This book provides the first coherent and comprehensive treatment of the thermodynamics and gas dynamics of the practical Stirling cycle. Invented in 1816, the Stirling engine is the subject of world-wide research and development on account of unique qualities – silence, indifference to heat source, low level of emissions when burning conventional fuels, and an ability to function in reverse as heat pump or refrigerator.

The working cycle embraces all aspects of engineering thermodynamics, heat transfer (conduction and convection) and unsteady, compressible fluid flow. A wide-ranging problem in thermodynamic design is reduced to manageable proportions through application of the key principle of *dynamic similarity*. This draws together and unifies a vast literature on the central issue of the thermal regenerator, permits a unified approach to computer simulation of the gas processes and allows reduction of a substantial problem in gas circuit design to the use of simple charts. Performance optimization is also treated. Comprehensive appendices include conversion factors and tables of the properties of gases and metals required for the design of practical gas circuits.

The student of engineering will discover an instructive and illuminating case study revealing the interactions of basic disciplines. The researcher will find the groundwork prepared for any type of computer simulation that is contemplated. Those involved in the use and teaching of solution methods for unsteady gas dynamics problems will find a comprehensive treatment of nonlinear (Method of Characteristics) and linear wave approaches, for the Stirling machine provides an elegant example of the application of each. The book will be indispensable for all those involved in researching, designing or manufacturing Stirling prime movers, coolers and related regenerative thermal machines.

THERMODYNAMICS AND GAS DYNAMICS OF THE STIRLING CYCLE MACHINE

THERMODYNAMICS AND GAS DYNAMICS OF THE STIRLING CYCLE MACHINE

ALLAN J. ORGAN
Engineering Department, University of Cambridge

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521131797

First published 1992
This digitally printed version 2010

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Organ, Allan J.
Thermodynamics and gas dynamics of the Stirling cycle machine/Allan J. Organ.
p. cm.
Includes bibliographical references.
ISBN 0-521-41363-X
1. Stirling engines—Design and construction. 2. Thermodynamics.
I. Title.
TJ765.074 1992
621.4′2—dc20 91-29146 CIP

ISBN 978-0-521-41363-3 Hardback
ISBN 978-0-521-13179-7 Paperback

This book is dedicated to the memory of my parents,
Margaret and Albert Allan

Contents

Foreword

by Theodore Finkelstein

Much progress has been made in the technology and understanding of Stirling cycle machines in the 40 or so years during which I have been active in Stirling machine research and development. Although series production is currently limited to special purpose machines, there is hope that the type may soon be in common use, as witnessed by the large-scale efforts by research and development groups in the USA, Japan and various European countries. As a consequence of these efforts, sponsored both by government and private industry, there has in recent years also been a surge of interest worldwide in theoretical aspects.

Stirling cycle machines present a curious paradox in that they are at the same time well known and unknown. They are well known because they have existed as engines for longer than most other engines in common use, and as heat pumps and refrigerators for about as long as current systems. Furthermore, there is no lack of literature references to the general – as well as to the detailed – aspects of Stirling technology. Yet the general knowledge and understanding of Stirling engines is still at such a low level that, even among experts, a wide divergence of opinion can be found, not only as to their basic applicability or desirable constructional features, but even as to the analytical approach appropriate for their design and optimization. This may be verified by anyone who attends one of the specialized meetings on the subject.

This dichotomy is the reason that, over the years, unrealistic applications have been proposed for Stirling machines, and prototypes have been constructed whose performance frequently fell short of theoretical predictions. Thus, the absence of commercial success in spite of evident advantages may possibly be correlated to the lack of adequate modelling techniques and of sound theoretical predictions of what these machines can actually accomplish.

An up-to-date, authoritative and comprehensive treatment of the

thermodynamic design of Stirling machines has therefore been overdue for a long time, and now it has finally arrived. The author of this specialized text-book is one eminently qualified for the task: Dr Organ is one of the leading theoreticians worldwide in this field, and his thorough and extensive treatise has been eagerly awaited by his co-workers in the subject. I have been privileged to know and exchange ideas with my friend and co-worker for a long time. Our relationship spans a quarter of a century, or longer if one counts correspondence conducted between us when he was working on Stirling machines in São Paulo, Brazil. Since then Dr Organ has returned to England and has become known as one of the few scientists worldwide who have devoted most of their working lives to the study of this area of scientific engineering. I am proud to say that since then he and I have maintained a two-man mutual help society engendered by a commonality of purpose, and agreement with each other's analytical, theoretical and practical approaches to Stirling cycle machines.

Dr Organ is one of the foremost innovators in this speciality, having originated several advanced concepts, including the analysis of the fluid circuit in terms not of *Eulerian* coordinates (grid boundaries fixed in the space domain) but of *Lagrange* coordinates (grid boundaries movable in the space domain). It was he who first pointed out that the finite difference methods applied to fixed boundary elements in Stirling machine analysis are unable to deal with propagating discontinuities. He has been a pioneer in the application of the Method of Characteristics and in the use of linear wave analysis. His unorthodox techniques have not been widely understood when first published. His contributions to the field have been so advanced that comparatively few co-workers have adopted his techniques at this time, and the publication of this book will undoubtedly make use of his advanced analytical approaches more widespread.

Since the work is based on advanced principles of analysis and fluid dynamics, one might easily miss the central message: a very significant question to ask is 'what should be the relative importance accorded in the first place to accurate modelling of a thermodynamic cycle, or in the second place to making an interpretation of the results of such modelling work so that it may be usable in machine design?' The development of Stirling cycle machines must necessarily rest upon design procedures that can be presented in a simple and easily digested form. Thus, the researcher should have available to him such design tools as Z-charts, multi-variate graphs and optimization diagrams which cover a wide range of design parameters and which can easily be applied to a candidate design. These distilled design data must all be presented in normalized form involving non-dimensional

groupings of parameters. An examination of the text will clearly show that its ultimate aim is the achievement of design data in this form.

The reader will readily appreciate that, in order to compare different mechanizations adequately one has to define a valid normalized criterion, referred to in this text as *specific cycle work*, applicable universally to elementary, Schmidt-type analyses, through advanced treatments to computerized 'number-crunching' solutions of the appropriate partial differential equations. This abstraction is essential for the purpose of making the simulation results accessible to the design engineer.

Dr Organ's investigational philosophy is best summed up in his own words: In a private communication to me he has stated (sic):

> . . . my view of cycle analysis and simulation is, and always has been, that it is a tool to be developed, tested, placed at the service of the designer and then forgotten so that we may get on with the problems of layout, thermal stress, wear and the production of specimen engines . . . the top priority is simplification of the approach to thermodynamic and flow design – the ultimate being elimination of the requirement for the computer in the process of designing a specific machine . . .

The gist of this approach is that the ultimate aim is not an elaborate computer code, however elegant and sophisticated it may be in terms of physical modelling techniques or 'structured' programming, but the end result – concise and reliable, simple design charts. These tools must be based on individually computed points, which may obscure the fact that a comprehensive 'number-crunching' capacity was utilized for individual data points. Electronic computer studies can be made to range over any number of parametric variations, and it is ultimately not the computer study but the resulting design chart which is the usable end product. The argument is eloquently made in this textbook and is supported by sound theoretical deductions and examples.

This overall strategy is by no means in conflict with the fact that the author has delved as deeply into the black art of computer programming for Stirling cycle machines as any other person known to me. The painstaking quality of Dr Organ's work is evident from even a cursory review of any section of the book. The accuracy of the material presented, combined with the diversity of carefully planned and always graceful illustrations make this book a pleasure to look through and a valuable addition to any reference library. It is refreshing to note that, in spite of the weighty subject the book is neither pedantic nor pompous, but that the wording has originality and even lightheartedness in places and is a pleasure to read.

This text may be used at two levels: A first use is for the experienced practitioner in the field who may concentrate on one particular topic, and

thus study the section devoted to it without reference to the other parts, since one of the outstanding merits of this book is that most parts segregated by subject matter are written in a stand-alone fashion. A second use will be for the student wishing to acquire an understanding, and a working knowledge of most of the practical approaches to the thermodynamic analysis of Stirling cycle machines. Thus this book should be suitable for reference as well as for a text of an advanced academic course. It is confidently expected to become a standard textbook and reference work on this topic.

The thermodynamic analysis of Stirling cycle machines is treated on the assumption that the reader has adequate background in the mathematical methods required, depending on the specific chapter. In some cases this includes such advanced topics as partial differential equations or matrix operations. Also presupposed is the fact that most serious applications of the theories presented require the use of up-to-date computers.

The text places into perspective and comprehensively deals with the main topic implied by the title. To this purpose it is structured so that first the reader is exposed to certain basics, then to progressively more advanced concepts until, towards the end of the book, he is thoroughly familiar with the fundamentals and methods deemed to be practical for the scientific analysis. Dr Organ does not attempt to make the book an all-inclusive reiteration of all that has been published before, unless it contributes to the current 'state of the art'. Thus the main text is not cluttered with lengthy algebraic procedures that are archaic. Instead, the reader is referred to the original sources in the literature. Similarly, the author consigns related, but not directly applicable, portions of the text to appendices, where the smooth flow of the textual presentation might otherwise be impaired. This includes the closed-form solutions first derived by Schmidt, and amplified considerably by other investigators, and which are the basis required for the theoretical reference case only.

A prominent feature of this book is the organized manner in which the theoretical cycle analysis is presented, leading up from the author's basic perception of the available avenues of approach to a rational analysis (Chapter 1) through a historical overview of the development of analytical tools in Chapter 2. The latter is not merely a recitation of scientific papers in the order in which they appeared, but a critical evaluation of the scientific foundation upon which they were based with reference to the conservation laws which were obeyed or violated in certain instances by previous investigators.

I am glad to benefit now from Dr Organ's in-depth research and shall improve my own analysis by utilizing his formulations. It is my sincere hope

that the publication of this book will also make it easier for other workers in this field to optimize Stirling cycle machine proportions based on the improved analytical techniques here presented. This should ultimately be reflected in the emergence of a new generation of advanced and improved types of Stirling cycle machine.

Los Angeles *Theodore Finkelstein*

Preface

An internationally-recognized authority on Stirling cycle machines† is on record as affirming that

> It is clear that for these engines the principal impediment to success is not theoretical analysis, but practical design and application . . .

Against this claim, it is common experience that the simple analyses (Schmidt, isothermal, adiabatic, basic finite cell) are useless for *ab initio* thermodynamic design, that the acquisition of design know-how of proven worth in the form of licences is prohibitively costly, and that the work of developing suitable analytical and computational tools from scratch is monumental.

If Stirling cycle machines are to become a commercial reality there is clearly an urgent need for a raising of the threshold from which the work of thermodynamic design may start. It is accordingly a purpose of this text to provide the design methods which have so far been lacking. There is first of all (Chap. 2) a review – and, more importantly, a classification – of existing analytical methods. This will serve to introduce the terminology and some essential concepts. It will also highlight a number of anomalies between the way certain principles of thermodynamics and gas dynamics are used in the context of Stirling machines and elsewhere.

Chap. 3 will establish a proper ideal reference cycle in place of the thermodynamically inconsistent reference standard commonly referred to. It will demonstrate that the margin between the efficiency of the ideal cycle and that of the practical counterpart is inevitable rather than the result of inappropriate design.

Chap. 4 demonstrates the benefits of applying to Stirling cycle analysis and design the principles of dynamic similarity. (While these principles are

† Naso V Chairman's introduction, Conference prospectus, 3rd. International Stirling Engine Conference, Rome, June 23–6 1986

regarded as indispensable in most branches of engineering – including the crucially pertinent areas of heat transfer and fluid flow – the benefits of working in terms of characteristic dimensionless groups have clearly not been apparent to the majority of Stirling analysts.) The chapter introduces for the first time the notion of *thermodynamic similarity* for use at the preliminary stages of design.

Crucial to the performance of Stirling machines is the regenerator, so it is ironic that detailed investigation into the inner workings of this component has tended to be a less attractive option than the 'black box' approach. Chap. 6 directs attention to the vast body of work dealing with flow through the wire gauze, very little of which has found its way into the science of Stirling machine design.

Chap. 5 develops an approach to cycle modelling having special relevance to the task of thermodynamic design: formulation in terms of the principles of Chap. 4 permits a single computational run to map the entire performance envelope of a whole class of thermodynamically similar machines. The result is equivalent to tens of thousands of simulations of individual machines operating at conditions specified in terms of absolute variables (rpm, p_{ref}, etc.). Performance is mapped graphically, and the result is a first step towards the thermodynamic design of Stirling machines by sole reference to charts. Chap. 6 extends the treatment to take into account the intricate transient conduction phenomenon internal to individual wires of the regenerator gauze.

A facility essential to any area of design is *scaling*: if it is not possible to create a new machine of arbitrarily increased (or decreased) size from a prototype of known performance, then the work of design and development becomes an inordinately costly and unattractive proposition. The key to scaling lies with dynamic similarity: Chap. 9 identifies the dimensionless combinations of variables which determine similarity of thermodynamic performance between geometrically identical machines of different sizes, and illustrates the rôle of the performance map in the scaling process.

Chap. 10 shows that the lumped-parameter approach to modelling reformulates directly in terms of the same combinations of dimensionless variables. It deals with the problem of specifying realistic coefficients of convective heat transfer for the variable-volume spaces.

Chap. 2 expressed concern at the fact that much computational work in the area of Stirling cycle machines makes use of numerical techniques which are considered unsuitable for solution of the same equations applied to other engineering problems. Chap. 11, with the aid of Appendix XI, introduces the Method of Characteristics, universally recognized as yielding correct numerical solutions to the equations of one-dimensional, unsteady, compressible

flow. The chapter applies the method to a searching test – a temperature discontinuity propagating in a heat exchanger passage. Comparison with the analytical solution reveals none of the arbitrary 'numerical diffusion' introduced by *ad hoc* fixed-grid schemes. Examples are given of simulations based on the Method of Characteristics, and the prospects discussed for the routine modelling of Stirling machines by this means.

The argument in favour of rigorous solution of the one-dimensional defining equations is weakened by recognition of the fact that there are respects in which these equations themselves fail to represent the gas processes of interest. The shortcoming is most marked in respect of flow through the regenerator where the assumption of one-dimensional flow cannot be justified on any grounds (Chap. 5). It would be unrealistic to consider looking into individual, local flow patterns with either fixed-grid or Characteristics methods, since several thousand subdivisions of the flow field would be called for, with no means of checking experimentally the realism of assumptions applied within individual computational cells. There is, on the other hand, a simplification which permits at least some features of the flow through the individual gauze to be recognized, and which allows each and every gauze (in a stack of some thousands) to contribute its individual effect to the overall computation. The assumption is that of linearity of pressure wave effects. Chap. 12 looks at the Stirling cycle gas circuit as a linear wave phenomenon and demonstrates the capability of linear methods to deal individually and collectively with an arbitrary number of regenerator gauzes.

In the search for a relatively straightforward integration scheme free of the vexed problem of artificial diffusion, Chap. 13 examines solutions based in Lagrange coordinates. There emerges immediately a gas process description so simple that it is virtually a 'one-line' cycle model. The Lagrange approach is amplified into a comprehensive cycle simulation in the form of a novel 'building bloc' architecture. One of several advantages of this form is the unprecedented scope it affords for checks of correct functioning. The simulation is applied to the Philips MP1002CA machine and comprehensive output obtained and discussed.

The ultimate use of cycle analysis is design of the *optimum* thermodynamic circuit. Given the problems of formulating a straightforward cycle analysis, the difficulties and pitfalls of extending to optimization promise to be numerous. Chap. 14 nevertheless makes a start on this most challenging of all aspects of thermodynamic modelling. Chap. 15 presents an analysis of the deceptively simple hot-air engine. Chap. 16 concludes.

It is pertinent to record that I have previously drafted two complete texts on the thermodynamics of Stirling cycle machines, discarding them wholesale

on coming to the realization that they were 'writing for the sake of writing' – mere compendia of thought current at the time. The present treatment differs in a number of respects, most importantly in uniting all the material around the principle of dynamic similarity (and its more flexible derivative, thermodynamic similarity). If unity had been the sole gain, a further attempt at a text would not have been justified. However, there has resulted in addition substantial simplification: that which previously appeared a monumental task of analytical and numerical modelling is now seen to lie within manageable bounds, the limits of which are set in terms of a modest number of characteristic dimensionless groups.

If the would-be designer will study the chapters which follow, he should at least be in a position to start work somewhat ahead of where the author began 28 years ago. In particular, if he takes on board the dynamic and thermodynamic similarity approaches, and calculates values of the characteristic dimensionless parameters for some specimen machine of known and acceptable performance, he may scale to a new design in the confidence that his new machine *has no option* but to deliver the predicted specific performance, since it is thermodynamically identical to its prototype. He must, of course, ensure that the new design does not introduce thermal shorting or seal leakage out of proportion to those of the prototype, and must bear in mind that when a machine is scaled, the quantity whose numerical value remains constant is *specific cycle work*: if this desired value of specific cycle work is attained in the new machine at very low rpm, (as determined by the scaling process) then power output is likely to be unacceptable; if at very high rpm, then inertia loads may be a problem. Guidance on dealing with these and other aspects of the fascinating problem of Stirling machine design is offered in the pages which follow.

My essential acknowledgement must be to my late father, Albert Allan: without the background of his enthusiasm for the toy hot-air engines he cherished as a child I should never have selected as my final year research project at the University of Birmingham a topic entitled 'An Appraisal of the Hot Air Engine'. Supervisors and heads of department who have encouraged and made possible my subsequent work in this area include Professor B. N. Cole, (at that time Reader in Plasticity at the University of Birmingham), Professor F. C. Hooper at the University of Toronto, Professor S. A. Tobias at the University of Birmingham, Professor W. B. Gosney of King's College, London, and Professors W. A. Mair, A. Shercliffe and J. Heyman of my present Department at Cambridge. Those familiar with the work of Dr T. Finkelstein will recognize his imprint on the aims, approach and style of every aspect of this text. My experimental and theoretical work has been

generously supported over the years by the Science and Engineering Research Council and by the Procurement Executive of the Ministry of Defence.

The literature review of Chap. 2 and the linear wave treatment of Chap. 12 were originally published in the *Proceedings of the Institution of Mechanical Engineers*, Part C, in 1987 and 1989 respectively, and are reproduced here with the permisson of the Council of the Institution. The Council also gives its approval for the use of all seven figures of Chap. 15.

The linear wave treatment was made possible through considerable assistance from Dr Ann Dowling of Cambridge University Engineering Department.

Reverting briefly to the opening theme, a statement by Glegg† is pertinent:

> Theory is practice. Every device you can invent behaves exactly as the theory of its operation in the context of its surroundings

With allowance for the terse wording, the claim identifies the indispensable rôle of analysis in design: to focus attention on the interaction between the natural laws underlying the behaviour of the physical system. When practical experiment identifies an unexpected feature of performance (frequently a shortfall) it is through manipulation of the symbolic model that the incomplete insight is most readily adjusted and a course of remedial action identified. Any who doubt may care to study developments in one of the most demanding of all areas of design – that of human powered flight. The recent sequence of emotive successes awaited Larrabee's elegant definition of the optimum low-speed propellor in terms of classical airscrew theory.‡

The imperatives of engineering design are to be reckoned with. Expensive disappointment is the price of imagining otherwise.

Cambridge University Engineering Department *AJO*

† Glegg G *A design engineer's pocket book*. Macmillan, London, 1983
‡ Burke J D The Gossamer Condor and Albatross: a case study in aircraft design. Report No. AV-R-80/549, AeroVironment Inc., Pasadena, California 91107, USA, June 1980 (published in the AIAA Professional Study Series)

Notation

Where possible, lower-case English letter characters represent variables having dimensions. A Greek character tends to denote a dimensionless quantity. Where a suitable Greek character is not available for the purpose, an English character in **bold** may be used.

English letter symbols

a	isentropic acoustic speed $= \sqrt{(\gamma RT)}$	m/s
	constant in equation for van der Waals' gas	m^6 Pa/kg^2
a, b	elements of 2×2 matrix, a_{11}, a_{12}, b_{11}, b_{12} etc.	
	constants for general use, as required	
b	constant in equation for van der Waals' gas	m^3/kg
A_w	wetted or exposed area	m^2
A_x	free-flow or cross-sectional area	m^2
B	Beale number, $W_{brake}/(p_{ref} V_{ref})$	
c	specific heat (of solid)	J/kg K
c_v, c_p	specific heats at constant volume and pressure respectively	J/kg K
C_d	coefficient of discharge	
C_f	friction factor, $\tau_{wall}/\frac{1}{2}\rho u^2$	
d	diameter	m
d_w	diameter of individual wire of regenerator matrix	m
D	substantial derivative	
D	diameter, nominal diameter of displacer	m
D^*	effective diameter	m
E	Euler number, 0.577215	

$f(\ )$, $g(\ )$	functions defining left- and right-running waves	Pa
f_{ve}, f_{vc}	variable volume – $f_{ve}(\phi)$, $f_{vc}(\phi)$ made dimensionless by dividing through by respective amplitude, V_E, V_C, so that values lie in range 0.0–1.0	
F	force	N
	friction parameter $(\frac{1}{2}C_f u^2/r_h)u/\lvert u\rvert$	m/s^2
F, G	complex constant with units of pressure	Pa
$\mathbf{F}_c$	dimensionless friction factor defined in connection with lumped-parameter formulation	
g	acceleration due to gravity	m/s^2
G	mean mass velocity, ρu	kg/m^2 s
h	coefficient of convective heat transfer	W/m^2 K
h	specific enthalpy	J/kg K
H	enthalpy	J/K
$\mathbf{H}$	$N_{ST} \cdot d\phi/2\pi$, e.g., $\mathbf{H}_{he} = N_{ST_{he}}\, d\phi/2\pi$ (expansion exchanger)	
i	square root of negative unity	
k	(dimensional) discharge coefficient defined by Finkelstein	1/m s
	thermal conductivity of gas	J/s m K
	permeability coefficient defined by Darcy's law	m^2
k_r	thermal conductivity of individual, solid element of regenerator	J/s m K
l	length	m
L	a fixed length – nominal distance between opposed faces of expansion and compression pistons in equivalent 'one-dimensional' Stirling cycle machine	m
L_j	length of jth flow passage element	m
L_d	nominal length of displacer	m
m	mass	kg
m'	mass flow rate	kg/s
M	mass of working fluid taking part in cycle	kg
N	revolutions per minute	1/min
N_c	Courant number, $u\Delta t/\Delta x$	
N_f	Fourier modulus, $\alpha\Delta t/\Delta x^2$	

N_{nu}	local, instantaneous Nusselt number, hd/k	
N_{ma}	local, instantaneous Mach number, u/a	
N_{pr}	local, instantaneous Prandtl number, $\mu c_p/k$	
N_{re}	local, instantaneous Reynolds number, $4\rho u r_h/\mu$	
$N_{re}{}^*$	a Reynolds number defined in Chapter 5	
N_{red}	local, instantaneous Reynolds number based on diameter, $\rho u d/\mu$	
N_s	revolutions per second	1/s
N_{st}	Stanton number, $h/c_p G$	
N_C	characteristic Courant number, $\Delta\phi/\Delta\lambda$	
N_F	characteristic Fourier number, $\alpha/L_{ref}{}^2 N_s$	
N_H	heat flux number $= p_{ref} L_{ref} N_s R_H / K_r T_{ref}$	
N_{MA}	characteristic pressure coefficient or Mach number, $N_s L_{ref}/\sqrt{(RT_{ref})}$	

NB In a review paper reported extensively in Chapter 2 the symbol N_F was used for N_{MA}, and the definition included constant terms, viz, $N_F = 2\pi N_s L_{ref}/\sqrt{(2RT_{ref})}$. The definition of Reynolds number was also different in the paper in question, *viz*: $N_{RE} = 8\pi p_{ref} L^2{}_{ref} N_s/\mu_{ref} RT_{ref}$. Thus, $N_F \approx 2\pi N_{MA}/\sqrt{2}$ and N_{RE}(Chap. 2) $\approx 8\pi N_{RE}$

N_{RE}	characteristic Reynolds number, $p_{ref} L^2{}_{ref} N_s/\mu_{ref} RT_{ref}$	
N_{SG}	Stirling number, $p_{ref}/N_s\mu = N_{RE}/N_{MA}{}^2$	
N_{ST}	a reference Stanton number, $hA_{ref}/c_p M N_s$	
N_{TC}	thermal capacity ratio $= T_{ref}\rho_r C_r/p_{ref}$	
N_α	diffusivity number $= N_s L_{ref}{}^2 \rho_r C_r/k_r$	
N_τ	rationalized temperature ratio, T_c/T_c	
n	polytropic index dimensionless number number of (items)	
p	absolute pressure	Pa, N/m^2
P	wetted perimeter	m
q^*	rate of heat exchange per unit mass	J/kg s
q'	rate of heat exchanged	J/s or W
$q_L{}'$	rate of heat exchange per unit length of duct	J/m s
Q	heat exchanged	J
r	crank pin offset	m
r_h	hydraulic radius, A_x/P	m
R_H	characteristic hydraulic radius, V_{ref}/A_{ref}	m

r	compression ratio, V_{min}/V_{max} (= inverse of expansion ratio, ε)	
r*	a ratio of volumes in the ideal cycle in which piston motion is discontinuous. = (vol. of comp. space at which const. vol. transfer begins)/V_C = $\mathbf{r}(\Sigma\mu_d + 1) - \Sigma\mu_d$	
R	specific gas constant	J/kg K
	thermal resistance, $1/hA$	K/W
$\underline{R}$	universal gas constant	J/kmol K
s	specific entropy	J/kg K
	stroke (of piston or displacer)	m
s_w	length of curve (e.g., of centre line of curved wire element)	m
S	entropy	J/K
t	independent variable, time	s
t_w	radial thickness of wall of cylindrical heat exchanger tube	m
T	absolute temperature	K
T_{su}	Sutherland temperature	K
u, U	velocity, 'superficial' velocity in porous materials (Chap. 5)	m/s
$\boldsymbol{u}$	velocity normalized by reference speed $N_s L_{ref}$ or a_o)	
U	internal energy	J
$U(\ )$	step function defined in text	
v	specific volume	m^3/kg
V	volume	m^3
V_E	volume displaced during stroke of expansion piston	m^3
V_C	volume displaced during stroke of compression piston	m^3
W	work or work/cycle	J
W'	work rate	J/s, W
x, y, z	independent variables, length	m
X_{VE}	V_E/r^3	
Y	regenerator gap width in hot-air engine	m
Y_d, Y_p, Y_{cl}	fixed lengths defined in Appendix V	m
$\mathbf{Z}$	matrix 2 × 2	

Greek characters

α	phase angle (rad) between volume variations. Equal to piston phase angle in opposed-piston machine	
	thermal diffusivity – $k/\rho c$	m^2/s
α_c	angle between axes of cylinders in V-configuration machine	
$\alpha_{¶}$	ratio of free flow area to frontal area	
ε	expansion ratio, V_{max}/V_{min} $(=1/\mathbf{r})$	
ε^*	corresponds to $\mathbf{r}^*$. Ratio V_C/(vol. of comp. space at which constant volume transfer begins) $= 1/\mathbf{r}^*$	
ϕ	crank angle $(=\omega t)$	
Φ	dissipation function	s^{-2}
Φ^*	dimensionless dissipation function, Φ/ω^2	
γ	ratio of specific heats, c_p/c_v	
λ	dimensionless length, x/L or x/L_{ref} (in coaxial machine) ratio piston swept volume/displacer swept volume	
λ_{bs}	ratio of bore to stroke	
λ_h	dimensionless hydraulic radius, r_h/L_{ref} or r_h/R_H	
η	efficiency (thermal or mechanical depending on subscript)	
θ	dimensionless discharge coefficient dimensionless time in Mach plane, $t/(L/a_o)$	
κ	volume ratio $V_C/V_E = r_c/r_e$ for machine with equal bores	
μ	ratio of a volume to a reference volume, V/V_{ref}	
	coefficient of dynamic viscosity	Pa s
μ_D	normalized dead space, V_D/V_{ref}, e.g., V_D/V_E	
$\mu_D{}^+$	'additional' dead space – amount by which the normalized compression-end dead space of a displacer-type (gamma) machine exceeds that of the otherwise identical coaxial (beta) machine. (Defined in text.)	
μ_v	ratio V_E/V_{ref} $(=1$ if $V_{ref} = V_E)$	

v, $v(N_\tau)$	reduced dead space $= \sum V_{di}T_{ref}/V_{ref}T_{di}$	
v^*, $v^*(N_\tau)$	approximate expression for v, $v(N_\tau)$ employed by some authors $= 2\sum V_D/(V_E(1+N_\tau))$	
π	a constant, 3.141592654	
Π	pressure ratio of ideal cycle $(1+\xi)/(1-\xi)$	
ρ	density	kg/m^3
	σ, σ' dimensionless mass, m/M, mass flow rate, $d(m/M)/d\phi$	
τ	normalized temperature, T/T_e	
τ	characteristic temperature ratio T_c/T_{ref}. *Subsequent to the review chapter (Chap. 2) a new characteristic temperature ratio is defined as $N_\tau = T_e/T_c$. This new variable is thus the inverse of the characteristic quantity used elsewhere in the literature.* (For reasons, see Chap. 2.)	
ξ	$\sqrt{[1/N_\tau^2 + \kappa^2 + (2/N_\tau)\kappa\cos\alpha]}/[1/N_\tau + \kappa + 2v(N_\tau)]$	
ζ	specific cycle work $- W/p_{ref}V_{ref}$ or W/MRT_{ref} etc.	
ω	angular speed	1/s

Other characters

$\P_v$	volumetric porosity (e.g. of regenerator matrix)	
$\wp$	dimensionless density, $\rho V_{ref}/M$	

Subscripts

brake	relates to work or power measured at brake
c, C	compression space
cv	control volume
d	dead (unswept) volume
ds	dynamic similarity
e, E	expansion space
gc	gas circuit – equivalent to wfc (below)
gen	generated (as in entropy generated)
h	relating to heat exchanger – he, expansion exchanger, hc compression

k	kinematics
mech	mechanical
n	nth item of
nom	nominal
o	optimum value of – e.g., κ_o, α_o
p	relating to pressure
r (or reg)	relating to regenerator
ref	a reference quantity
Sch	relating to Schmidt analysis
Su	Sutherland
sw	swept
th	thermal
ts	thermodynamic similarity case
w (or wetted)	wetted (as in heat exchanger wetted area)
	wall (as in T_w, t_w)
	wire (as in d_w)
wfc	working fluid circuit
x, y, z	in x, y, z coordinate direction
underline	mean value