something that you have an intuition about; how could you prove it? Awad?

Awad: Um, first of all, a hexagon has six sides, you know, and then, like, if you take any of these shapes, you know, it won't make it; I mean it has to be like, see, all these lines are going this way and everything, but these don't do that.

Awad had his notebook propped open on his lap against the desk and glanced at it as he gestured with his pen and hand. I said something about what I thought Awad was getting at and asked if someone else who also thought it was impossible could expand on Awad's explanation. But, instead, Martin disagreed with Awad:

Lampert: OK. So the fact that a hexagon has six sides that you started out saying there, and the relationship between these shapes makes it hard to

Cambridge University Press
978-0-521-17495-4 - Talking Mathematics in School: Studies of Teaching and Learning
Edited by Magdalene Lampert and Merrie L. Blunk
Excerpt
More information

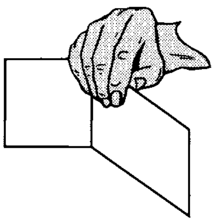

Figure 1.2. Martin's hexagon

make it with two pieces. Could you, could somebody expand on what Awad said, or add, say your own thoughts.

Martin: Yeah, well, I know that you can. I just did it, it's not, it doesn't look like a hexagon, but it is, because it has six sides.

Everyone started talking at their tables. Several students addressed Martin directly. Shahroukh's voice could be heard above the din aggressively agreeing that Martin's figure did indeed have six sides and so it was a hexagon.

Shahroukh: Yeah! Yeah!
Lampert: OK.
Student: Let's see it.
Lampert: Now . . .
Students: Yeah, it has six sides . . .
Shahroukh: It has six sides, that's all you need!

Someone mentioned "six angles" and I asked Martin to hold up the figure so I and everyone else could see what he had done.

Students: Six angles . . . it has to have six angles . . . then . . .
Lampert: Let's see. Could you hold that up again, Martin, so that I could draw it on the board.

Martin held up the two pieces in one hand (see Figure 1.2). I drew his "solution" on the board. What I drew looked something like Figure 1.3. The class seemed disturbed by Martin's idea. There ensued a half-hour-long discussion, moderated by the teacher, in which assertions were made, using various kinds of evidence, about whether this was or was not a hexagon. The discussion veered into extended arguments about how to identify and measure the angles in the figure. There was a lot of disagreement about where the angles in the figure should be measured in relation to the "inside" and the "outside" of the figure, and this

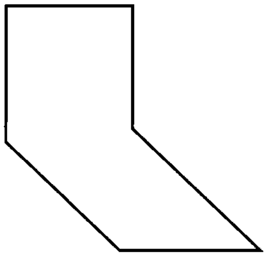

Figure 1.3. Drawing of Martin's hexagon

seemed to relate to how many angles the figure could be said to have, and thus to whether it was or was not a hexagon.

I asked Martin why he thought people might have been disturbed at his solution. He responded with a recognition that the figure that he had constructed in response to the puzzle was a little "strangely shaped" and it did not have equal sides. Does a hexagon have to have equal sides?

Lampert: Martin, why do you think people are a little disturbed by your idea?
Martin: Because, um, I, I usually think of a hexagon, I don't know if this is true of other people, but I usually think of a hexagon as having all *equal* sides, not being, you know, strangely shaped like that.

Much of the class seemed engaged with Martin's assertion, talking in their groups about whether he was right or wrong. I saw an opportunity to provoke some thinking and talking about important mathematical ideas, so I invited other students to comment on the assertion. Martin turned away from me to grab a dictionary and several students raised their hands. I asked a question about what Martin had said and several students indicated that they wanted to speak. I called on Dorota.

Lampert: OK and is that [i.e., equal sides] the only thing something has to have in order to be a hexagon? [pause, as Martin murmurs something inaudible] You're gonna look in the dictionary? Dorota?
Dorota: Six angles and six sides.

Next I asked a question that I thought would focus her and the rest of the class on an important geometrical idea and take us into more general mathematical territory. My intervention brought a chorus of student comments but not unanimity.

Lampert: OK, now Dorota said six angles and six sides. And what I would challenge you to say is, to figure out is, does every figure that has six sides also have six angles?
Students: No! No! Yes! . . . (many children talking at one time)

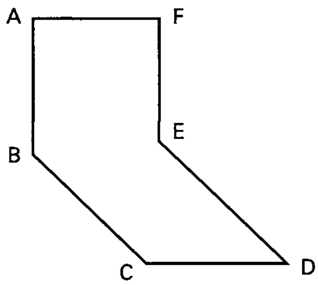

Figure 1.4. Labeled drawing

At this point I asked Martin to read the dictionary definition, but what he read did not address the disagreement on the table. He interpreted the dictionary definition as evidence that his assertion was correct, and he used it to make a distinction between regular and irregular hexagons.

Martin: I've found the definition . . . it says a plane figure having six sides, and six angles, and *that* [pointing to the board] is a hexagon. It's an irregular hexagon because it isn't shaped like, you know, the honeycomb figure that everyone pictures it as [Martin firmly closes the dictionary and returns it to the shelf].

Meanwhile Shahroukh asserted that the figure Martin had drawn was *not* a hexagon because it did not have six angles. As he spoke to me and the class, he hedged a bit and provoked me to focus him and the class on particular parts of the drawing on the board. I labeled some of the angles as a way of coordinating our communication while introducing a mathematical convention.

Lampert: Shahroukh, why at the beginning did you think it did not have six angles?
Shahroukh: Well, as soon as I saw it, I thought that two and two on the top and the bottom, that's four. And then, after I looked at it and then I saw two on the side, that's six . . .
Lampert: OK.
Shahroukh: . . . but, then, if you . . . well . . .
Martin: See, Dr. Lampert . . .
Lampert: I'm gonna label these also, so that we can talk about them a little better. [Teacher labels the figure as shown in Figure 1.4.]

Shahroukh did not miss a beat, incorporating the labels I had added in his attempt to explain his disagreement. Shahroukh continued:

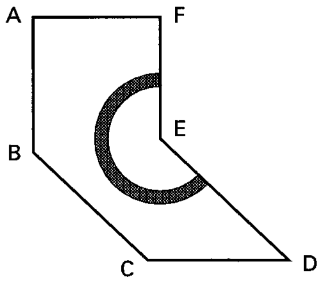

Figure 1.5. Angle E measured inside

Shahroukh: Why did I think it did not have six angles? Because when I saw it I thought of it as having it straight, you know, and then I thought there's C and D and A and F, and so that's four, and I started going, no, it doesn't have, it doesn't have six angles . . .

While he was addressing the class on what I considered to be an important matter, several other students were also talking to each other. I reminded the class that this was "large group." The routine for this part of the class, in place since the beginning of the year, was that only one person could be speaking at a time.[1]

Lampert: Wait, excuse me, we're having large group now and I don't think people are paying attention [to Shahroukh]. So you didn't think at first that B and E were angles?

From here, the discussion moved to a focus on angle E – was it inside or outside the figure? Arguments for which of these made sense were connected with questions about how to measure it. One student came up and gestured that angle E was as shown in Figure 1.5. Around the room, I could see by the ways other students were gesturing in their talk with one another that many disagreed. Another student came up and said you would need to measure it as in Figure 1.6, and a third said, no, it should be done "the smaller way," gesturing as in Figure 1.7. The discussion continued for another 20 minutes.

I present this small story of classwork here as an example of the kind of "math talk" that the authors of the studies reported in this book are seeking to understand. In the chapters that follow, the reader will find a variety of theoretical and analytical frames for addressing questions like: What are these students and this teacher talking about? (A juxtaposition of two blocks? A figure on the board? A set of mathematical abstractions?) What are the units of analysis we should use to answer this

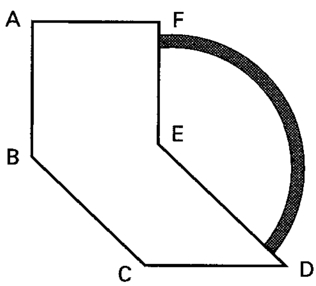

Figure 1.6. Angle E measured outside

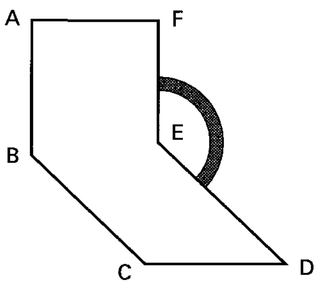

Figure 1.7. Angle E measured outside "the smaller way"

question? (The whole lesson? A thematic exchange? A single utterance?) What do speaker and spoken to do to check on whether they are talking about the same thing? Should they be talking about the same thing? What evidence exists that anyone is listening? Or understanding what is said? What are these students learning and how are they learning it? What is this teacher teaching and how is she teaching it?

Relating Talk and Learning

Much of what the fifth-grade students say in this geometry lesson might be called "inarticulate," and yet the teacher often seems to know what they are talking about, or acts as if they are talking about something mathematically coherent, as in the exchange between Awad and his teacher at the beginning of the story above. Some of my early insights into mathematical talk in classrooms actually came from recognizing the parallels between this teacher–student talk and the adult–child talk that goes on between caregivers and young children. There is some

Cambridge University Press
978-0-521-17495-4 - Talking Mathematics in School: Studies of Teaching and Learning
Edited by Magdalene Lampert and Merrie L. Blunk
Excerpt
More information

pointing and naming, some talking about objects that are present to both speakers, and some just talking. James Britton (1970) explains the importance of such primitive exchanges, not only as occasions when children learn the names of things and how to form sentences, but also as a process of organizing experience. As the caregiver or the teacher makes the child's utterances more explicit and experimentally fills in more of the situation than the child has expressed, she or he presents the ways in which society organizes time, space, and activity. For example, in the math lesson above, when Awad said, pointing to the figures in his notebook, "A hexagon has six sides, you know, and then, like, if you take any of these shapes, you know, it won't make it; I mean it has to be like, see, all these lines are going this way and everything, but these don't do that." I responded with a more mathematically coherent statement. I inserted the word *relationship* into my revoicing of what he had said: "O.K. So the fact that a hexagon has six sides that you started out saying there, and the relationship between these shapes makes it hard to make it with two pieces." With this expansion, the mathematics I was talking came closer to what would be heard outside the classroom among mature speakers, shaping what Awad and the others in the room might attend to.[2]

The contribution of Russian activity theory to the way we formulate the relationship between thought and language underscores this connection between words and worldview. This theory has been used by many scholars in the past 10 years to examine classroom discourse in all areas of the curriculum. Ideas like those I first encountered reading Britton in the 1970s are currently elaborated in reference to the work of Luria, Vygotsky, and Bakhtin, so that conversation and culture have become inseparable foci for investigation in classroom research (Newman, Griffin, & Cole, 1989; Moll, 1990; Hicks, 1996; Cazden, 1996). Activity theory has the child taking a central role in making language meaningful as it is acquired. The speaker of a new language is not a receiver of conventional definitions and ways of knowing, but an "appropriator." Like Martin with his strangely shaped hexagon, a student in school takes words from the culture of mathematics to make sense of something in experience, but also might push beyond conventional assumptions about how mathematical language can be used. The students responding to Martin reconstruct what the teacher has to offer as they negotiate what will stand as a "hexagon" and an "angle" in their talk. Deborah Hicks (1996, p. 108) boldly calls this process "learning": "As the child moves within the social world of the classroom, she appropriates (internalizes) but also reconstructs the discourses that constitute the social world of her classroom.

This creative process is what I would term learning." The practice of learning and the practice of teaching in school complement one another in the context of jointly constructed activity. Just as the child appropriates from the culture of the classroom, the teacher puts things out there to be appropriated, functioning as a partner in the conversation but with "a special mission and power" to ensure the classroom culture is rich in "offers, challenges, alternatives, and models, including languaging" (Bauersfeld, 1995, p. 283). In the example above, the activity is getting Martin's invention named and getting its angles identified, and the vehicles for this activity are talk, gesturing, and drawing. In these media, students and teacher are explaining, arguing, and demonstrating in ways that intend to convince the listener of a particular point of view.

Why Study Mathematical Talk in School?

Most classrooms are language rich, even those where the teacher does most of the talking. In a few classrooms, the teaching and learning of distinctly *mathematical* talk by having students engage in such talk is a deliberate pedagogical focus. The studies of teaching and learning included in this book have all been done in settings where the teacher's intention was that students should learn how to "talk mathematics." Why should we be interested in these classrooms? The work reported here occurs at the intersection of researchers' renewed attention to classroom discourse and the latest wave of reform in mathematics education. In the same year (1990) that I taught the lesson on tangrams, calls for the reform of mathematics in school classrooms began to be heard. In a broad-reaching proposal to change teaching in K–12 schools, the National Council of Teachers of Mathematics (1991, p. 3) asserted:

We need to shift

- toward classrooms as mathematical communities – away from classrooms as simply collections of individuals;
- toward logic and mathematical evidence as verification – away from the teacher as the sole authority for right answers;
- toward mathematical reasoning – away from merely memorizing procedures;
- toward conjecturing, inventing, and problem solving – away from an emphasis on mechanistic answer-finding;
- toward connecting mathematics, its ideas, and its applications – away from treating mathematics as a body of isolated concepts and procedures.

The message of this and other concurrent reform proposals is that students should learn from being more directly engaged in doing and talking about mathematics.

The emphasis and, indeed, even the substance of the new curriculum and pedagogy are formed from ideas about what mathematicians and people who use mathematics do when they are working on problems (NRC, 1988; Steen, 1990). They work in communities, they reason toward solutions and use logic to support their conclusions, they invent and make "educated guesses" called conjectures before they find solutions, and they connect elements of what they are doing with one another and with ideas outside the domain. If school lessons are to involve "communities" of learners doing this kind of mathematical work rather than individuals acquiring skills and remembering rules, classrooms will not be silent places where each learner is privately engaged with ideas. If students are to employ logic and mathematical evidence, they will need to compose speech acts or written artifacts that expose their reasoning. If they are to conjecture and connect, they will need to communicate. The idea that such classroom activities should be a goal of educational reform is based in part on current research in the learning sciences and research that examines the application of psychological theories to curriculum and instruction (see Romberg, 1990; Zarrinia, Lamon, & Romberg, 1987). The sorts of shifts in instructional practice advocated by the NCTM *Standards* are expected to produce increased understanding and improve practical competence.

How Shall We Study Mathematical Talk in School?

In order to measure whether certain kinds of classroom talk result in more and more desirable mathematical understanding, we need to investigate the kinds of curriculum and instruction that will support that talk. The studies in this book are an attempt to begin to understand these matters more clearly from the perspective of practice. The authors argue that we need a rich vision of what "talk" can be and a socioculturally complex concept of learning in order to relate the two. To develop this vision, they look carefully at a few classrooms in which teachers tried to work in the spirit of the reforms, where mathematical talk is primary among the activities that are structured to give students opportunities to learn. We acknowledge that these classrooms are not typical. The Third International Mathematics and Science Study, conducted in 1995, included a close look at teaching in classrooms in three of the participating