AXIOMATIC THEORIES OF TRUTH

At the centre of the traditional discussion of truth is the question of how truth is defined. Recent research, especially with the development of deflationist accounts of truth, has tended to take truth as an undefined primitive notion governed by axioms, while the liar paradox and cognate paradoxes pose problems for certain seemingly natural axioms for truth. In this book, Volker Halbach examines the most important axiomatizations of truth, explores their properties, and shows how the logical results impinge on the philosophical topics related to truth. For instance, he shows how the discussion of topics such as deflationism depends on the solution of the paradoxes. His book is an invaluable survey of the logical background to the philosophical discussion of truth, and will be indispensable reading for any graduate and professional philosopher in theories of truth.

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AXIOMATIC THEORIES
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Contents

Preface viii

Part I Foundations 1

1 Definitional and axiomatic theories of truth 3

2 Objects of truth 9

3 Tarski 15

4 Truth and set theory 25
   4.1 Definitions and axiomatizations 25
   4.2 Paradoxes and typing 27

5 Technical preliminaries 29
   5.1 Peano arithmetic 29
   5.2 Truth and satisfaction 35
   5.3 Translations and the recursion theorem 36

6 Comparing axiomatic theories of truth 39

Part II Typed truth 49

7 Disquotation 53

8 Classical compositional truth 63
   8.1 The conservativity of compositional truth 68
   8.2 Conservativity and model theory 82
   8.3 Nonstandard models 83
   8.4 Lachlan’s theorem 89
   8.5 Satisfaction classes and axiomatic theories of truth 98
   8.6 Compositional truth and elementary comprehension 101
   8.7 Positive truth 116

9 Hierarchies 123
   9.1 Tarski’s hierarchy axiomatized 125
   9.2 Illfounded hierarchies 129
Part III  Type-free truth  135

10 Typed and type-free theories of truth  140
11 Reasons against typing  146
12 Axioms and rules  149
13 Axioms for type-free truth  152
14 Classical symmetric truth  159
14.1 The Friedman–Sheard theory and revision semantics  162
14.2 Proof theory of the Friedman–Sheard theory  175
14.3 The Friedman–Sheard axiomatization  185
14.4 Expressing necessitation via reflection  188
14.5 Without satisfaction  192
15 Kripke–Feferman  195
15.1 Fixed-point semantics  202
15.2 Completeness and consistency  212
15.3 Proof theory of the Kripke–Feferman system  217
15.4 Extensions  225
16 Axiomatizing Kripke’s theory in partial logic  228
16.1 Partial Kripke–Feferman  231
16.2 Proof-theoretic analysis of partial Kripke–Feferman  244
17 Grounded truth  257
18 Alternative evaluation schemata  263
19 Disquotation  267
19.1 Maximal consistent sets of disquotation sentences  267
19.2 Maximal conservative sets of disquotation sentences  272
19.3 Positive disquotation  274
19.4 The semantics of positive disquotation  277
19.5 Proof theory of positive disquotation  280

Part IV  Ways to the truth  287

20 Classical logic  289
20.1 The costs of nonclassical logic  291
20.2 The internal logic of the Kripke–Feferman theory  295
20.3 Expressive power in nonclassical logic  300
20.4 Containing nonclassical logic  303
21 Deflationism 306
  21.1 Disquotationalism 307
  21.2 Conservativity 312
22 Reflection 322
  22.1 Reflection principles 322
  22.2 Reflective closure 326
23 Ontological reduction 330
24 Applying theories of truth 333
  24.1 Truth in natural language 333
  24.2 Extending schemata 335

Index of systems 343
Bibliography 345
Index 357
Preface

This book has four parts. In the first part I sketch some mathematical preliminaries, fix notational conventions, and outline some motivations for studying axiomatic theories of truth. Deeper philosophical investigation, however, is postponed to the last part when the significance of the formal results is discussed. The axiomatic theories of truth and the results about them are then given in the two central parts. The first of them is devoted to typed theories, that is, to theories where the truth predicate applies provably only to sentences not containing the truth predicate. In the third part of the book I discuss type-free theories of truth and how inconsistency can be avoided without Tarski’s object and metalanguage distinction. In the fourth and final part, the philosophical implications of the formal results are evaluated.

I have tried to make the book usable as a handbook of axiomatic truth theories, so that one can dip into various sections without having read all the preceding material. To this end I have also included many cross references and occasionally repeated some explanations concerning notation. It should be possible to read the final part on philosophical issues without having read the two formal parts containing the formal results. However, this last part presupposes some familiarity with the notation introduced in Chapters 5 and 6 in the first part. Of course, when discussing philosophical issues I will refer back to the formal results obtained in the two previous parts, and the reader who is interested in the last part only and skips the two formal parts will have to take my word for them.

All parts of the book should be accessible to a reader who has some acquaintance with the proofs of the Gödel incompleteness theorems and therefore with the basic concepts of recursion theory and metamathematics. In sections where I have used techniques from other areas of logic like model theory, I have defined all notions I use and have made most proofs so explicit that they should be accessible to readers not used to these techniques. I also assume very little with respect to proof theory: I do not use methods from ordinal analysis as I felt unable to provide an introduction to this branch of proof theory in a short chapter. In many research papers axiomatic truth theories are analysed by relating them to well-investigated subsystems.
of second-order number theory. The truth theories can then be compared via well-known results about these subsystems. Here I have attempted, whenever possible, to relate the theories of truth to one another directly, without the detour via the second-order systems, obviating the need to appeal to well-documented or folklore results about them.

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