CLASSICAL SOLUTIONS IN QUANTUM FIELD THEORY

Classical solutions play an important role in quantum field theory, high energy physics, and cosmology. Real time soliton solutions give rise to particles, such as magnetic monopoles, and extended structures, such as domain walls and cosmic strings, that have implications for the cosmology of the early universe. Imaginary time Euclidean instantons are responsible for important nonperturbative effects, while Euclidean bounce solutions govern transitions between metastable states.

Written for advanced graduate students and researchers in elementary particle physics, cosmology, and related fields, this book brings the reader up to the level of current research in the field. The first half of the book discusses the most important classes of solitons: kinks, vortices, and magnetic monopoles. The cosmological and observational constraints on these are covered, as are more formal aspects, including BPS solitons and their connection with supersymmetry. The second half is devoted to Euclidean solutions, with particular emphasis on Yang–Mills instantons and on bounce solutions.

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To Carolyn, Michael, and Cate

Contents

Preface

Semiclassical methods based on classical solutions play an important role in quantum field theory, high energy physics, and cosmology. Real-time soliton solutions give rise both to new particles, such as magnetic monopoles, and to extended structures, such as domain walls and cosmic strings. These could have been produced as topological defects in the very early universe. Confronting the consequences of such objects with observation and experiment places important constraints on grand unification and other potential theories of high energy physics beyond the standard model. Imaginary-time Euclidean instanton solutions are responsible for important nonperturbative effects. In the context of quantum chromodynamics they resolve one puzzle—the U(1) problem—while raising another—the strong CP problem—whose resolution may entail the existence of a new species of particle, the axion. The Euclidean bounce solutions govern transitions between metastable vacuum states. They determine the rates of bubble nucleation in cosmological first-order transitions and give crucial information about the evolution of these bubbles after nucleation. These bounces become of particular interest if there is a string theory landscape with a myriad of metastable vacua.

This book is intended as a survey and overview of this field. As the title indicates, there is a dual focus. On the one hand, solitons and instantons arise as solutions to classical field equations. The study of their many varieties and their mathematical properties is a fascinating subfield of mathematical physics that is of interest in its own right. Much of the book is devoted to this aspect, explaining how the solutions are discovered, their essential properties, and the topological underpinnings of many of the solutions. However, the physical significance of these classical objects can only be fully understood when they are seen in the context of the corresponding quantum field theories. To that end, there is also a discussion of quantum effects, including those arising from the interplay of fermion fields with topologically nontrivial classical solutions, and of some of the phenomenological consequences of instantons and solitons.

The first half of this book focuses on real-time classical solutions. I focus in particular on three classes of solitons—kinks, vortices, and magnetic monopoles—in one, two, and three spatial dimensions, respectively. Several chapters are devoted to their classical properties and many aspects of their quantum behavior. These are followed by a chapter that discusses the cosmological consequences of domain walls and cosmic strings—the dimensionally extended manifestations of kinks and

vortices—and of magnetic monopoles, and the implications of these for proposed high energy theories. Finally, there is a chapter discussing solitons in the BPS limit, including the connections with supersymmetry and duality.

After considering solitons, I turn to Euclidean solutions. Although these are solutions of classical equations, they are associated with tunneling processes that are truly quantum mechanical phenomena. An introductory chapter presenting an overview of this connection is followed by two chapters on Yang–Mills instantons. The first of these is primarily concerned with the mathematical properties of these solutions and their interpretation in terms of vacuum tunneling. Fermions are introduced in the second chapter, which discusses the physical consequences flowing from the instantons. A final chapter describes the bounce solutions and vacuum transitions.

Of necessity, some topics had to be omitted. In particular, Q-balls, nontopological solitons whose existence is based on the possession of a conserved charge rather than on topology, are not covered, nor are skyrmions, a fascinating class of topological solitons.

My goal has been to make the book accessible to advanced graduate students and other newcomers to the field, but also useful for more experienced researchers. I assume that the reader has had an introductory course in quantum field theory and some familiarity with non-Abelian gauge theories, but only the mathematical background of a typical physics graduate student. The homotopy theory needed to understand the topological underpinnings of the solitons is presented and explained. An appendix discusses roots, weights, and other necessary properties of Lie groups and algebras, building on the familiar results associated with SU(2).

I owe much to the colleagues and students with whom I have collaborated in research in this field. I thank Claude Bernard, Xingang Chen, Norman Christ, Huidong Guo, Alan Guth, Jim Hackworth, Conor Houghton, Roman Jackiw, Tom Kibble, Alex Kusenko, Bum-Hoon Lee, Choonkyu Lee, Hakjoon Lee, Kimyeong Lee, Sang-Hoon Lee, Arthur Lue, Ali Masoumi, Dimitrios Metaxas, Chris Miller, Doug Rajaraman, Alex Ridgway, Jon Rosner, Koenraad Schalm, and Piljin Yi. I am grateful to the late Sidney Coleman, from whom I learned field theory and much more.

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