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Preface

The development of first-order logic programming can be traced back to resolution theorem proving and further to the Skolem-Herbrand-Gödel Theorem, a fundamental theorem of first-order logic. This book proceeds by providing an extrapolation of this development to a higher-order setting. More precisely, it presents a largely theoretical foundation of a form of higher-order resolution, and combined functional and logic programming.

The Clausal Theory of Types is a higher-order logic for which the Skolem-Herbrand-Gödel Theorem and resolution can be generalized. It is a clausal extensional sub-logic of Church's formulation of the Simple Theory of Types which includes equality. It is "higher-order" in the sense that it allows quantification of variables of all types, embeddable predicates, representations of functions and predicates by λ -abstractions, and equations involving abstractions.

All of these features enable its Horn clause form to be a concise logic and functional programming language which has a sound and complete declarative and operational semantics. Clausal Theory of Types logic programs incorporate higher-order functional evaluation as an elementary operation through unification or conditional higher-order rewriting, higher-order relational proofs through backtrack search, and the separation of their declarative specifications.

This extrapolation entails largely self-contained discussions on the development of resolution and logic programming, the simply typed λ -calculus, higher-order logics and Henkin-Andrews general models, higher-order forms of term rewriting and equational unification, and higher-order versions of the Resolution Theorem and fixed point, model theoretic, and operational results in logic programming. There is also a survey of results on higher-order unification, and approaches to solving whether higher-order matching is decidable.

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Sections of the book are based on talks I gave at l'Institut National de Recherche en Informatique et en Automatique (INRIA), Rocquencourt, France in January 1990, University of Cambridge Computer Laboratory in February and November 1990, the Fourth International Conference on Rewriting Techniques and Applications at Como, Italy in April 1991, and the Sixth International Workshop on Unification at Schloß Dagstuhl, Germany in July 1992. Comments by the participants at these meetings and students at

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lectures on the Clausal Theory of Types led to my changing the presentation of parts of the book.

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D.A.W.

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