CHAPTER 1

Introduction

1.1 The role of rights in social choice
This book is concerned with the direct implications of incorporating a mild libertarian requirement on social choice. The general problem of social choice can be viewed as how society can best choose amongst a set of alternative social states and it is the strong belief that libertarian rights should be properly incorporated into the social choice framework that has motivated the research undertaken for this book. In this context a basic and a weak libertarian premise is that if two states of the world, \( x \) and \( y \), differ only with respect to a person \( b \)'s business alone, then if person \( b \) prefers \( x \) to \( y \), \( y \) should not be chosen if \( x \) is available.

Before formally incorporating rights into a framework of social choice (which will be dealt with in Chapter 2) I wish to consider a major objection to the above libertarian premise. Briefly it is that rights should be treated as some ‘control’ an individual has over certain states of affairs and such alternatives as he dismisses by this personal control should not be considered as available options in social choice. Rights exercising determines the set of options open for social choice. One of the leading proponents of this view is Nozick (1974):

The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice can be made by a social choice mechanism based upon a social ordering, if there are any choices left to make! (p. 166)

Given that an individual has a right to a choice over a pair of alternatives, what is socially important, it is argued, is that the choice is his. The outcome of his choice may well be socially undesirable but this is irrelevant, the value in society of individual rights comes from the allowed freedom to choose. Social choice must therefore be constrained by the personal exercising (through personal control) of rights rather than having rights explicitly incorporated into the social choice procedure itself.

I wish to separate two aspects of this argument for independent criticism (though the two aspects are not independent from each other).

(1) Social choice should not concern itself with alternatives which are dismissed by rights exercising (through direct control of an individual or by some procedure implementing the right in accordance with the
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individual’s desires). Given that \(x\) and \(y\) differ only with respect to a person \(b\)’s business alone and person \(b\) prefers \(x\) to \(y\), \(y\) should never be considered in any social choice problem from an available set of options.

(2) Social choice should not implement a right by a procedure based on the individual’s desires but allow that individual to implement his right through his own personal control over that decision. Given that \(x\) and \(y\) differ only with respect to a person \(b\)’s business alone then person \(b\) must be allowed full control, through his own personal action, to dismiss one of these alternatives from the choice set.

The main difference between these aspects may be made clearer by noting that in (1) the implementation of a right may not be directly controlled by the individual himself though this implementation will be carried out independently of a social evaluation of the alternatives; and in (2), the alternative which is dismissed through the personal control of the individual can still be in the initial set of options evaluated by the social choice procedure. This allows a social judgement of the form: it is better that \(x\) is chosen over \(y\) when this is the result of person \(b\)’s personal control over alternatives which differ solely with respect to his own business.

Let us consider (1) first.

Nozick, illustrating the point, has provided the following example:

If I have a right to choose to live in New York or Massachusetts, and I choose Massachusetts, then alternatives involving my living in New York are not appropriate objects to be entered in a social ordering. (Nozick, 1973, p. 62).

Sen has argued against this illustration of the argument in (1):

If I believe that it is a better society which — given other things — lets Nozick decide where he wishes to live, then I must assert that it is socially better that Nozick should be permitted to live in Massachusetts as desired by him. If Nozick is forced out of Massachusetts, then one would wish to say not only that Nozick’s rights have been violated, but that society is worse off — given other things — by stopping Nozick from living where he wishes. (Sen, 1976, p. 230)

It is important that by saying society is better off one does not mean that the alternative dismissed by the implementation of a right is, in itself, socially undesirable. Society is better off because this alternative is dismissed as a result of the implementation of an individual’s right. The existence of this right is of social value in itself, independent of the nature of the outcome of the implementation of this right. It is better for Nozick to live in Massachusetts precisely because he desires this. Such a social comparison between Nozick living in Massachusetts or in New York cannot be made if Nozick living in New York is not admitted into the set of alternatives available for such social comparison.
1.1 The role of rights in social choice

Rights are hardly ever ‘absolute’ in the sense that one can never universally say that under all logically possible circumstances a right must be implemented (either through direct control of the individual himself, or otherwise), for example the right to free expression through peaceful marches in public streets. In most circumstances this right should be implemented. However, if it is the British fascist organisation ‘the National Front’ planning such a march this right may well be justifiably withheld, since the consequential social disruption in terms of physical violence and disorder is so costly that it outweighs the social value of free expression. If all states which involve the National Front not marching are excluded from the set of states available for social comparison then any possibility of a social judgement that the march should be banned becomes impossible – under any circumstances.

An island which has a population of 1800 people has enjoyed self-determination for 150 years. They should have a right to continue to self-determine their lives. However, if the cost of defending this ‘self-determination’ involves over three million lives being lost, then there is a strong case for violating their rights. This is not to say that the islanders’ self-determination is not of value and not worth some cost to secure it. My only point is that there exist circumstances (albeit extreme in this example) where the value of the right is outweighed by other social considerations. If all states of the world where the islanders do not have self-determination were excluded from the set of social options available for choice, then choice must involve only states which give the islanders the right to self-determination and this should be sought at all costs.

Now let us consider the argument presented in (2) above. Should rights be specified as some personal control over certain states of affairs? Consider the following example taken from Sen (1983a, Section VII).

There is a person, Ed, who is unconscious after a car accident. His companion who is unhurt knows about Ed’s moral beliefs and strengths of conviction. The doctors tell the companion that she can treat Ed in one of two ways, A and B, and while she is certain that both would be effective, she is also certain that A would be very much better for Ed in terms of side-effects. However, Ed’s companion knows that though Ed would agree that treatment A is better for his health, he would have very strong objections to it since its development involved cruelty to animals. Thus Ed’s companion knows, without doubt, that Ed would prefer treatment B. It seems reasonable to argue that Ed’s liberty would be served better if the doctor gave treatment B even though Ed himself is not exercising any direct control over the particular choice. Sen states that this choice would better serve Ed’s ‘indirect liberty’. Examples of ‘indirect liberty’ are not uncommon to find, and to not characterise liberty as taking account of this would yield a very limited notion of liberty indeed.
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The social choice characterisation of liberty compares what emerges with what a person would have chosen, whether or not he actually does the choosing. (Sen, 1983a, Section VII)

Thus the social choice framework permits analysis of indirect liberty while the interpretation of rights as ‘control’ of an individual over his personal affairs does not.

1.2 Some clarifying matters

Before embarking on the more formal analysis of rights in social choice, it is necessary to make clear some assumptions which will be kept, without question, throughout the book.

Firstly, individuals’ preferences will be assumed to be honest reflections of their respective ‘desires’. The problem of extracting such honest preferences will not be discussed. Thus individuals trying to manipulate social choice by deliberate misspecification of their preferences will never arise. This problem, though important, could not possibly be dealt with in an adequate way without taking up a disproportionate amount of space within the present book, the object of which is to examine the implications of rights given the honest preferences in social choice.

Secondly, none of the libertarian conditions presented here are intended to summarise, in totality, the concept of liberty in society. The argument in Section 1.1 is that the libertarian premise reflects an aspect of liberty which should be accepted by all libertarians, and indeed some non-libertarians as well.

Finally, the assignment of rights in this monograph, in terms of allocating each individual’s pairs of alternatives which differ with respect to each individual’s business alone, is assumed to be decided on the basis of non-utility information, in particular the nature of the alternatives concerned. It is not assumed that the rights assignments are determined according to the nature of individual preferences.
CHAPTER 2

The Gibbard paradox

2.1 Introduction

This chapter formalises a framework for rights in social choice. It is seen that under some circumstances rights of individuals can conflict with each other in the sense that whatever is chosen from a set of alternatives, somebody’s rights will be seen to be violated. The chapter proceeds to analyse different ways of resolving this problem.

2.2 A framework for rights in social choice

If two states of the world $x$ and $y$ differ only with respect to a person $b$'s business alone, then, if person $b$ prefers $x$ to $y$, $y$ should not be chosen if $x$ is available. This, as has been argued in Chapter 1, constitutes an acceptance of a mild form of personal liberty in social choice. How can this best be formalised into a framework of social choice?

Gibbard (1974) formalised rights in terms of issues. He argued that since a person $b$ is granted rights over a pair of social alternatives because they differ with respect to his business alone, we can thus isolate, and explicitly define, this personal issue (and other peoples' personal issues) by defining a social alternative in terms of component ‘feature alternatives’, one for each issue in the state of the world. Following Gibbard, a formal framework for analysis will now be stated.

Social alternatives Each alternative will be represented by a number of distinct features. Assuming there are $n$ of them we can represent a typical alternative as $x = (x_1, x_2, \ldots, x_n)$. For each issue $i = 1, 2, \ldots, n$ let there be a finite set $M_i$ of feature alternatives for issue $i$, each with at least two elements. Assuming the features are technologically separable from each other, the set $X$ of all technologically possible alternatives is the cartesian product $M_1 \times M_2 \times \ldots \times M_n$; it consists of every $n$-tuple $(x_1, x_2, \ldots, x_n)$, with $x_1 \in M_1, x_2 \in M_2, \ldots, x_n \in M_n$. The issues may be ‘personal’ to some individual, e.g. a person’s wall colour, or may be ‘public’ like the issue of nuclear disarmament. Within this issue framework it is assumed that the number of issues exceeds the number of individuals.
2. The Gibbard paradox

**Individual preferences** Assume there are $m \geq 2$ individuals $b$, each having a preference ordering $R_b$ (reflexive, complete and transitive) over $X$. Let $P_b$ and $I_b$ be the asymmetric and symmetric factors of $R_b$ respectively. Let \{\{R_b\}\} represent a profile for society, one ordering $R_b$ for each individual $b$.

**The social choice function** Assume there exists a choice function $C(S, \{R_b\})$ defined over $X$ which is a functional relation such that the choice set $C$ is non-empty for every non-empty subset $S$ of $X$, and for every logically possible set of preference profiles $\{R_b\}$. A condition of ‘unrestricted domain’ is incorporated into the definition of $C(S, \{R_b\})$. This does not mean that choice need be based on preferences of individuals.

We must now incorporate rights into this framework by formalising a libertarian condition. This can be done in two steps. The first is to define a rights assignment or ‘rights system’. The second is to define a ‘rights implementation rule’. Consider, first, first, two general definitions.¹

**Rights system** A rights system $\mathcal{R}$ is an assignment of ordered pairs of alternatives to individuals. A rights system will be a set of ordered triples of the form $(x, y; b)$; if $(x, y; b) \in \mathcal{R}$ it will be said that $\mathcal{R}$ assigns $(x, y)$ to $b$.

**Rights implementation format** A right to $x$ over $y$ for $(x, y; b) \in \mathcal{R}$ is implemented if $y \notin C(S: x \in S, \{R_b\})$ whenever some condition, to be defined, relating $b, x$ and $y$ is fulfilled.

We now choose a specific rights system and specific rights implementation rule, and define a libertarian condition as the combination of the two. The following two definitions collectively define Gibbard’s libertarian condition $L$, the one he postulates ‘most naturally’ follows from the issue framework developed.

**Standard rights system (SRS)** For every individual $b$ there is at least one private issue $j$ such that, for every pair of alternatives $x$ and $y$,

$$\forall i (i \neq j): x_i = y_i, \text{then } (x, y; b) \in \mathcal{R}$$

Pairs of alternatives $\{x, y\}$ such that $\forall i (i \neq j): x_i = y_i$, will be termed ‘$j$-variants’. Note that we have assumed the number of issues exceeds the number of individuals.

**Standard rights implementation rule (SRI)** If $(x, y; b) \in \mathcal{R}$ and $xP_b y$ then $y \notin C(S: x \in S, \{R_b\})$.

¹ This distinction is similar to, but not the same as, the distinction Gibbard (1974) makes between the assignment of rights and the exercising of rights.
2.2 A framework for rights in social choice


Note that, given this condition, choice must now take account of certain preferences of individuals.

This condition is not meant to summarise all that a libertarian thinks should hold in society, but only one aspect of what he believes should hold. Indeed, many non-libertarians might agree on such a condition.

Let us consider an example that will clarify the above definitions and that will illustrate the apparent attractiveness of this formulation of rights. Let there be a two person society. Let person 1 be called Wales and person 2 be called Scotland. Let there be four alternatives, each with two feature alternatives.

\[ D_1D_2 \] (Wales devolved from the United Kingdom, Scotland devolved from the United Kingdom)

\[ D_1N_2 \] (Wales devolved from the United Kingdom, Scotland not devolved from the United Kingdom)

\[ N_1N_2 \] (Wales not devolved from the United Kingdom, Scotland not devolved from the United Kingdom)

\[ N_1D_2 \] (Wales not devolved from the United Kingdom, Scotland devolved from the United Kingdom)

This example fits well into the issue framework. Wales is assigned the issue of Welsh devolution (issue 1) and Scotland is assigned the issue of Scottish devolution (issue 2). For each issue, \( j = 1, 2 \), there are two feature alternatives, devolution and staying in the United Kingdom. Rights are assigned according to SRS on the basis that if two alternatives differ only with respect to one party’s issue \( j \), then that party should be assigned this pair (i.e. over \( j \)-variants). Thus the rights system is:

\[ \langle D_1D_2, N_1D_2; Wales \rangle \quad \langle D_1D_2, D_1N_2; Scotland \rangle \]

\[ \langle N_1D_2, D_1D_2; Wales \rangle \quad \langle D_1N_2, D_1D_2; Scotland \rangle \]

\[ \langle D_1N_2, N_1N_2; Wales \rangle \quad \langle N_1D_2, N_1N_2; Scotland \rangle \]

\[ \langle N_1N_2, D_1N_2; Wales \rangle \quad \langle N_1N_2, N_1D_2; Scotland \rangle \]

The standard rights implementation rule implies that if Wales prefers \( x \) to \( y \), where \( x \) is a 1-variant of \( y \) then \( y \) should not be chosen when \( x \) is available. Similarly, if Scotland prefers \( x \) to \( y \) where \( x \) is a 2-variant of \( y \), then \( y \) should not be chosen when \( x \) is available.

However, Gibbard has shown that this apparently appealing rights system and the ‘natural’ rights implementation rule results in an empty choice set for some logically possible preferences of society.
2. The Gibbard paradox

Theorem 2.2.1 (see Gibbard (1974), Theorem 1) No choice function $C(S, \{R_b\})$, satisfies condition $L$.

The proof may be illustrated with reference to the above example. Let the preferences of the two parties (Wales and Scotland) be:

<table>
<thead>
<tr>
<th>Wales</th>
<th>Scotland</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1D_2$</td>
<td>$D_1D_2$</td>
</tr>
<tr>
<td>$D_1N_2$</td>
<td>$D_1N_2$</td>
</tr>
<tr>
<td>$N_1N_2$</td>
<td>$N_1D_2$</td>
</tr>
</tbody>
</table>

Descending order of strict preference

According to SRS and SRI, and thus condition $L$:

1. Wales should dictate that $N_1N_2$ and $D_1D_2$ should not be chosen.
2. Scotland should dictate that $D_1N_2$ and $N_1D_2$ should not be chosen.

Thus no alternative in the set $\{N_1N_2, N_1D_2, D_1D_2, D_1N_2\}$ should be chosen.

This impossibility result is often called the ‘Gibbard paradox’ after Farrell (1976).

2.3 Respecting rights consistently

Condition $L$ is the combination of SRS and SRI. These two aspects of condition $L$ help us to classify two broad approaches to solving the Gibbard paradox. The first, which will be classified under the heading ‘rights system modification’, solves the paradox by weakening the specification of SRS. The second, classified under the heading ‘rights implementation modification’, solves the paradox by weakening the specification of SRI.

2.3.1 Rights system modification

The standard rights system, which Gibbard uses, is very strong. Consider the following three aspects:

1. Rights are assigned with respect to issues.
2. All individuals are allocated at least one personal issue.
3. Each individual, given an issue $i$, is assigned all pairs that differ with respect to their $j$th feature, i.e. an individual $b$ allocated an issue $i$ is assigned all $i$-variants.

The paradox is robust to a significant weakening of aspects (2) and (3). It may be the case that in some specific examples not all individuals should be allocated an issue. However, there need be only two individuals that are allocated issues for the impossibility result to go through, as should be clear from
2.3 Respecting rights consistently

the preceding section. It may also be the case that an individual, given an issue \( j \), should not have rights over all \( j \)-variants. For example, a person \( b \)'s issue \( j \) may be ‘volume of his record player’ and \( M_j = \{\text{very noisy, noisy, quiet, very quiet}\} \). Though, everything else equal, the choice between ‘quiet’ and ‘very quiet’ should be up to the individual, there might be some concern (e.g. from a next-door neighbour) if he is given rights to choose, everything else equal, between ‘very noisy’ and ‘noisy’. However, two individuals who are allocated rights with respect to just two feature alternatives in their respective \( M_j \) is sufficient for the impossibility result to go through — as should again be clear from the preceding section. Further weakening of aspects (2) and (3) would simply weaken condition \( L \) into being an uninteresting condition in social choice.

But what about aspect (1)? In the devolution example cited earlier the issue framework does seem the appropriate way to formulate a rights assignment. Are all case examples so well suited to an issue framework? Consider the following example taken from Barnes (1980).

There are two individuals, Mr Box and Mr Cox, who are the sole members of the ‘Tolpuddle Philanthropical Association’. Mr Cox, who is steward of the wine cellar, is decisive when it comes to choosing wines; and Mr Box, being secretary for local charities, is decisive in matters of making donations to worthy Tolpuddle causes. At the annual general meeting the association must decide how to allocate its few remaining funds. Four proposals are listed on the agenda:

A That monies be donated to the Tolpuddle Martyrs memorial fund.
B That monies be donated to the Tolpuddle Arts Council.
C That a good bottle of malmsey be bought and drunk.
D That a good bottle of Quininto du Noval be bought and drunk.

‘Natural’ considerations of rights would construct the following rights system for this committee:

\[
\langle A, B; \text{Mr Box} \rangle \quad \langle D, C; \text{Mr Cox} \rangle \\
\langle B, A; \text{Mr Box} \rangle \quad \langle C, D; \text{Mr Cox} \rangle
\]

It should be clear that in this case, and conceivably many others, the natural assignment of rights and the definition of alternatives do not fit well into the issue framework as formalised by Gibbard. There is no possibility of forming the notion of \( j \)-variants in this case. Note also that in the above case the rights system, together with SRI, will not lead to an empty choice set for any logically possible set of preferences of the two individuals.

Thus in some cases at least, a solution to the Gibbard paradox can be well constructed by not defining a rights system with respect to issues but over the social alternatives themselves. This is the approach that Sen (1970a, b) uses in
2. The Gibbard Paradox

Constructing his libertarian condition. Consider the rights system that Sen adopts.1

Sen’s rights system (SENRS) For each person \( b \) there is at least one pair of social alternatives \( x \) and \( y \) such that \((x, y; b)\) and \((y, x; b)\) are in \( R \).

SENRS together with SRI becomes Sen’s ‘liberalism’ condition:

Condition \( L^S \) (Sen’s liberalism condition, Sen (1970a, b)). SENRS and SRI respectively exist and operate in society.

By assigning rights directly over pairs of social alternatives, rather than indirectly via issues, a greater degree of freedom is gained in the specification of the rights system. With the definition of SENRS no stipulation is made over the relationship between the alternatives people are decisive over. Thus we can assume, to take an unnecessarily restrictive example, that no alternative in a pair over which a person \( b \) is decisive is in a pair of alternatives over which another person \( c \) is decisive. It follows that condition \( L^S \), with any configuration of preferences, cannot dismiss all the alternatives from the choice set.

In fact, there are many types of rights assignments, consistent with SENRS, that will guarantee possibility. All we need to be careful about is that they are ‘coherent’.

Coherent rights assignment (Suzumura (1978), p. 331, and Sen (1976), p. 243) Each person \( b \) is assigned pairs of alternatives \( \{x, y\} \) such that no matter how they order them there is a full ordering \( R \) of \( X \) of which each \( b \)’s preference over each respective \( \{x, y\} \) is a subrelation.

Suzumura (1978) proves that coherent rights assignments are necessary and sufficient for the resolution of the Gibbard paradox, given SRI.1 The

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1 Sen does not construct his libertarian condition specifically to avoid the Gibbard paradox, but to illustrate the paradox of the ‘Paretian—Liberal’. What is important here is that his rights system can be seen explicitly to avoid the Gibbard paradox. Sen’s result can thus be seen as a conflict between a consistent formulation of rights and the Pareto principle. Note also that it is not necessary to use either the SRS or SENS but we can construct a rights system which is a mixture of the two, e.g. one person’s rights may be allocated to be consistent with SENRS only, and another allocated rights also consistent with SRS. Possibility may still be assured so long as only one person is allocated rights that are also consistent with SRS.