

CONTENTS

PREFACE.....	xiii
CHAPTER 1. INTRODUCTION	1
1.1. Computably Enumerable Sets	1
1.2. Degrees.....	2
1.3. Finite Sequences and Trees	3
CHAPTER 2. SYSTEMS OF TREES OF STRATEGIES	5
2.1. An Overview	7
2.2. Systems of Trees.....	14
2.3. Basic Modules	17
2.4. The Path Generating Function λ	18
2.5. Links	21
2.6. Consistency.....	23
2.7. Blocks	24
2.8. The Weight Function	27
2.9. Directing and Action Sentences: An Example	29
2.10. Directing and Action Sentences: Properties.....	31
2.11. The Framework Theorem.....	33
2.12. A Sampling of Lemmas.....	36
CHAPTER 3. Σ_1 CONSTRUCTIONS	41
3.1. Σ_1 Constructions	41
3.2. The Friedberg–Mućnik Theorem	47
3.2.1. The Basic $R_{\Phi,i}$ -Module.....	49
3.2.2. T^I -Analysis.....	49
3.2.3. T^0 -Analysis.....	49
3.2.4. The Construction	50
3.2.5. The Verification	50
3.3. A Low Computably Enumerable Degree.....	50
3.3.1. The Basic P_Φ -Module	51
3.3.2. The Basic N_e -Module	52
3.3.3. T^I -Analysis.....	52

3.3.4.	T^0 -Analysis	52
3.3.5.	The Construction	52
3.3.6.	The Verification	52
3.4.	A Properly d-c.e. Degree	53
3.4.1.	The Basic $R_{\Phi, \Psi, W}$ -Module	54
3.4.2.	T^1 -Analysis	55
3.4.3.	T^0 -Analysis	55
3.4.4.	The Construction	56
3.4.5.	The Verification	56
3.5.	Summary	56
CHAPTER 4.	Δ_2 CONSTRUCTIONS	57
4.1.	The Δ_2 Level	57
4.2.	Avoiding Upward Cones	58
4.2.1.	The Basic P_{Φ} -Module	59
4.2.2.	The Basic N_{Φ} -Module	59
4.2.3.	T^2 -Analysis	60
4.2.4.	T^1 -Analysis	60
4.2.5.	T^0 -Analysis	60
4.2.6.	The Construction	61
4.2.7.	The Verification	61
4.3.	The Sacks Splitting Theorem	61
4.3.1.	The Basic P_x -Module	62
4.3.2.	The Basic $N_{\Phi, i}$ -Module	62
4.3.3.	T^2 -Analysis	63
4.3.4.	T^1 -Analysis	63
4.3.5.	T^0 -Analysis	63
4.3.6.	The Construction	64
4.3.7.	The Verification	64
4.4.	Backtracking	64
4.5.	Permitting	66
4.5.1.	The Basic $P_{\Phi, i}$ -Module	68
4.5.2.	T^2 -Analysis	69
4.5.3.	T^1 -Analysis	69
4.5.4.	T^0 -Analysis	70
4.5.5.	The Construction	70
4.5.6.	The Verification	70
4.5.7.	Further Remarks	71
4.6.	Summary	71
CHAPTER 5.	Π_2 CONSTRUCTIONS	73
5.1.	Π_2 Constructions	73
5.2.	A High Computably Enumerable Degree	75

CONTENTS		ix
5.2.1.	The Basic P_Φ -Module	76
5.2.2.	The Basic Q_x -Module	77
5.2.3.	T^2 -Analysis	77
5.2.4.	T^1 -Analysis	77
5.2.5.	T^0 -Analysis	78
5.2.6.	The Construction	78
5.2.7.	The Verification	78
5.2.8.	Further Remarks	78
5.3.	The Jump Inversion Theorem	78
5.3.1.	The Basic Q_e -Module	80
5.3.2.	The Basic N_e -Module	80
5.3.3.	T^2 -Analysis	80
5.3.4.	T^1 -Analysis	80
5.3.5.	T^0 -Analysis	81
5.3.6.	The Construction	81
5.3.7.	The Verification	81
5.4.	The Minimal Pair Theorem	81
5.4.1.	The Basic $P_{\Phi,i}$ -Module	84
5.4.2.	The Basic $N_{\Phi,\Psi}$ -Module	84
5.4.3.	T^2 -Analysis	85
5.4.4.	T^1 -Analysis	85
5.4.5.	T^0 -Analysis	85
5.4.6.	The Construction	85
5.4.7.	The Verification	86
5.4.8.	Further Remarks	86
5.5.	Embedding the Pentagon	86
5.5.1.	The L Requirement	87
5.5.2.	The J Requirement	87
5.5.3.	The Basic $N_{\Psi,\Theta}$ -Module	88
5.5.4.	The Basic $P_{A,\Phi}$ -Module and the Basic $P_{C,\Phi}$ -Module	88
5.5.5.	The Basic Q_Φ -Module	88
5.5.6.	T^2 -Analysis	89
5.5.7.	T^1 -Analysis	89
5.5.8.	T^0 -Analysis	89
5.5.9.	The Construction	90
5.5.10.	The Verification	90
5.5.11.	Further Remarks	90
5.6.	Summary	91
CHAPTER 6.	Δ_3 CONSTRUCTIONS	93
6.1.	Δ_3 Constructions	93
6.1.1.	Properties of Δ_3 Constructions	94
6.1.2.	The Restraint Requirements	95

6.1.3.	The Correction Requirements	96
6.1.4.	Bounding A above by V	97
6.2.	The Density Theorem	98
6.2.1.	The R Requirement	98
6.2.2.	The Basic N_Φ -Module	98
6.2.3.	The Basic P_Ψ -Module	99
6.2.4.	T^3 -Analysis	100
6.2.5.	T^2 -Analysis, T^1 -Analysis	101
6.2.6.	T^0 -Analysis	101
6.2.7.	The Construction	101
6.2.8.	The Verification	101
6.3.	Summary	102
CHAPTER 7. Σ_3 CONSTRUCTIONS		103
7.1.	Module-Respecting Derived Assignments	103
7.2.	Strong Minimal Pairs	104
7.3.	The Strong Minimal Pair Theorem	106
7.3.1.	The Basic P_Φ -Module	106
7.3.2.	The Basic Q_Ψ -Module	106
7.3.3.	The Basic $M_{\Phi,W}$ -Module	107
7.3.4.	T^3 -Analysis	108
7.3.5.	T^2 -Analysis	109
7.3.6.	T^1 -Analysis	109
7.3.7.	T^0 -Analysis	110
7.3.8.	The Construction	110
7.3.9.	The Verification	110
7.4.	Some Comments	111
CHAPTER 8. PATHS AND LINKS		113
8.1.	Paths	113
8.2.	Switching and Nonswitching Extensions	117
8.3.	Links	119
8.4.	Free Nodes	124
8.5.	Relationships Between Links	126
8.6.	Framework Lemmas	132
8.6.1.	Monotonic Sentence Decomposition	132
8.6.2.	Derived Assignments	133
8.7.	Level Analysis Lemmas	136
8.7.1.	The Σ_1 Level	136
8.7.2.	The Δ_2 Level	137
8.7.3.	The Δ_3 Level	138
8.8.	Restraint Lemmas	140
8.9.	Miscellaneous Lemmas	144

CONTENTS

xi

CHAPTER 9. BACKTRACKING	147
9.1. Backtracking	147
9.2. Properties of PL Sets	149
9.3. Extenders	152
9.4. Backtracking and Links	154
9.5. Set Approximation	155
9.6. Constraint	158
9.7. REA Sets	159
9.8. Level-Specific Backtracking	161
CHAPTER 10. HIGHER-LEVEL CONSTRUCTIONS	163
10.1. Jump Posets with Least Element	163
10.2. The Requirements	164
10.2.1. T^ℓ -Analysis	166
10.2.2. T^i -Analysis, $i < \ell$	166
10.3. The Construction	166
10.3.1. The Construction	167
10.4. Proof	168
CHAPTER 11. INFINITE SYSTEMS OF TREES	173
11.1. Infinite Systems: A Brief Analysis	173
REFERENCES	175