Sparse Image and Signal Processing: Wavelets, Curvelets, Morphological Diversity

This book presents the state of the art in sparse and multiscale image and signal processing, covering linear multiscale transforms, such as wavelet, ridgelet, or curvelet transforms, and non-linear multiscale transforms based on the median and mathematical morphology operators. Recent concepts of sparsity and morphological diversity are described and exploited for various problems such as denoising, inverse problem regularization, sparse signal decomposition, blind source separation, and compressed sensing.

This book weds theory and practice in examining applications in areas such as astronomy, biology, physics, digital media, and forensics. A final chapter explores a paradigm shift in signal processing, showing that previous limits to information sampling and extraction can be overcome in very significant ways.

MATLAB and IDL code accompany these methods and applications to reproduce the experiments and illustrate the reasoning and methodology of the research available for download at the associated Web site.

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Frontmatter
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SPARSE IMAGE AND SIGNAL PROCESSING

Wavelets, Curvelets, Morphological Diversity

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Acronyms

1-D, 2-D, 3-D	one-dimensional, two-dimensional, three-dimensional
AAC	advanced audio coding
AIC	Akaike information criterion
BCR	block-coordinate relaxation
BIC	Bayesian information criterion
BP	basis pursuit
BPDN	basis pursuit denoising
BSS	blind source separation
CCD	charge-coupled device
CeCILL	CEA CNRS INRIA Logiciel Libre
CMB	cosmic microwave background
COBE	Cosmic Background Explorer
CTS	curvelet transform on the sphere
CS	compressed sensing
CWT	continuous wavelet transform
dB	decibel
DCT	discrete cosine transform
DCTG1, DCTG2	first-generation discrete curvelet transform, second-generation discrete curvelet transform
DR	Douglas-Rachford
DRT	discrete ridgelet transform
DWT	discrete wavelet transform
ECP	equidistant coordinate partition
EEG	electroencephalography
EFICA	efficient fast independent component analysis
EM	expectation maximization
ERS	European remote sensing
ESA	European Space Agency
FB	forward-backward
FDR	false discovery rate
FFT	fast Fourier transform

FIR	finite impulse response
FITS	Flexible Image Transport System
fMRI	functional magnetic resonance imaging
FSS	fast slant stack
FWER	familywise error rate
FWHM	full width at half maximum
GCV	generalized cross-validation
GGD	generalized Gaussian distribution
GLESP	Gauss-Legendre sky pixelization
GMCA	generalized morphological component analysis
GUI	graphical user interface
HEALPix	hierarchical equal area isolatitude pixelization
HSD	hybrid steepest descent
HTM	hierarchical triangular mesh
ICA	independent component analysis
ICF	inertial confinement fusion
IDL	interactive data language
IFFT	inverse fast Fourier transform
IHT	iterative hard thresholding
iid	independently and identically distributed
IRAS	Infrared Astronomical Satellite
ISO	Infrared Space Observatory
IST	iterative soft thresholding
IUWT	isotropic undecimated wavelet (starlet) transform
JADE	joint approximate diagonalization of eigen-matrices
JPEG	Joint Photographic Experts Group
KL	Kullback-Leibler
LARS	least angle regression
LP	linear programming
lsc	lower semicontinuous
MAD	median absolute deviation
MAP	maximum a posteriori
MCA	morphological component analysis
MDL	minimum description length
MI	mutual information
ML	maximum likelihood
MMSE	minimum mean squares estimator
MMT	multiscale median transform
MMV	multiple measurements vectors
MOLA	Mars Orbiter Laser Altimeter
MOM	mean of maximum
MP	matching pursuit
MP3	MPEG-1 Audio Layer 3
MPEG	Moving Picture Experts Group
MR	magnetic resonance
MRF	Markov random field
MSE	mean square error

MS-VST	multiscale variance stabilization transform
NLA	nonlinear approximation
OFRT	orthonormal finite ridgelet transform
OMP	orthogonal matching pursuit
OSCIR	Observatory Spectrometer and Camera for the Infrared
OWT	orthogonal wavelet transform
PACS	Photodetector Array Camera and Spectrometer
PCA	principal components analysis
PCTS	pyramidal curvelet transform on the sphere
PDE	partial differential equation
pdf	probability density function
PMT	pyramidal median transform
POCS	projections onto convex sets
PSF	point spread function
PSNR	peak signal-to-noise ratio
PWT	partially decimated wavelet transform
PWTS	pyramidal wavelet transform on the sphere
QMF	quadrature mirror filters
RIC	restricted isometry constant
RIP	restricted isometry property
RNA	relative Newton algorithm
SAR	Synthetic Aperture Radar
SeaWiFS	Sea-viewing Wide Field-of-view Sensor
SNR	signal-to-noise ratio
s.t.	subject to
STFT	short-time Fourier transform
StOMP	Stage-wise Orthogonal Matching Pursuit
SURE	Stein unbiased risk estimator
TV	total variation
UDWT	undecimated discrete wavelet transform
USFFT	unequispaced fast Fourier transform
UWT	undecimated wavelet transform
UWTS	undecimated wavelet transform on the sphere
VST	variance-stabilizing transform
WMAP	Wilkinson Microwave Anisotropy Probe
WT	wavelet transform

Notation

Functions and Signals

$f(t)$	continuous-time function, $t \in \mathbb{R}$
$f(\mathbf{t})$ or $f(t_1, \ldots, t_d)$	d-dimensional continuous-time function, $\mathbf{t} \in \mathbb{R}^d$
$f[k]$	discrete-time signal, $k \in \mathbb{Z}$, or kth entry of a finite-dimensional vector
$f[\mathbf{k}]$ or $f[k, l, \ldots]$	d-dimensional discrete-time signal, $\mathbf{k} \in \mathbb{Z}^d$
$\bar{f}$	time-reversed version of f as a function ($\bar{f}(t) = f(-t), \forall t \in \mathbb{R}$) or signal ($\bar{f}[k] = f[-k], \forall k \in \mathbb{Z}$)
$\hat{f}$	Fourier transform of f
f^*	complex conjugate of a function or signal
$H(z)$	z transform of a discrete filter h
lhs $= O($rhs$)$	lhs is of order rhs; there exists a constant $C > 0$ such that lhs $\leq C$rhs
lhs $\sim$ rhs	lhs is equivalent to rhs; lhs $= O($rhs$)$ and rhs $= O($lhs$)$
$\mathbf{1}_{\{\text{condition}\}}$	1 if condition is met, and zero otherwise
$L_2(\Omega)$	space of square-integrable functions on a continuous domain Ω
$\ell_2(\Omega)$	space of square-summable signals on a discrete domain Ω
$\Gamma_0(\mathcal{H})$	class of proper lower-semicontinuous convex functions from $\mathcal{H}$ to $\mathbb{R} \cup \{+\infty\}$

Operators on Signals or Functions

$[\cdot]_{\downarrow 2}$	down-sampling or decimation by a factor 2
$[\cdot]_{\downarrow 2^e}$	down-sampling by a factor 2 that keeps even samples
$[\cdot]_{\downarrow 2^o}$	down-sampling by a factor 2 that keeps odd samples
$\breve{\cdot}$ or $[\cdot]_{\uparrow 2}$	up-sampling by a factor 2, i.e., zero insertion between each two samples
$[\cdot]_{\uparrow 2^e}$	even-sample zero insertion
$[\cdot]_{\uparrow 2^o}$	odd-sample zero insertion

$[\cdot]_{\downarrow 2,2}$	down-sampling or decimation by a factor 2 in each direction of a two-dimensional image
$*$	continuous convolution
$\star$	discrete convolution
$\odot$	composition (arbitrary)

Matrices, Linear Operators, and Norms

$.^{\mathrm{T}}$	transpose of a vector or a matrix
$\mathbf{M}^*$	adjoint of $\mathbf{M}$
Gram matrix of $\mathbf{M}$	$\mathbf{M}^*\mathbf{M}$ or $\mathbf{M}^{\mathrm{T}}\mathbf{M}$
$\mathbf{M}[i, j]$	entry at ith row and jth column of a matrix $\mathbf{M}$
$\det(\mathbf{M})$	determinant of a matrix $\mathbf{M}$
$\mathrm{rank}(\mathbf{M})$	rank of a matrix $\mathbf{M}$
$\mathrm{diag}(\mathbf{M})$	diagonal matrix with the same diagonal elements as its argument $\mathbf{M}$
$\mathrm{trace}(\mathbf{M})$	trace of a square matrix $\mathbf{M}$
$\mathrm{vect}(\mathbf{M})$	stacks the columns of $\mathbf{M}$ in a long column vector
$\mathbf{M}^+$	pseudo-inverse of $\mathbf{M}$
$\mathbf{I}$	identity operator or identity matrix of appropriate dimension; $\mathbf{I}_n$ if the dimension is not clear from the context
$\langle \cdot, \cdot \rangle$	inner product (in a pre-Hilbert space)
$\|\cdot\|$	associated norm
$\|\cdot\|_p$	$p \geq 1$, ℓ_p norm of a signal
$\|\cdot\|_0$	ℓ_0 quasi-norm of a signal; number of nonzero elements
$\|\cdot\|_{\mathrm{TV}}$	discrete total variation (semi)norm
$\overline{\nabla}$	discrete gradient of an image
$\overline{\mathrm{div}}$	discrete divergence operator (adjoint of $\overline{\nabla}$)
$\vert\vert\vert \cdot \vert\vert\vert$	spectral norm for linear operators
$\|\cdot\|_{\mathrm{F}}$	Frobenius norm of a matrix
$\otimes$	tensor product

Random Variables and Vectors

$\varepsilon \sim \mathcal{N}(\mu, \mathbf{\Sigma})$	ε is normally distributed with mean μ and covariance $\mathbf{\Sigma}$
$\varepsilon \sim \mathcal{N}(\mu, \sigma^2)$	ε is additive white Gaussian with mean μ and variance σ^2
$\varepsilon \sim \mathcal{P}(\lambda)$	ε is Poisson distributed with intensity (mean) λ
$\mathbb{E}[.]$	expectation operator
$\mathrm{Var}[.]$	variance operator
$(\varepsilon; \mu, \sigma^2)$	normal probability density function of mean μ and variance σ^2
$(\varepsilon; \mu, \sigma^2)$	normal cumulative distribution of mean μ and variance σ^2

Preface

Often, nowadays, one addresses public understanding of mathematics and rigor by pointing to important applications and how they underpin a great deal of science and engineering. In this context, multiple resolution methods in image and signal processing, as discussed in depth in this book, are important. Results of such methods are often visual. Results, too, can often be presented to the layperson in an easily understood way. In addition to those aspects that speak powerfully in favor of the methods presented here, the following is worth noting. Among the most cited articles in statistics and signal processing, one finds works in the general area of what we cover in this book.

The methods discussed in this book are essential underpinnings of data analysis, of relevance to multimedia data processing and to image, video, and signal processing. The methods discussed here feature very crucially in statistics, in mathematical methods, and in computational techniques.

Domains of application are incredibly wide, including imaging and signal processing in biology, medicine, and the life sciences generally; astronomy, physics, and the natural sciences; seismology and land use studies, as indicative subdomains from geology and geography in the earth sciences; materials science, metrology, and other areas of mechanical and civil engineering; image and video compression, analysis, and synthesis for movies and television; and so on.

There is a weakness, though, in regard to well-written available works in this area: the very rigor of the methods also means that the ideas can be very deep. When separated from the means to apply and to experiment with the methods, the theory and underpinnings can require a great deal of background knowledge and diligence – and study, too – to grasp the essential material.

Our aim in this book is to provide an essential bridge between theoretical background and easily applicable experimentation. We have an additional aim, namely, that coverage be as extensive as can be, given the dynamic and broad field with which we are dealing.

Our approach, which is wedded to theory and practice, is based on a great deal of practical engagement across many application areas. Very varied applications are used for illustration and discussion in this book. This is natural, given how ubiquitous the wavelet and other multiresolution transforms have become. These transforms have become essential building blocks for addressing problems across most of data, signal, image, and indeed information handling and processing. We can characterize our approach as premised on an *embedded systems* view of how and where wavelets and multiresolution methods are to be used.

Each chapter has a section titled "Guided Numerical Experiments," complementing the accompanying description. In fact, these sections independently provide the reader with a set of recipes for quick and easy trial and assessment of the methods presented. Our bridging of theory and practice uses openly accessible and freely available as well as very widely used MATLAB toolboxes. In addition, interactive data language is used, and all code described and used here is freely available.

The scripts that we discuss in this book are available online (http://www.SparseSignalRecipes.info) together with the sample images used. In this form, the software code is succinct and easily shown in the text of the book. The code caters to all commonly used platforms: Windows, Macintosh, Linux, and other Unix systems.

In this book, we exemplify the theme of *reproducible research*. Reproducibility is at the heart of the scientific method and all successful technology development. In theoretical disciplines, the gold standard has been set by mathematics, where formal proof, in principle, allows anyone to reproduce the cognitive steps leading to verification of a theorem. In experimental disciplines, such as biology, physics, or chemistry, for a result to be well established, particular attention is paid to experiment replication. Computational science is a much younger field than mathematics but is already of great importance. By reproducibility of research, here it is recognized that the outcome of a research project is not just publication, but rather the entire environment used to reproduce the results presented, including data, software, and documentation. An inspiring early example was Don Knuth's seminal notion of literate programming, which he developed in the 1980s to ensure trust or even understanding for software code and algorithms. In the late 1980s, Jon Claerbout, of Stanford University, used the Unix Make tool to guarantee automatic rebuilding of all results in a paper. He imposed on his group the discipline that all research books and publications originating from his group be completely reproducible.

In computational science, a paradigmatic end product is a figure in a paper. Unfortunately, it is rare that the reader can attempt to rebuild the authors' complex system in an attempt to understand what the authors might have done over months or years. Through our providing software and data sets coupled to the figures in this book, the reader will be able to reproduce what we have here.

This book provides both a means to access the state of the art in theory and a means to experiment through the software provided. By applying, in practice, the many cutting-edge signal processing approaches described here, the reader will gain a great deal of understanding. As a work of reference, we believe that this book will remain invaluable for a long time to come.

The book is aimed at graduate-level study, advanced undergraduate study, and self-study. The target reader includes whoever has a professional interest in image and signal processing. Additionally, the target reader is a domain specialist in data analysis in any of a very wide swath of applications who wants to adopt innovative approaches in his or her field. A further class of target reader is interested in learning all there is to know about the potential of multiscale methods and also in having a very complete overview of the most recent perspectives in this area. Another class of target reader is undoubtedly the student – an advanced undergraduate project student, for example, or a doctoral student – who needs to grasp theory and application-oriented understanding quickly and decisively in quite varied application fields as well as in statistics, industrially oriented mathematics, electrical engineering, and elsewhere.

The central themes of this book are *scale*, *sparsity*, and *morphological diversity*. The term *sparsity* implies a form of parsimony. *Scale* is synonymous with *resolution*.

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