COMPLEX VARIABLE THEORY AND TRANSFORM CALCULUS

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AND

TRANSFORM CALCULUS

WITH TECHNICAL APPLICATIONS

BY

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PREFACE TO THE SECOND EDITION

In 1936–7 when the MS. of the first edition was prepared, the degree of rigour seemed to be adequate, but certain pure mathematicians (and physicists!) who reviewed the book disagreed. In the interim, the standard of technical mathematics has improved, and it is now possible to be more rigorous than before. Accordingly the chapters on Complex Integration Theory have been rewritten, amplified, and made rigorous enough for all but the pure mathematician, *to whom the book is not addressed*. Sections of the text set in small type deal with more recondite topics, and may be omitted in the first reading, reference being made to them when required. The rest of the book has been revised completely and brought up to date. Certain of the old sections have been removed to make way for more important subject matter, e.g. repeated impulses, Fourier transforms, and frequency spectra have been added to Chapter X. In solving ordinary differential equations, use is made of either a list of p-multiplied Laplace transforms or the Mellin inversion theorem, according to the problem under consideration. The sections involving partial differential equations have been recast. The approach is via Laplace transform, thereby permitting the initial conditions to be incorporated easily. The Mellin theorem is used for inversion.* Practical data for loaded and unloaded submarine telegraph cables have been given. The calculated and actual shapes of received signals, together with diagrams of the circuits used at both ends of the cable, are reproduced in Chapter XIII. The data for loaded cables are due to Mr A. L. Meyers, and for unloaded cables to the Author. There are additional sections on electrical filters. By aid of a new theorem,† the solution for a dissipative filter can be expressed concisely in the form of a definite integral. The number and variety of the problems to be

* Iterated use of the Laplace transform for solving partial differential equations is exemplified in [266].

† N. W. McLachlan, *Math. Gaz.* 30, 85, 1946.

worked out by the reader has been increased, while the reference list has been extended. An Appendix on convergence of many of the series which occur in the text is given, and should be useful. The list of p-multiplied Laplace transforms covers merely what is needed for the text, since an extended list is available elsewhere [235 *a*, *b*].

The present work and that entitled *Modern Operational Calculus* [236] are complementary, and may be used together. The latter proceeds via real variable and Laplace transform method, which carries the subject to a stage where it may profitably be taken over by the complex variable method, as exemplified herein. Complex integration is needed in solving many of the technical problems involving partial differential equations, and in deriving asymptotic formulae.

N. W. M.

MAY 1952

PREFATORY NOTE

Drs A. J. Macintyre and C. Strachan read parts of the MS. of the second edition, while Profs. T. J. Higgins and E. J. Scott read proofs. I wish to thank these gentlemen for their very helpful criticisms and suggestions. I welcome the opportunity to thank Prof. P. Humbert for his kind gesture in obtaining publication of the lists of transforms in references [235 *a*, *b*].

In this new impression, I am much indebted to Prof. A. Erdélyi for his valuable criticism and suggestions. I have made some minor alterations in the text.

APRIL 1962 N.W.M.

PREFACE TO THE FIRST EDITION

The purpose of this book is to provide a modern treatment of the so-called operational method, and to illustrate its application to problems in various branches of technology. Although it is written primarily for the mathematical technologist,* certain parts of the text may be useful to others....

The reader may wonder why p is used outside the Laplace integral (1) § 8·11. The reasons for this are as follows: (i) By retaining p, the operational forms† of various functions are identical with those obtained by the Heaviside method. Such forms are of long standing and widespread use, so that an alteration now would be inexpedient;‡ (ii) The operational form of t^n is $n!/p^n$. Thus if t and p are considered to have dimensions d and d^{-1}, respectively, and if $f(t)$ and its operational form can be expanded in absolutely convergent series, the corresponding terms are identical dimensionally. This is useful for checking purposes.

The book is divided into four parts, (I) Complex Variable, (II) Operational Calculus, (III) Technical Applications and examples to be worked out by the reader, (IV) Appendices and list of references. Each of the first three parts is preceded by a foreword,§ which the reader should peruse carefully before commencing to study the part in question. *Part I must be understood thoroughly.* After reading each chapter, the corresponding problems at the end of Part III ought to be worked out....When Part I has been assimilated, a knowledge of the early parts of chaps. VIII–X will enable the reader to pass on to Part III. To avoid interpolated explanations of cognate points in the text, a number of Appendices is given in Part IV. Frequent reference is made to these throughout the book....

* A person who uses mathematics to solve technical problems of various kinds, e.g. acoustical, aeronautical, chemical, electrical, mechanical, thermal, etc. The term also applies to the mathematician engaged in industrial and applied research work.

† Designated p-multiplied Laplace transforms in the second edition.

‡ This reason is now inapplicable (1952).

§ Omitted from the second edition.

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Frontmatter
More information

The technologist is not fitted by training, nor has he the time, to delve into rigour to the last epsilon. Just as the mathematician does not need to be versed in thermodynamics and internal combustion engine design to drive a motor car, the technologist need not know how to prove all the theorems he uses. But like the mathematical motorist, he must be acquainted with the highway code. In other words some rigour is needed, and I hope that in this volume a happy mean has been struck between the demands of the mathematician on the one hand, and the requirements of the practical man on the other.

It may be argued by some, that, on the whole, the text is difficult, because complex integration plays such an important part therein. Looking back half a century, we find that engineers regarded the differential and integral calculus as a mystery beyond the reach of the majority. Nowadays the engineering student takes the calculus (at least the small and inadequate dose administered to him) in his stride. Consequently if this book is the means of introducing complex integration to the mathematical technologist who reads the English language, it will justify itself in this respect alone. Moreover, after the customary lapse of valuable time, those who teach the 'young technical idea' will no longer be panic-stricken by a subject which has graced the curricula of continental technical institutions for many years.

Symbols. A new symbol, namely $\supset$, has been introduced to signify 'Laplace transform of', for reasons stated in [131]*. The round end points to the transform, and the open end to the corresponding function t, e.g. $f(t) \supset \phi(p)$. m, n, r are used for integers, while μ, ν denote unrestricted numbers. $R(\nu)$ means 'the real part of ν'. Symbols in heavy type indicate per unit length or area, as the case may be. $\sim$ signifies that the right-hand side is an asymptotic formula for the left-hand side when the variable is large enough. $\simeq$ signifies 'is approximately equal to', $\neq$ signifies 'is not equal to', $O(1/z^2)$ signifies 'is of order $1/z^2$', $\exp\{f(z)\}$ signifies $e^{f(z)}$, $\to \pm 0$ means 'tends to zero' from the positive or the negative side, $\to \pm \infty$ means 'tends to infinity' from the positive or the negative side, $\sqrt{i^{\frac12}} = i = e^{\frac14 \pi i} = (1+i)/\sqrt{2}$ and kindred symbols are illustrated in Fig. 21 (*c*) on p. 63.

* The numbers in [ ] are references on pp. 366–377.

Acknowledgments. I have been exceedingly fortunate in enlisting the help of a number of friends, namely, Professors W. N. Bailey, W. G. Bickley, T. A. A. Broadbent, E. T. Copson, Drs J. M. Jackson, A. T. McKay and Mr A. L. Meyers. They have read various parts of the MS. critically and/or corrected proofs. I owe a great deal to their valuable comments, and accord them my warmest thanks. I am also much indebted to Professor A. R. Collar and Dr R. A. Fraser for suggestions regarding § 11·41* et seq. Best thanks for the loan of blocks are due to the Delegates of the Oxford Press (Figs. 23, 49*a*, 56, 57), the Editors of the *Philosophical Magazine* (Figs. 44, 45), and the Editor of the *Wireless Engineer* (Figs. 42, 43, 46–48, 58, 60). Permission has kindly been granted by the Council of the Physical Society to reproduce the table on p. 272 from a paper by Dr A. T. McKay, and by the Bell Telephone Laboratories to reproduce Fig. 56 from a paper by L. J. Sivian in the *Bell System Technical Journal*.

N. W. M.

LONDON
MARCH 1939

* The numbers in this preface refer to the first edition.