

# 1

## Fundamentals of nuclear fusion



### 1.1. The energy problem and nuclear fusion

#### *1.1.1. Energy and human societies*

How can man have ruled the world, setting himself at the centre and behaving as if all other animals and plants are there for his use? There are many sayings which seek to distinguish mankind from other animals; for example, ‘man is an animal which is able to laugh’, or ‘man is a thinking reed’. It is convenient to suppose that one effect is the direct result of just one cause. But generally speaking each effect stems from many causes. And an explanation which seems to appeal to just one concept may in fact be using that concept to stand for many others.

To say that ‘man is the only animal which uses fire’ is a case in point for the science and technology of energy. Although other animals are afraid of fire, man uses it; with fire he guards himself from the attack of wild animals, obtains warmth and cooks his food. We may express this more generally by saying that man is the only animal to use energy constructively in the normal course of his way of life. In their daily lives, human beings use many different kinds of energy, all of which we may represent by the single concept of ‘fire’. So if we say man is what he is because he uses fire, what we are really saying is that man depends on energy.

A unit of energy, 1 Q, is equivalent to  $1.05 \times 10^{21}$  J or  $10^{18}$  BTU. From AD 1 to 1850, world energy consumption per years has been estimated at 0.004 Q. During this period, energy was obtained by various means: gathering fallen branches, making charcoal, collecting colza oil. The wood, charcoal and oil were supplied each year by the growth of plants and so constituted a renewable form of energy. The energy problem would not arise in the world of today if human societies still used renewable energy sources alone. In the past, man lived as he has always lived; science and technology as we know them did not exist. Nowadays we are inclined, with sentimental nostalgia, to see the life of our ancestors as happy in consequence. The reality was different. Many lived their lives in vain,

at the mercy of disease; the luxury of the rich and noble few was supported by the toil of the many poor serfs.

In the eighteenth and nineteenth centuries, the industrial revolution occurred in Britain and continental Europe. Machines were substituted for human labour, and manufacturing changed from small numbers of hand-made products to mass production in factories. To begin, the energy sources substituted for human labour were wind and water power; and these, like fallen branches and charcoal, are classified as renewable energy, based on solar energy radiated to the earth.

In 1781 James Watt developed his steam engine, using coal as the heat source and transposing the motion of the piston into rotation. The use of coal as fuel brought to an end the old world based on renewable energy. Fossil fuels such as coal and oil are not renewable: they are finite stores of old solar energy locked in the remains of organic matter which grew by absorbing the radiation of the sun. Nowadays man uses only about 1% of the energy resources of forests: 99% of wood rots in vain, without being changed into coal or oil.

Once human societies learnt how to use non-renewable fossil fuels, world energy consumption per year averaged over the period from 1850 to 1950 increased by 0.04 Q, i.e. energy consumption was raised one order of magnitude, from 0.004 Q over the period AD 0–1850 to 0.04 Q in the next 100 years. For the 100 years from 1950–2050, average annual energy consumption is estimated to increase by another order of magnitude.

Energy is a source of negative entropy; energy sources supply human societies with heat fluxes of higher temperature. These energy fluxes are then thrown away, being released to the environment at low temperatures. In the process, however, human society increased negative entropy. Up to 1850, energy consumption was low and human society was almost closed with respect to energy. If a system is closed, it has maximum entropy and the distribution of energy is Maxwellian. Before the industrial revolution, wealth and power were distributed among the people according to such a Maxwellian distribution: the lives of the few who were rich and aristocratic were supported by their many poor retainers. After the industrial revolution, as energy consumption gradually increased, the difference in income between rich and poor decreased. In modern economically advanced countries, it now approaches equality. The energy consumption of a country may now be considered directly proportional to its gross national product.

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In the twentieth century, oil has increasingly been substituted for coal. This is not because there is a shortage of coal, but rather because oil or natural gas is cleaner and more convenient. Coal deposits which can be mined economically are estimated at 100 Q, compared to 11 Q for oil. At present, annual energy consumption is estimated at 0.3 Q, and the total energy required over the next 100 years is about 70 Q. Most of this energy is currently consumed in advanced countries, but the instinctive desires of human beings for more comfortable and convenient lifestyles will increase energy consumption in developing countries. Even if advanced countries manage to save energy, the energy needed over the next 100 years cannot be supplied by oil, a fact already reflected in the dependence of the price of oil on international relations.

In the twentieth century mankind has been able to rely on energy supplied by the combustion of oil, but if we continue to do so in the face of the finite stock of this unrenovable supply of energy, social confusion will result. So if mankind is to maintain a civilised lifestyle in the future, new energy sources must be developed within the next 100 years.

The energy given out by the sun is enormous; only a fraction of it (but still a great amount) reaches the earth. Our ancestors used the sun as the source of energy for agriculture. Can we sustain civilised life for the foreseeable future by using the sun's energy, rather than fossil fuels, as the energy source for giant industries? The sun radiates energy at  $9.3 \times 10^{21}$  kcal per second,  $4.1 \times 10^{13}$  kcal per second reaches the earth,  $5 \times 10^{-9}$  of the total. Over a year, the earth receives  $1.3 \times 10^{21}$  kcal, or  $5 \times 10^3$  Q per year. If we consume 1 Q per year, the energy received from the sun per year corresponds to 5000 years annual consumption; the amount consumed is thus very small compared to the amount received. However, the density of the sun's energy on a plane normal to the earth's surface is as low as 2 cal/cm<sup>2</sup> min, assuming none is absorbed between the sun and the earth. Because of this low energy, a large area is needed in order to collect a significant amount of the sun's energy. Generally speaking, it is difficult to utilise low-density energy. When a power station is constructed, the energy pay rate, here defined as energy pay rate

$$= \frac{\text{total amount of energy generated during operational life}}{\text{total energy consumed in constructing the station}}$$

must be greater than one, from the technological point of view (leaving aside the economics of the matter). Operational life is usually

taken to be 20 years. The total energy consumed includes mining the iron ore, turning it into steel, transporting to the site and then constructing and commissioning the plant. Fuel energy – consumed in extraction, transportation and treatment of waste products – must also be included. If a large power supply system is to be constituted by the same kind of station, not one but many stations must be constructed in succession: one station is needed to create the energy by which another is constructed during its operational life. Consequently, the total energy in the denominator of the pay rate equation is frequently doubled. The pay rate will seldom be greater than one if the station uses low-density energy as input; the station must be large, so the value of the denominator becomes large too.

When low-density energy is used, it is desirable to operate the station at low temperatures: water at 40°C can be used for heating, and water at 75°C for cooling. But it is difficult to make the energy pay rate greater than one when solar energy is used to heat the water to such temperatures. Needless to say, the energy pay rate will never exceed one if solar energy is to provide high temperatures, such as those for the generation of electricity (1400°C for an oil-powered station, 800°C for a nuclear one). Natural (soft) energy sources are out of the question as far as high-output stations are concerned.

### **1.1.2. Energy in classical mechanics**

In classical mechanics, the kinetic energy of a body of mass  $m$  with velocity  $v$  is given by

$$KE = \frac{1}{2}mv^2. \quad (1.1)$$

If the body is located in the field of a conservative force, such as gravity, it will have a potential energy given by

$$U = mgh \quad (1.2)$$

where  $h$  is the height above ground level in a field of gravitational strength  $g$ . The sum of the kinetic energy and the potential energy is the total energy  $E$ , and by the law of conservation of energy,

$$E = KE + U = \frac{1}{2}mv^2 + mgh = \text{constant}. \quad (1.3)$$

If the body is released at height  $h$ , its potential energy decreases and kinetic energy increases as it falls; at ground level, all its energy is in the form of kinetic energy. In order to extend the law of conservation of energy to what happens once the body hits the ground, the concept of heat energy must be introduced. The heat

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energy  $J$  absorbed by the ground after impact is equal to the total energy  $E$ . The mass of the body is constant, whether it is at rest at height  $h$ , falling, or hitting the ground; in classical mechanics, energy and mass are conserved separately.

1.1.3. *Chemical energy*

All matter consists of molecules, the ultimate units of chemical character. A molecule consists of atoms, and atoms consist of a nucleus with a positive electric charge and electrons with a negative charge. The greater part of the mass of an atom is in the nucleus: the mass of an electron is much smaller than that of a nucleus. Further, the nature of the atom depends on the nucleus: if its mass or charge is different, it is a different atom. The chemical nature of an atom, however, depends solely on the charge of the nucleus. When the charges of two nuclei are the same but the masses are different, they are called isotopes and are classified as the same atom. For the complete atom, the nucleus is said to have a charge  $Ze$ , where  $e$  is the charge of an electron and  $Z$  is a positive integer known as the atomic number, and  $Z$  electrons, each with charge  $-e$ , revolving around the nucleus. The chemical energy of an atom stems from the potential energy of these electrons.

Suppose a molecule  $A$  consists of an atom  $X$  and an atom  $Y$ . The nuclei of atoms  $X$  and  $Y$  are neutralised by electrons when they form molecule  $A$ . If the two nuclei approach each other within a distance of less than  $10^{-10}$  m, the electrons (which revolve separately around their own nucleus when the nuclei are distant from one another) change their orbits to ones that revolve around the two nuclei together. If the potential energy of the electrons moving in the new orbits is smaller than the sum of the potential energies of the electrons separated into two atoms, the two atoms will constitute one molecule  $A$ . Take the chemical reaction



where  $A$ ,  $B$  and  $C$  are molecules. The reaction proceeds because the sum of the electron energies of the two molecules  $A$  and  $B$  is larger than that of the molecule  $C$ . The difference  $E_c$  in the energies of the electrons is released and called the heat of reaction.

In the twentieth century, human beings have used energy from the combustion of coal and oil. This thermal energy has come from the changes in the potential energy of electrons revolving around nuclei resulting from reactions such as eq. (1.4).

#### 1.1.4. Atomic energy

In 1945, atomic energy was released by the atomic bombs dropped on Hiroshima and Nagasaki. From the mid-1950s atomic energy could be used for peaceful purposes. Atomic energy and chemical energy have quite different origins; the theoretical foundations of atomic energy were laid by 'the special theory of relativity' developed by Albert Einstein in 1905.

In classical mechanics, mass, momentum and energy are conserved separately before and after interactions. The total energy of a body of mass  $m$ , according to the special theory of relativity, is given by

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}. \quad (1.5)$$

In eq. (1.5),  $m_0$  is the rest mass of the body and  $c$  is the speed of light in a vacuum. The mass of the body is not constant but is related to its velocity  $v$  by

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}. \quad (1.6)$$

Equation (1.5) reduces to

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2, \quad (1.5')$$

if  $v/c$  in eq. (1.5) is expanded as a series, provided that  $v$  is smaller than  $c$ . The second term on the right-hand side of eq. (1.5') is the kinetic energy, while the first term is a constant, called the rest energy. Because there is a term in eq. (1.5') which includes the rest mass  $m_0$  for the energy  $E$ , mass and energy are not conserved separately, but are related to each other. Equation (1.5') shows that a new energy  $m_0 c^2$  must appear when the rest mass  $m_0$  disappears. A mass of 1 kg corresponds to an energy of  $9 \times 10^{16}$  J, which is quite a large amount. The mass of the nucleus decreases when the mass changes into energy, because the majority of the mass of the atom is located in the nucleus. In chemical reactions, the nucleus does not change. Atomic energy, on the other hand, uses the change in mass of the nucleus.

#### 1.1.5. The Coulomb force and the nuclear force

Let us investigate nuclei in more detail. A nucleus is made up of more fundamental constituents called elementary particles. Although many elementary particles have been observed up to now, the electron, proton and neutron are the most important ones.

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An electron has a mass of  $9.10988 \times 10^{-31}$  kg and charge of  $-1.6 \times 10^{-19}$  C, and revolves around the nucleus. The nucleus consists of protons, whose mass is  $1.673 \times 10^{-27}$  kg and charge is  $1.6 \times 10^{-19}$  C, and neutrons, whose mass is  $1.675 \times 10^{-27}$  kg and charge is zero. Protons, together with neutrons, are called nucleons. A hydrogen atom has one electron, which revolves around a nucleus consisting of a single proton. In the case of a helium atom, two electrons revolve around a nucleus consisting of two protons and two neutrons. A helium nucleus is expressed as  ${}^4_2\text{He}$ . The upper-left number is the mass number (the mass of the helium nucleus has four times the mass of the hydrogen nucleus, and the mass number is the number of nucleons that constitute the nucleus) and the lower-left number is the atomic number or the charge number (the charge of the helium nucleus has twice the charge of a proton nucleus and the charge number is the number of protons in the nucleus).

The neutron is electrically neutral while the proton has a positive charge. A repulsive force, called the Coulomb force, acts on any two particles with the same kind of charge. The force  $F$  acting on two bodies whose charges are respectively  $Z_1e$  and  $Z_2e$  is

$$F = \frac{Z_1 Z_2 e^2}{4\pi\epsilon r^2}. \quad (1.7)$$

Here  $\epsilon = 8.854 \times 10^{-12}$  F/m is the dielectric constant in a vacuum. The potential energy  $U$  of the two charges whose distance is  $r$  is given by

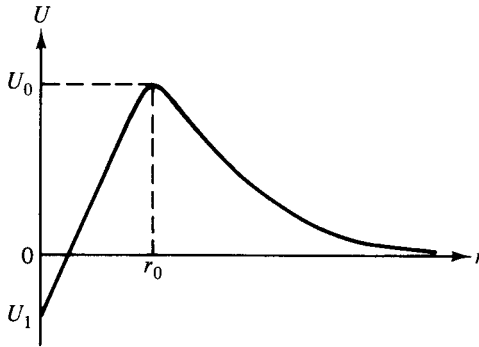
$$U = - \int_{\infty}^r \frac{Z_1 Z_2 e^2}{4\pi\epsilon r^2} dr = \frac{Z_1 Z_2 e^2}{4\pi\epsilon r}. \quad (1.8)$$

Here the potential energy is chosen to be zero at infinity. Take any two protons. At  $r_0 = 5 \times 10^{-15}$  m, which is the radius of a nucleus,  $U = 0.29$  MeV. Here  $1 \text{ eV} = 1.602 \times 10^{-19}$  J is the kinetic energy obtained by an electron which is accelerated by an electric potential of 1 V. When the distance between the centres of the two nuclei becomes less than  $5 \times 10^{-15}$  m, a strong attractive force called the nuclear force acts on the nuclei. This nuclear force binds protons and neutrons together in a nucleus. Figure 1.1 shows the relation between the potential energy  $U$  and the distance  $r$  between the two protons. Potential energy increases by the Coulomb force as  $r$  decreases from  $\infty$  to  $r_0$ . Due to the nuclear force, potential energy decreases in the region within  $r_0$ .  $U_0$  in Fig. 1.1 is  $U = 0.29$  MeV for two protons at  $r = r_0$ , as given by eq. (1.8). The value of  $U_1$  is negative because the potential energy due to the nuclear force is larger than

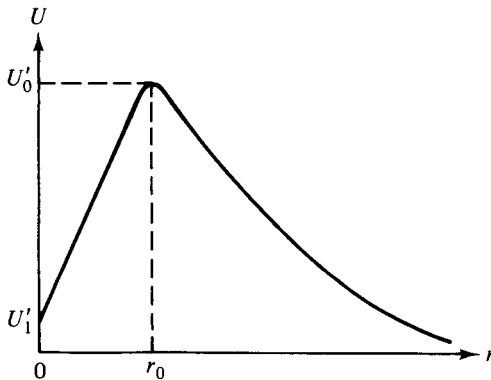
that due to the Coulomb force. Two protons become stable when they form one nucleus, rather than two separate protons at  $r = \infty$ .

Instead of two protons, let us now consider two nuclei, whose charges are  $Z_1e$  and  $Z_2e$  respectively. The Coulomb force between the nuclei is expressed as  $F = Z_1Z_2e^2/4\pi\epsilon r^2$ . On the other hand, the distance  $r_0$  inside which the nuclear force acts, and the nuclear force itself for these two nuclei, are not much different from those for the two protons. In other words, the nuclear force does not depend much on  $Z$ . Thus the potential energy for these two nuclei varies as shown in Fig. 1.2. The value of  $U'_0$  in Fig. 1.2 is  $Z_1Z_2$  times that of  $U_0$  in Fig. 1.1. As the difference between  $U_0 - U_1$  and  $U'_0 - U'_1$  is not large,  $U'_1$  becomes positive (see Fig. 1.2) if  $Z_1Z_2$  is large. In

**Fig. 1.1.** Potential energy due to the Coulomb force and the nuclear force between two protons.



**Fig. 1.2.** Potential energy due to the Coulomb force and the nuclear force between two nuclei with large  $Z$ .



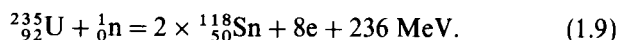


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such a case, the two separate nuclei at  $r = \infty$  are more stable than one large nucleus formed by the two nuclei. In summary, one large nucleus becomes stable when it is separated into two, while two small nuclei become stable when they are combined to form one large nucleus. The nucleus of  $Z = 26$ , i.e.  ${}_{26}\text{Fe}$ , at the centre of the periodic table, is the most stable.

1.1.6. *The nuclear fission reaction*

The uranium nucleus is large and unstable. Because it has no charge, a neutron can approach a nucleus easily. If a uranium nucleus absorbs a neutron, the nucleus becomes unstable, resulting in nuclear fission. The typical fission reaction of uranium is given by



Here  $\text{n}$  is the neutron and  $\text{e}$  is the electron. Equation (1.9) describes a uranium nucleus which is split into two equal Sn nuclei. The atomic weight of  ${}_{92}^{235}\text{U}$  is 235.124 (1 atomic weight unit is  $1.6603 \times 10^{-27}$  kg), i.e. the atomic mass of  ${}_{92}^{235}\text{U}$  is  $3.903764 \times 10^{-25}$  kg. As the atomic weight of  ${}_{50}^{118}\text{Sn}$  is 117.94, the atomic mass of  ${}_{50}^{118}\text{Sn}$  is  $1.958158 \times 10^{-25}$  kg. The mass of  $-4.201 \times 10^{-28}$  kg is the difference between double the mass of tin and the sum of the masses of uranium and of a neutron. This is the mass deficiency in the nuclear fission of  ${}_{92}^{235}\text{U}$  and corresponds to an energy of  $3.78 \times 10^{-11}$  J = 236 MeV. An atomic power station uses this mass deficiency to generate electricity.

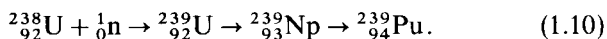
In 1951, JRR-1, Japan's first fission reactor, began operation. When the reactor reached the critical state, the radio announcer said 'a second fire now burns in our country'. The 'first fire' is the combustion through which man used energy in his daily life. Atomic energy is the second fire, newly obtained. The atomic energy extracted from light-water reactors uses the mass deficiency in the fission of uranium nuclei.

The amount of uranium in mineral deposits is estimated at  $2.5 \times 10^{10}$  kg. Two isotopes  ${}_{92}^{235}\text{U}$  and  ${}_{92}^{238}\text{U}$  are found in natural uranium. Only  ${}_{92}^{235}\text{U}$  undergoes nuclear fission in the light-water reactor; natural uranium includes 0.7 % of  ${}_{92}^{235}\text{U}$ . The thermal efficiency of the atomic power station is about 20 %. This low efficiency comes from the safety margin of the reactor, which operates at a low temperature. In light-water reactors, electric energy of only 7 Q can be released by use of the natural uranium of  $2.5 \times 10^{10}$  kg. Atomic energy as the 'second fire' cannot support human civilisation

for long, if we take into account that human beings will use 1 Q of energy per year in the near future.

In seawater, uranium is found in concentrations of 0.0033 ppm. The total amount is  $4.4 \times 10^{12}$  kg, which is 200 times that of the uranium as a mine resource. If we develop techniques to collect the dilute uranium from seawater, nuclear fission energy could help us to sustain our civilised lifestyle for a longer time.

When  ${}^{238}_{92}\text{U}$  absorbs a high-speed neutron,  ${}^{238}_{92}\text{U}$  is changed to  ${}^{239}_{94}\text{Pu}$  as



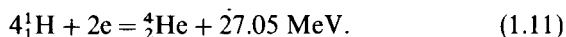
Plutonium undergoes nuclear fission too. In the fast breeder reactor,  ${}^{238}_{92}\text{U}$  is reacted with  ${}^{235}_{92}\text{U}$  and changed to  ${}^{239}_{94}\text{Pu}$ . Taking into account that the breeding rate (change rate of  ${}^{238}_{92}\text{U}$  to  ${}^{239}_{94}\text{Pu}$ ) is 60 % and the thermal efficiency can be increased to 30 % with increase in safety (due to the improvement of the wall materials), we can expect that a fission energy of 300 Q can be extracted from the uranium as a mine resource. It is hoped that the technology of the fast breeder reactor will soon be perfected (the Super-phoenix of 1.2 GW is already operating in France), together with a fuel cycle which includes the extraction of unburnt uranium and plutonium.

The nuclear fission reactor can confine fission products with strong radio-activities (plutonium is especially dangerous to human beings; it is the explosive of the atomic bomb) in the fuel rod; by contrast, a coal power plant exhausts polluted materials into the atmosphere. The safe management and utilisation of a large amount of radio-active fission products over a longer period is the most important and most difficult technological task involved in using the fission reactor.

### *1.1.7. Energy from nuclear fusion*

The sun is a giant sphere with radius  $7 \times 10^8$  m and mass  $2 \times 10^{30}$  kg. It radiates  $9.3 \times 10^{21}$  kcal of energy per second; it would have burnt itself out in 3000 years if it had been burning coal. Before Einstein many wondered why the sun continues to shine. The special theory of relativity explained its enduring life.

The nucleus of hydrogen is a single proton. When it combines with other protons to form a larger nucleus, it becomes more stable. Consider the following nuclear reaction:



The atomic weight of  ${}^1_1\text{H}$  is 1.008 while the atomic weight of  ${}^4_2\text{He}$  is