Student Solution Manual for
Essential Mathematical Methods for the Physical Sciences

This Student Solution Manual provides complete solutions to all the odd-numbered problems in Essential Mathematical Methods for the Physical Sciences. It takes students through each problem step by step, so they can clearly see how the solution is reached, and understand any mistakes in their own working. Students will learn by example how to select an appropriate method, improving their problem-solving skills.

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Essential Mathematical Methods for the Physical Sciences

Student Solution Manual

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Contents

Preface page vii

1 Matrices and vector spaces 1

2 Vector calculus 27

3 Line, surface and volume integrals 41

4 Fourier series 56

5 Integral transforms 72

6 Higher-order ordinary differential equations 85

7 Series solutions of ordinary differential equations 105

8 Eigenfunction methods for differential equations 116

9 Special functions 128

10 Partial differential equations 138

11 Solution methods for PDEs 149

12 Calculus of variations 166

13 Integral equations 182

14 Complex variables 192

15 Applications of complex variables 200

16 Probability 214

17 Statistics 231
Preface

For reasons that are explained in the preface to *Essential Mathematical Methods for the Physical Sciences* the text of the third edition of *Mathematical Methods for Physics and Engineering (MMPE)* (Cambridge: Cambridge University Press, 2006) by Riley, Hobson and Bence, after a number of additions and omissions, has been republished as two slightly overlapping texts. *Essential Mathematical Methods for the Physical Sciences (EMMPS)* contains most of the more advanced material, and specifically develops mathematical methods that can be applied throughout the physical sciences; an augmented version of the more introductory material, principally concerned with mathematical tools rather than methods, is available as *Foundation Mathematics for the Physical Sciences*. The full text of *MMPE*, including all of the more specialized and advanced topics, is still available under its original title.

As in the third edition of *MMPE*, the penultimate subsection of each chapter of *EMMPS* consists of a significant number of problems, nearly all of which are based on topics drawn from several sections of that chapter. Also as in the third edition, hints and outline answers are given in the final subsection, but only to the odd-numbered problems, leaving all even-numbered problems free to be set as unaided homework.

*This book is the solutions manual for the problems in EMMPS. For the 230 plus odd-numbered problems it contains, complete solutions are available, to both students and their teachers, in the form of this manual; these are in addition to the hints and outline answers given in the main text. For each problem, the original question is reproduced and then followed by a fully worked solution. For those original problems that make internal reference to the main text or to other (even-numbered) problems not included in this solutions manual, the questions have been reworded, usually by including additional information, so that the questions can stand alone. Some further minor rewording has been included to improve the page layout.*

In many cases the solution given is even fuller than one that might be expected of a good student who has understood the material. This is because we have aimed to make the solutions instructional as well as utilitarian. To this end, we have included comments that are intended to show how the plan for the solution is formulated and have provided the justifications for particular intermediate steps (something not always done, even by the best of students). We have also tried to write each individual substituted formula in the form that best indicates how it was obtained, before simplifying it at the next or a subsequent stage. Where several lines of algebraic manipulation or calculus are needed to obtain a final result, they are normally included in full; this should enable the student to determine whether an incorrect answer is due to a misunderstanding of principles or to a technical error.
Preface

As noted above, the original questions are reproduced in full, or in a suitably modified stand-alone form, at the start of each problem. Reference to the main text is not needed provided that standard formulae are known (and a set of tables is available for a few of the statistical and numerical problems). This means that, although it is not its prime purpose, this manual could be used as a test or quiz book by a student who has learned, or thinks that he or she has learned, the material covered in the main text.