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THE ROTATION OF THE EARTH
A GEOPHYSICAL DISCUSSION

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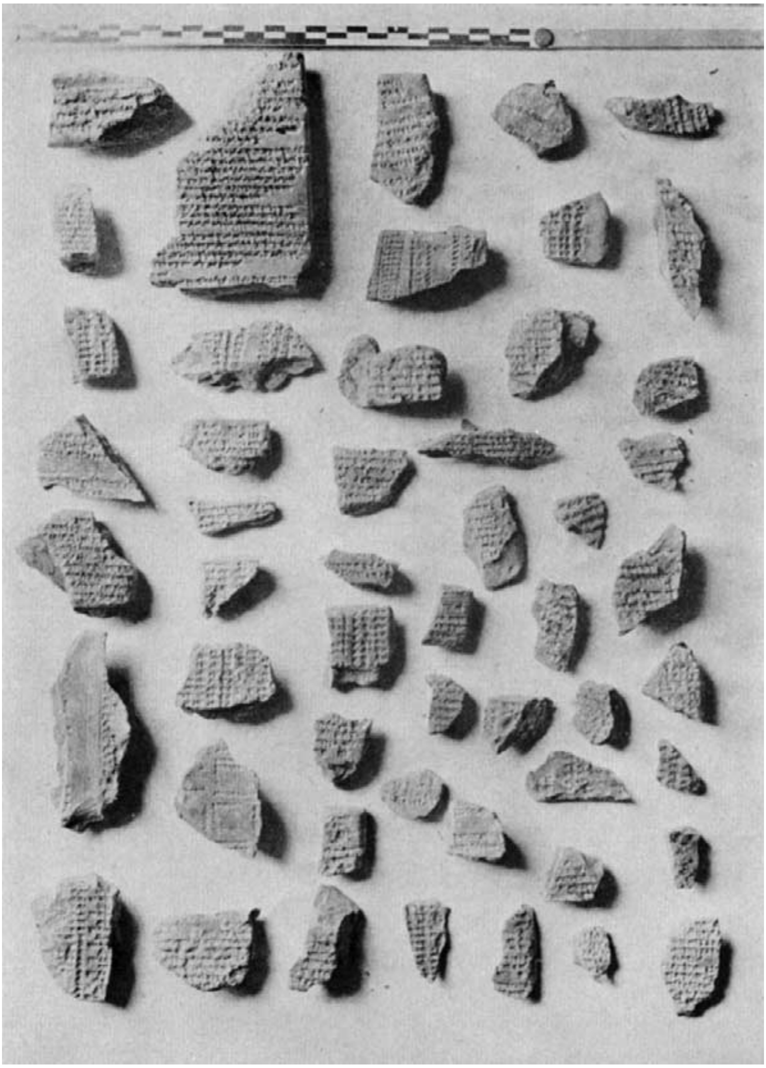

Tidal friction (see Chapter 11). From Neugebauer (1957), Plate 6: Warka fragments.

THE ROTATION OF THE EARTH

A GEOPHYSICAL DISCUSSION

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TO

SIR HAROLD JEFFREYS

Cambridge University Press
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Walter H. Munk and Gordon J. F. MacDonald
Frontmatter
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PREFACE

'Disturbed by Newcomb's suspicions of the Earth's irregularities as a time-keeper, I could think of nothing but precession and nutation, and tides and monsoons, and settlements of the equatorial regions, and meltings of the polar ice' (Thomson, 1876). Lord Kelvin had just returned from a visit abroad to deliver the Presidential Address before the British Association. He was scheduled to speak on recent scientific progress in America. Instead he devoted his talk entirely to the rotation of the Earth. For us, too, the subject has been irresistible.

This book is an account of certain irregularities in the rotation of the Earth which are not ordinarily included in the gravitational theory. Such irregularities are a nuisance to the astronomer. They complicate his time-scale and limit the accuracy with which he can predict eclipses and other events. He has now circumvented the problem by legislation: by redefining time in terms of the length of year rather than the length of day.

It has become increasingly clear that irregularities in rotation are largely caused by events on, and in, the Earth; conversely, that such events can be effectively studied by means of the measured irregularities. It is the purpose of this book to make this method of study readily accessible to the geophysicist, and so make a geophysical asset out of an astronomical nuisance.

The astronomers were the first to attempt geophysical interpretations of the irregularities they had discovered. The earliest discussions usually appear in the form of brief geophysical supplements to the astronomical papers, and they reflect the faith that the relative simplicity of celestial mechanics can be carried over into the interpretation. Irregularities are fitted by a few sine waves, one speaks of the 'proper motion' of observatories, and the Himalayan complex is suddenly raised a foot. Unfortunately, the terrestrial mechanics is more involved than the celestial mechanics: 'Es sind . . . nicht mehr die einfachen Verhältnisse der Himmelsmechanik massgebend, sondern wir befinden uns hier bereits auf dem verschlungenen Gebiete der Geophysik' (Klein and Sommerfeld, 1903).

The subject-matter appears to have been fashionable during the late nineteenth century and was treated at length by Routh, George Darwin, Kelvin, and other great Victorians. During that era physicists were still aware of their external surroundings and had not yet become obsessed with the atom. The last part of Thomson and Tait's *Treatise on Natural Philosophy* contains the first systematic account. The subject was reopened in the light of modern geophysical knowledge by Jeffreys. His contributions dominate the subject.

At the moment the astronomical evidence poses a dozen related problems; these are summarized in ch. 1. Seven have been examined with some degree of success only during the last ten years, and new evidence has turned up for two others. Several problems considered as solved in the 1920's are now wide open. The entire subject needs to be tidied. In doing so we do not imply that the rate of progress in this field has leveled off. To the contrary, we expect new problems to be posed and solved problems to be unsolved at an accelerating rate. The introduction of the cesium frequency standard into the Time Service late in 1955 has already left its mark; moon cameras, satellites and computers add a new dimension.

The diversity of the subject is appalling. It touches on every branch of geophysics. By the time it is covered, information will have been gained concerning wind and air masses, atmospheric, oceanic and bodily tides, sea level, rigidity and anelasticity of the Earth's mantle, and motion in its fluid core. In each case the information is limited to certain integral quantities taken over the entire globe. This is the weakness of the method—and its strength. In principle such integrated quantities can be evaluated by appropriate summations of individual station values. For competitive accuracy one usually finds that the stations are too unevenly distributed, and too few. This is true now; we doubt whether it will ever be any different.

Here we have attempted to give only as much information about astronomical instruments and methods as is required for an intelligent use of the data. With regard to geodynamics, we have stated assumptions, sketched the derivations, and given the formulae for the actual calculations. Cited references may be helpful in further developments of the theory. The geophysical discussion is intended for a reader without special training in the various branches of this science. Errors and omissions will undoubtedly be found. We would appreciate hearing about them.

It is a pleasure to record our gratitude to those who have made helpful comments; among them F. Birch, H. Bondi, G. Clemence, C. Eckart, W. Elsasser, F. Gilbert, R. Haubrich, W. Markowitz, R. Revelle, L. Slichter and H. Urey. We are grateful to Gretchen Chambers and Janice Von Herzen for the preparation of the manuscript. Elizabeth Strong has taken a most active part in all phases of the investigations leading to this book. Our research (without which there would have been no incentive to prepare this book) has been generously supported by the Office of Naval Research, the National Science Foundation, the Guggenheim Foundation, and the Institute of Geophysics, University of California. Contrast this with the earliest American work pertinent to the subject: transit observations in an observatory surreptitiously smuggled into the Naval Depot of Charts and Instruments (Dupree, 1957, p. 62).

NOTATION

Only the symbols that appear throughout the book are here identified; a notation that is used in one section only is defined locally. Several symbols have different meanings in different chapters. This is impossible to avoid in a subject covering dynamics, geophysics, and astronomy. When a choice had to be made between an established convention and some degree of ambiguity, our decision was with convention.

Cartesian tensors are employed; subscripts i, j, k, l, m, are reserved for this purpose. Thus m_i designated m_1, m_2, m_3. But U_n is a potential of degree n, and *not* a tensor; nor is the load deformation $\boldsymbol{\psi}_L$, nor the cross-correlation R_{uv}. Bold type indicates complex numbers, $\mathbf{m} = m_1 + im_2$. Some parameters appear in dimensional, dimensionless, complex, and operational forms: e.g., $\tilde{\mu}$ is the (dimensional) rigidity, μ a dimensionless rigidity, $\boldsymbol{\mu} = \mu_{\text{real}} + i\mu_{\text{imaginary}}$ a complex rigidity, and $\hat{\mu}$ a rigidity operator.

A, B, C	the principal moments of inertia; $C = 8{\cdot}068 \times 10^{44}$ g cm^2, $C - A = C - B = 2{\cdot}6 \times 10^{42}$ g cm^2
BIH	Bureau International de L'Heure (§ 9.1)
C_{ij}	inertia tensor (3.1.4)
$\hat{D}$	the operator $\mathrm{d}/\mathrm{d}t$
E	energy
ET	Ephemeris Time (§ 8.2, 11.2)
G	gravitational constant, $6{\cdot}670 \times 10^{-8}$ cm^3 g^{-1} sec^{-2}
GET	Newcomb's Great Empirical Term (§ 11.2)
H	precessional constant, $3{\cdot}273 \times 10^{-3}$ (2.3.1); also magnetic field strength
H_i	absolute angular momentum (3.1.3)
$H(x)$	Heaviside step function
IRM	isothermal remnant magnetization (§ 12.2)
ILS	International Latitude Service
K	kinetic energy
$K_☾$	general lunar coefficient (7.4.3)
K–V	Kelvin–Voigt body (§ 5.11)
L_i	torque

$L_{\odot}$, $L_{☾}$	L_3 due to Sun and Moon, respectively
$\Delta L_{\odot}$, $\Delta L_{☾}$, $\Delta L_{☿}$	longitude discrepancies of Sun, Moon, and Mercury (11.2.1, 11.2.3)
M, $M_{\oplus}$	mass of Earth, 5·976 × 10^{27} g
M	abbreviation for Maxwell body (§ 5.11)
NRM	natural remnant magnetization (§ 12.2)
O	order symbol; $y = 0(x)$ implies $\lim_{x\to 0} \lvert y/x \rvert < \infty$
PZT	photographic zenith tube
Q	(§ 4.3)
R	autocorrelation (§ A.2)
S	surface; dS surface element; also the power spectrum (§ A.2)
S_n	surface spherical harmonic of degree n (5.10.1)
T	length of record; also $ET_{\odot}$, the Ephemeris Time determined by the Sun
TRM	thermal remnant magnetization (§ 12.2)
U_i	velocity vector relating to non-rotating coordinates
UT	Universal Time (§ 8.2, 11.2)
U_n, V_n, W_n	spherical solid harmonic of degree n
U, V, W	spherical solid harmonic of degree 2
V	volume; dV volume element
WWD	weighted discrepancy difference (11.2.4)
X_i	non-rotating coordinate
a	radius of Earth, 6·371 × 10^8 cm
a_{ij}	anelastic strain (§ 4.1)
a_n^m, b_n^m	spherical harmonic coefficients of degree n and order m
$a_{☾}$, $b_{☾}$, $c_{☾}$	terms in the longitude of the Moon; similarly for Sun and Mercury (11.2.3, 11.2.6)
b	tidal amplitude factor (7.4.2)
$\mathbf{c}_n^m$	complex spherical harmonic coefficient $a_n^m + ib_n^m$
c_{ij}	perturbations in inertia tensor (6.1.1)
c/s, c/year	cycles per second, cycles per year
d_{ij}	total rate of strain (§ 4.1)
f	frequency in cycles per unit time; as subscript, referring to 'fluid' (§. 5.4)

f_c	Coriolis frequency, $2\Omega \cos \theta$
f_0	resonance (or Chandler) frequency (1/14) cycles per month
f_N	Nyquist frequency (A.2.15)
$f_☾, f_\odot, f_☿$	discrepancies in the longitudes of the Moon, Sun, and Mercury (11.2.3, 11.2.4)
g	acceleration of gravity, 980 g cm^{-2}
g_{ij}	metric strain tensor of material in the relaxed state (§ 4·1)
h, h'	Love numbers
h_i	relative angular momentum
i	imaginary unit
i, j, k, l, m	tensor subscript indices
k, l, k', l'	Love numbers
l.o.d.	length of day
m	a mass
m_i	direction cosines of rotation-axis
$\binom{m}{n}$	indicating order m and degree n of special harmonic
$n_☾, n_\odot, n_☿$	orbital velocities of Moon, Sun, and Mercury
n_i	outwardly directed normal to a surface element, dS
p_n^m	spherical harmonic of order m and degree n (A.1.1)
p	hydrostatic pressure
p_{ij}	stress tensor
q	surface load, g cm^{-2}; also the angular velocity of the Earth relative to the 'mean Sun', 15° per mean solar hour
r	distance from center of Earth
$r(\tau)$	lag window (§ A.2)
s	$ds = \sin \theta \, d\theta \, d\lambda$ is surface element on unit sphere; also frequency divided by resonance frequency
s_{ij}	frictional stress
t	time
Δt	time interval between tabulated (or observed) values; also $ET_\odot - UT$ (11.2.2)
u_i	velocity vector relative to rotating coordinates
x_i	rotating coordinates
Γ	length of the mean solar day, 86,400 sec

Ψ_i	modified excitation function (6.3.3)
Ω	mean diurnal rotation, $7{\cdot}292 \times 10^{-5}$ radians $\sec^{-1}$
α, β, γ	damping factors
$\beta(\lambda, t)$	tidal phase (7.4.2)
δ_{ij}	Kroenecker delta or substitution tensor (§ 3.1)
$\delta(x)$	Dirac delta function
ε	ellipticity
ε_{ij}	elastic strain
ε_{ijk}	alternating tensor (3.1.2)
η	dynamic viscosity
θ	colatitude
ι	isostatic factor
κ	transfer function (§ 6.4); also electrical conductivity
λ	east longitude; also Lamé's constant
μ	rigidity
ν	kinematic viscosity; also degress of freedom (A.2.8)
ξ	elevation of sea surface above mean sea level
ρ	density; $\bar{\rho} = 5{\cdot}53$ g cm^{-3} is mean density of Earth, $\rho_w = 1{\cdot}025$ g cm^{-3} density of sea water
σ	frequency in radians per unit time
σ_0	resonance (or Chandler) frequency, $(2\pi/14)$ radians per month
σ_r	resonance (or Eulerian) frequency, $(2\pi/10)$ radians per month
τ	a time constant; also the amount by which the Earth is slow
τ_0	finite strength
τ_{ij}	elastic stress
υ	surface tension
ϕ_i	excitation function (§ 6.1, 6.3)
$\psi_i, \psi_{i(L)}, \psi_{i(D)}$	excitation functions for rigid Earth, load and rotational deformations (§ 6.3)
ω_i	angular velocity
$\mathscr{C}$	'global' functions defined in (A.1)
☾ ☉ ☿ ⊕	subscripts referring to Moon, Sun, Mercury, and Earth

☾	in cos ☾, sin ☾: the longitude of the 'mean Moon', (table 7.4.1)
⊙	in cos ⊙, sin ⊙: the longitude of the 'mean Sun' measured from the beginning of the year (not from 21 March)
$\langle x \rangle$	time average of x