6. ELEMENTS OF SPATIAL STRUCTURE
CAMBRIDGE GEOGRAPHICAL STUDIES

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Elements of spatial structure
A QUANTITATIVE APPROACH

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