

## VERIFICATION AND VALIDATION IN SCIENTIFIC COMPUTING

Advances in scientific computing have made modeling and simulation an important part of the decision-making process in engineering, science, and public policy. This book provides a comprehensive and systematic development of the basic concepts, principles, and procedures for verification and validation of models and simulations. The emphasis is placed on models that are described by partial differential and integral equations and the simulations that result from their numerical solution. The methods described can be applied to a wide range of technical fields, such as the physical sciences, engineering, and technology, as well as to a wide range of applications in industry, environmental regulations and safety, product and plant safety, financial investing, and governmental regulations.

This book will be genuinely welcomed by researchers, practitioners, and decision-makers in a broad range of fields who seek to improve the credibility and reliability of simulation results. It will also be appropriate for either university courses or independent study.

WILLIAM L. OBERKAMPF has 39 years of experience in research and development in fluid dynamics, heat transfer, flight dynamics, and solid mechanics. He has worked in both computational and experimental areas, and taught 30 short courses in the field of verification and validation. He recently retired as a Distinguished Member of the Technical Staff at Sandia National Laboratories.

CHRISTOPHER J. ROY is an Associate Professor in the Aerospace and Ocean Engineering Department at Virginia Tech. After receiving his PhD from North Carolina State University in 1998, he spent five years working as a Senior Member of the Technical Staff at Sandia National Laboratories. He has published numerous articles on verification and validation in the area of computational fluid dynamics. In 2006, he received a Presidential Early Career Award for Scientists and Engineers for his work on verification and validation in computational science and engineering.



# VERIFICATION AND VALIDATION IN SCIENTIFIC COMPUTING

WILLIAM L. OBERKAMPF CHRISTOPHER J. ROY





CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo, Mexico City

Cambridge University Press The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521113601

© W. L. Oberkampf and C. J. Roy 2010

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2010

Printed in the United Kingdom at the University Press, Cambridge

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Oberkampf, William L., 1944–
Verification and validation in scientific computing / William L. Oberkampf, Christopher J. Roy.
p. cm.

Includes index.

ISBN 978-0-521-11360-1 (hardback)

Science – Data processing.
 Numerical calculations – Verification.
 Computer programs – Validation.
 Decision making – Mathematical models.
 Roy, Christopher J. II. Title.

Q183.9.O24 2010

502.85 - dc22 2010021488

ISBN 978-0-521-11360-1 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.



To our wives, Sandra and Rachel



#### Contents

	Preface		page x1	
	Ack	Acknowledgments		
1	Introduction			
	1.1	Historical and modern role of modeling and simulation	1	
	1.2	Credibility of scientific computing	8	
	1.3	Outline and use of the book	15	
	1.4	References	17	
Part I	Fui	19		
2	Fundamental concepts and terminology		21	
	2.1	Development of concepts and terminology	21	
	2.2	Primary terms and concepts	32	
	2.3	Types and sources of uncertainties	51	
	2.4	Error in a quantity	57	
	2.5	Integration of verification, validation, and prediction	59	
	2.6	References	75	
3	Modeling and computational simulation		83	
	3.1	Fundamentals of system specifications	84	
	3.2	Fundamentals of models and simulations	89	
	3.3	Risk and failure	115	
	3.4	Phases of computational simulation	116	
	3.5	Example problem: missile flight dynamics	127	
	3.6	References	137	
Part II	Coc	de verification	145	
4	Software engineering		146	
	4.1	Software development	147	
	4.2	Version control	151	
	4.3	Software verification and validation	153	
	4.4	Software quality and reliability	159	
	4.5	Case study in reliability: the T experiments	161	



viii Contents

	4.6 Software engineering for large software projects	162
	4.7 References	167
5	Code verification	170
	5.1 Code verification criteria	171
	5.2 Definitions	175
	5.3 Order of accuracy	180
	5.4 Systematic mesh refinement	185
	5.5 Order verification procedures	192
	5.6 Responsibility for code verification	204
	5.7 References	205
6	Exact solutions	208
	6.1 Introduction to differential equations	209
	6.2 Traditional exact solutions	210
	6.3 Method of manufactured solutions (MMS)	219
	6.4 Physically realistic manufactured solutions	234
	6.5 Approximate solution methods	239
	6.6 References	244
Part III	<b>Solution verification</b>	249
7	Solution verification	250
	7.1 Elements of solution verification	250
	7.2 Round-off error	252
	7.3 Statistical sampling error	258
	7.4 Iterative error	260
	7.5 Numerical error versus numerical uncertainty	283
	7.6 References	284
8	Discretization error	286
	8.1 Elements of the discretization process	288
	8.2 Approaches for estimating discretization error	297
	8.3 Richardson extrapolation	309
	8.4 Reliability of discretization error estimators	317
	8.5 Discretization error and uncertainty	322
	8.6 Roache's grid convergence index (GCI)	323
	8.7 Mesh refinement issues	329
	8.8 Open research issues	334
	8.9 References	338
9	Solution adaptation	343
	9.1 Factors affecting the discretization error	343
	9.2 Adaptation criteria	349
	9.3 Adaptation approaches	356
	9.4 Comparison of methods for driving mesh adaptation	360
	9.5 References	366



		Contents	ix
Part IV	Mod	lel validation and prediction	369
10	Model validation fundamentals		
	10.1	Philosophy of validation experiments	372
	10.2	Validation experiment hierarchy	388
	10.3	Example problem: hypersonic cruise missile	396
	10.4	Conceptual, technical, and practical difficulties	
		of validation	401
	10.5	References	405
11	Design and execution of validation experiments		409
	11.1	Guidelines for validation experiments	409
	11.2	Validation experiment example: Joint Computational/	
		Experimental Aerodynamics Program (JCEAP)	422
	11.3	Example of estimation of experimental measurement	
		uncertainties in JCEAP	437
	11.4	Example of further computational–experimental synergism	
		in JCEAP	455
	11.5	References	465
12	Model accuracy assessment		469
	12.1	Elements of model accuracy assessment	470
	12.2	Approaches to parameter estimation and validation metrics	479
	12.3	Recommended features for validation metrics	486
	12.4	Introduction to the approach for comparing means	491
	12.5	Comparison of means using interpolation of experimental	
		data	500
	12.6	Comparison of means requiring linear regression of the	
		experimental data	508
	12.7	Comparison of means requiring nonlinear regression of the	
		experimental data	514
	12.8	Validation metric for comparing p-boxes	524
	12.9	References	548
13	Predi	ctive capability	555
	13.1	Step 1: identify all relevant sources of uncertainty	557
	13.2	Step 2: characterize each source of uncertainty	565
	13.3	Step 3: estimate numerical solution error	584
	13.4	Step 4: estimate output uncertainty	599
	13.5	Step 5: conduct model updating	622
	13.6	Step 6: conduct sensitivity analysis	633
	13.7	Example problem: thermal heating of a safety component	638
	13.8	Bayesian approach as opposed to PBA	664
	13.9	References	665



x Contents

Part V	Plan	ning, management, and implementation issues	671
14	Planning and prioritization in modeling and simulation		
	14.1	Methodology for planning and prioritization	673
	14.2	Phenomena identification and ranking table (PIRT)	678
	14.3	Gap analysis process	684
	14.4	Planning and prioritization with commercial codes	690
	14.5	Example problem: aircraft fire spread during crash landing	691
	14.6	References	694
15	Maturity assessment of modeling and simulation		696
	15.1	Survey of maturity assessment procedures	696
	15.2	Predictive capability maturity model	702
	15.3	Additional uses of the PCMM	721
	15.4	References	725
16	Development and responsibilities for verification, validation and		
	uncei	tainty quantification	728
	16.1	Needed technical developments	728
	16.2	Staff responsibilities	729
	16.3	Management actions and responsibilities	738
	16.4	Development of databases	747
	16.5	Development of standards	753
	16.6	References	755
	Appendix: Programming practices		757
	Index		762



#### **Preface**

Modeling and simulation are used in a myriad of ways in business and government. The range covers science, engineering and technology, industry, environmental regulations and safety, product and plant safety, financial investing, design of military systems, governmental planning, and many more. In all of these activities models are built that are mental constructs of how we believe the activity functions and how it is influenced by events or surroundings. All models are abstractions of the real activity that are based on many different types of approximation. These models are then programmed for execution on a digital computer, and the computer produces a simulation result. The simulation result may have high fidelity to the actual activity of interest, or it may be complete nonsense. The question is: how can we tell which is which? This book deals with various technical and procedural tools that can be used to assess the fidelity of modeling and simulation aspects of scientific computing. Our focus is on physical processes and systems in a broad range of the natural sciences and engineering.

The tools discussed here are primarily focused on mathematical models that are represented by differential and/or integral equations. Many of these mathematical models occur in physics, chemistry, astronomy, Earth sciences, and engineering, but they also occur in other fields of modeling and simulation. The topics addressed in this book are all related to the principles involved in assessing the credibility of the models and the simulation results. We do not deal with the specific details of modeling the physical process or system of interest, but with assessment procedures relating to the fidelity of the models and simulations. These procedures are typically described by the terms *verification* and *validation*.

We present the state of the art in verification and validation of mathematical models and scientific computing simulations. Although we will discuss the terminology in detail, *verification* can simply be described as "solving the equations right" and *validation* as "solving the right equations." Verification and validation (V&V) are built on the concept of quantitative accuracy assessment. V&V do not answer the entire question of simulation credibility, but they are key contributors. V&V could be described as the processes that provide evidence of the correctness and/or accuracy of computational results. To measure correctness, one must have accurate benchmarks or reference values with which to compare. However, the majority of simulations of complex processes do not have a computable or measurable reference value. For these situations we must rely on numerical error estimation



xii Preface

and estimation of the effects of all of the contributors to uncertainty in system responses. In verification, the primary benchmarks are highly accurate solutions to specific, although limited, mathematical models. In validation, the benchmarks are high-quality experimental measurements of system response quantities of interest. These experimental measurements, and the detailed information of the system being tested, should also have carefully estimated uncertainty in all of the quantities that are needed to perform a simulation of the experiment.

Mathematical models are built and programmed into software for the purpose of making predictions of system responses for cases where we do not have experimental data. We refer to this step as *prediction*. Since prediction is the usual goal of modeling and simulation, we discuss how accuracy assessment results from V&V activities enter into prediction uncertainty. We discuss methods for including the estimated numerical errors from the solution of the differential and/or integral equations into the prediction result. We review methods dealing with model input uncertainty and we present one approach for including estimated model uncertainty into the prediction result. The topic of how to incorporate the outcomes of V&V processes into prediction uncertainty is an active area of current research.

Because the field of V&V for models and simulations is in the early development stage, this book does not simply provide a prescriptive list of steps to be followed. The procedures and techniques presented will apply in the majority of cases, but there remain many open research issues. For example, there are times where we point out that some procedures may not be reliable, may simply not work, or may yield misleading results.

As the impact of modeling and simulation has rapidly increased during the last two decades, the interest in V&V has also increased. Although various techniques and procedures have been developed in V&V, the philosophical foundation of the field is *skepticism*. Stated differently, if the evidence for computer code correctness, numerical error estimation, and model accuracy assessment are not presented as part of a prediction, then the V&V perspective presumes these activities were not done and the results should be questioned. We feel this is the appropriate counter balance to commonly unsubstantiated claims of accuracy made by modeling and simulation. As humankind steadily moves from decision making primarily based on system testing to decision making based more heavily on modeling and simulation, increased prudence and caution are in order.



### Acknowledgments

Although only two names appear on the cover of this book, we recognize that if other people had not been there for us, and many others had not helped, this book would have never been written. These people provided training and guidance, created opportunities, gave advice and encouragement, corrected us when we were wrong, and showed the way to improved understanding of the subject. Although there were many pivotal individuals early in our lives, here we only mention those who have contributed during the last decade when the idea for this book first came to mind.

Timothy Trucano, Frederic Blottner, Patrick Roache, Dominique Pelletier, Daniel Aeschlimam, and Luís Eça have been critical in generously providing technical insights for many years. We have benefited from their deep knowledge of verification and validation, as well as a number of other fields. Jon Helton and Scott Ferson have guided our way to an understanding of uncertainty quantification and how it is used in risk-informed decision making. They have also provided key ideas concerning how to connect quantitative validation results with uncertainty estimates in model predictions. Without these people entering our technical lives, we would not be where we are today in our understanding of the field.

Martin Pilch created opportunities and provided long-term funding support at Sandia National Laboratories, without which we would not have been able to help advance the state of the art in V&V. He, along with Paul Hommert, Walter Rutledge, and Basil Hassan at Sandia, understood that V&V and uncertainty quantification were critical to building credibility and confidence in modeling and simulation. They all recognized that both technical advancements and changes in the culture of people and organizations were needed so that more reliable and understandable information could be provided to project managers and decision makers.

Many colleagues provided technical and conceptual ideas, as well as help in working through analyses. Although we cannot list them all, we must mention Mathew Barone, Robert Croll, Sharon DeLand, Kathleen Diegert, Ravi Duggirala, John Henfling, Harold Iuzzolino, Jay Johnson, Cliff Joslyn, David Larson, Mary McWherter-Payne, Brian Rutherford, Gary Don Seidel, Kari Sentz, James Stewart, Laura Swiler, and Roger Tate. We have benefited from the outstanding technical editing support through the years from Rhonda Reinert and Cynthia Gruver. Help from students Dylan Wood, S. Pavan Veluri, and John Janeski in computations for examples and/or presentation of graphical results was vital.



xiv

#### Acknowledgments

Reviewers of the manuscript have provided invaluable constructive criticism, corrections, and suggestions for improvements. Edward Allen, Ryan Bond, James Carpenter, Anthony Giunta, Matthew Hopkins, Edward Luke, Chris Nelson, Martin Pilch, William Rider, and William Wood reviewed one or more chapters and helped immeasurably in improving the quality and correctness of the material. Special recognition must be given to Tim Trucano, Rob Easterling, Luís Eça, Patrick Knupp, and Frederick Blottner for commenting on and correcting several draft chapters, or in some cases, the entire manuscript. We take full responsibility for any errors or misconceptions still remaining.

We were blessed with encouragement and patience from our wives, Sandra and Rachel. They tolerated our long hours of work on this book for longer than we deserved.