Computation, Proof, Machine: Mathematics Enters a New Age

Mathematics Enters a New Age

Computation is revolutionizing our world, even the inner world of the "pure" mathematician. Mathematical methods – especially the notion of proof – that have their roots in classical antiquity have seen a radical transformation since the 1970s, as successive advances have challenged the priority of reason over computation.

Like many revolutions, this one comes from within. Computation, calculation, algorithms – all have played an important role in mathematical progress from the beginning – but behind the scenes, their contribution obscured in the enduring mathematical literature. To understand the future of mathematics, this fascinating book returns to its past, tracing the hidden history that follows the thread of computation. Along the way it invites us to reconsider the dialogue between mathematics and the natural sciences, as well as the relationship between mathematics and computer science. It also sheds new light on philosophical concepts, such as the notions of analytic and synthetic judgment. Finally, it brings us to the brink of the new age, in which machine intelligence offers new ways of solving mathematical problems previously inaccessible.

This book is the 2007 Winner of the Grand Prix de Philosophie de l’Académie Française.

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