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Roberto Cipolla
University of Cambridge

Peter Giblin
University of Liverpool



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Preface

Computer Vision is the automatic analysis of sequences of images for the purpose of recovering three-dimensional surface shape. In recent years, several branches of mathematics, both ancient and modern, have been applied to computer vision. Projective geometry, which in its mathematical form dates back at least two centuries, is used to describe the relationship between points and lines in different images of the same object. Differential geometry, which is even older, though it received its definitive modern look in the first half of the nineteenth century, is used to describe the shape of curves and surfaces. More recently developments in singularity theory have enriched the field of geometry by making possible a wealth of detail only dreamed of fifty years ago. Likewise, developments in the speed and power of computers over the last decade have turned other dreams into reality, and made possible real-world applications of mathematical theory.

The goal of this book is to reconstruct surfaces from their ‘apparent contours’, that is the outlines which they present to us when we view them from a distance. It is not obvious that these apparent contours contain enough information to reconstruct an unmarked smooth surface at all. It is even less obvious that without accurate knowledge of the observer’s motion they contain this information; in fact, at the time of writing, we do not know in generality whether this is true. We have, however, successfully implemented the reconstruction when the observer’s motion is only partly known – that is, when it is constrained to be of a special kind called circular motion. Other work on more general motion is in progress.

Chapter 1 is introductory, and Chapters 2 to 4 introduce the mathematical ideas and techniques necessary to the study of surfaces and their apparent contours under viewer motion. In Chapters 5 and 6 we bring the mathematics to life with the latest techniques in photogrammetry and computer vision. We describe the real-time implementation of the theory with real

image sequences. We show that in practice apparent contours can be used effectively to reconstruct both motion and surface shape.

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Several students have helped in producing figures. Gordon Fletcher was instrumental in creating many of the figures from Chapters 2, 3 and 4, using the ‘Liverpool Surfaces Modelling Package’ written by Richard Morris. Paulo Mendonca and Kenneth Wong have helped in the real-time implementation of the theory and in obtaining the results described in Chapters 5 and 6.

The first author acknowledges the support of the Engineering and Physical Science Research Council. The second author acknowledges the hospitality of the Mathematics Department of Brown University, where some of this book was written, and also the Fulbright Commission for a Senior Fulbright Scholarship 1997/8. He also acknowledges the European grants VIVA and VANGUARD.

The front cover is based on a reconstruction of Benvenuto Cellini’s bronze sculpture, Perseus, which is undergoing restoration at the Uffizi Gallery in Florence. We are very grateful to Dr Francesca d’Uva and Dr Tina Guiducci of Burson Marsteller and Cassa di Risparmio di Firenze and to Francesco Porta of Datanord Multimedia for supplying the images.