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Walter H. Munk and Gordon J. F. MacDonald
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CHAPTER 1

PREVIEW

A preview of the treatment might be helpful. Following a qualitative discussion of the irregularities in rotation (ch. 2), the following four chapters are devoted to fundamentals. The solution to any problem must satisfy (1) the equations governing the dynamics of rotating bodies and (2) the equations governing the appropriate stress–strain relations. In ch. 3 the dynamic equations are presented in a form sufficiently general to impose no restrictions on deformation. The stress–strain relations are discussed in ch. 4. In most problems the stress–strain relations can be introduced in the form of dimensionless parameters, the Love numbers (ch. 5). Perturbation methods are given in ch. 6. For a reading of the subsequent part of this book, dealing with the observations and their interpretation, the chapters on fundamentals may not be prerequisite.

The remaining chapters are devoted to a discussion of irregularities in the rotation of the Earth. The irregularities fall into two categories: (1) a wobble of the Earth and (2) changes in the rate of rotation or, more simply, changes in the length of day (l.o.d.). The evidence with regard to wobble is outlined in the upper half of fig. 1.1, with regard to l.o.d. in the lower half. It has been customary to discuss problems involving wobble separately from those involving the l.o.d., possibly because the methods of observation are so different. But the two subjects are closely related, and much is gained when they are treated side by side.

The methods of observation are dealt with in chs. 7 and 8. The remaining chapters present a discussion of the irregularities organized according to frequency. Ch. 9 deals with irregularities with a time-scale on the order of one year or less. Evidence for the wobble comes from observations by the International Latitude Service (ch. 7), for the l.o.d. from a comparison of astronomic time with precise clock time (ch. 8). The annual wobble is largely due to seasonal shifts in air mass. The annual variation in the l.o.d. is caused by winds, shorter period terms by bodily tides. Ch. 10

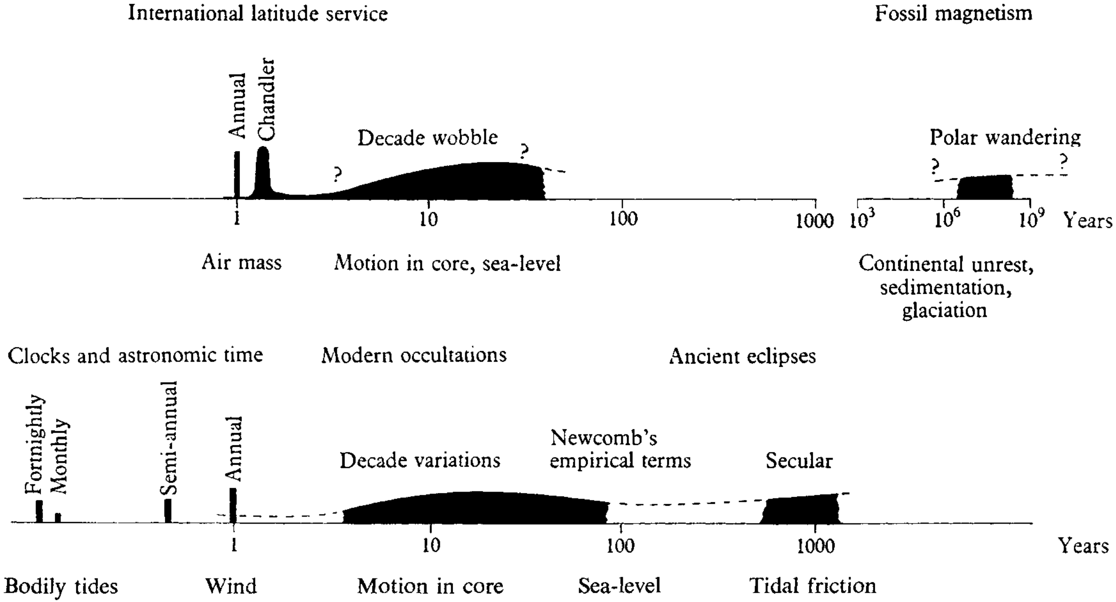

Fig. 1.1. The spectrum of rotation. The *wobble* components (*top*) and *length of day* components (*bottom*) are schematically arranged according to their time-scale in years. Vertical lines indicate discrete frequencies, shaded portions indicate a continuous, or noisy, spectrum. Principal source of the observations is shown above lines, presumable geophysical cause beneath lines.

deals with the 14-month Chandler wobble, the period of which is governed by the ellipticity and rigidity of the Earth; the wobble is generated by random impulses of unknown origin and damped by some unknown imperfections from elasticity, or by some other means.

The longer periods are discussed in ch. 11. The evidence from latitude observations is inconclusive. Evidence for the l.o.d. comes largely from modern occultations and ancient eclipses. Remarkably large irregular variations in the l.o.d. with a decade time-scale may be due to electromagnetic coupling of the mantle to a turbulent, fluid core. Fluctuations with a century time-scale (Newcomb's empirical terms) may be due to changes in the Earth's moment of inertia. Changes over the last few thousand years are predominantly the result of tidal friction, but here again changes in inertia (presumably associated with a variable sea-level) must play an important part.

Ch. 12 deals with wobble on a geologic time-scale. Paleomagnetism and other indirect evidence is sometimes interpreted to indicate very large displacements of the pole during the last few hundred million years. Vertical unrest of continents and convective motion in the Earth's mantle have been suggested as causes. For a discussion of this problem the dynamics has to be extended to the case of an anelastic Earth.

The clearcut distinction made in the literature between the regular variations of short period (annual, semi-annual etc.) and the irregular variations of long period is not suitable from the geophysical point of view. The short-period spectral 'lines' must be imbedded in a continuous spectrum caused by weather; long-period spectral lines, such as the one associated with the 18.6-year tide, must be superimposed on the continuous spectrum of the variations. The concept of the 'power spectrum' is central to our discussion. A brief account of the method is given in the Appendix.

The literature on the subject is enormous, but there appears to have been no previous attempt to give a unified account of the spectrum of both wobble and rotation. The first systematic account of the wobble problem appeared in the *Treatise on Natural Philosophy* (Thomson and Tait, 1883) and in the *Theorie des Kreisels* (Klein and

Sommerfeld, 1903). Jeffreys's (1959) book *The Earth* contains a summary of short-period wobble in ch. 7, and of the effect of tidal friction on long-period changes in the l.o.d. in ch. 8. Essentially the same subject-matter is covered in ch. 16 and by ch. 6 of vol. II, *Physics of the Earth Series* (Lambert, 1931; Lambert *et al.* 1931). Changes in the l.o.d. of all periods are discussed by Spencer-Jones (1956) in the *Handbuch der Physik* and in ch. 1 of *The Earth as a Planet* (Kuiper, 1954), and by Gondolatsch (1953). Melchior (1957) has given the most up-to-date summary of the wobble problem.

CHAPTER 2

PRECESSION, NUTATION AND WOBBLE

1. Wobble and precession

Consider time-exposures of stars with a camera pointing vertically upward, that is, opposite to local gravity.* On the photographs the star trails appear as portions of concentric circles. For two positions on the Earth, the *poles of rotation*, the concentric circles would be centered on the photographs. The *celestial poles* vertically above the poles of rotation are those two points in the sky where a star would have no diurnal motion. (Polaris is near the celestial north pole.) The *rotation axis* (or *celestial axis*) extends through the poles of rotation from one celestial pole to the other.

If the positions are checked after a month, it will be found that the poles of rotation have moved a few feet from their previous positions. By the time a year has passed, the poles will have moved in a roughly elliptical path with a mean diameter of twenty feet.

A stake is driven in the ground somewhere near the center of the ellipse. It marks the *pole of reference*. The *axis of reference* extends from the center of the Earth through this pole. The pole of rotation revolves about the pole of reference. With respect to an observer on a fixed star, the rotation axis remains fixed, and the reference pole (averaged for diurnal motion) revolves about the rotation pole.

In the discussion so far the celestial pole has been considered as fixed in its position near Polaris.† But over a period of many years the celestial pole shifts appreciably. In 5000 years it will be near α Cephei; 5000 years ago it was near α Draconis and the southern cross could be seen in England. These changes in the orientation of the rotation axis *in space* are associated with the precession of the equinox. There are also shorter-period changes in the position of the celestial pole, the forced nutations. Such changes in the orientation of the Earth's rotation axis in space are altogether different from the wobble of the Earth relative to this axis.

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* The small effect of tidal deflexions of the local vertical is discussed in § 7.4.
† Neglecting 'sway,' § 6.6.

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It would be unfortunate if wobble and precession could be detected only by an observer at the pole. In practice the measurements of appropriate angles at any latitude will do. The angles which are to be determined are the declination of a star and the latitude of a place. Fig. 2.1 shows the situation for the idealized cases of wobble only, and precession (or forced nutation) only. The celestial pole P is near Polaris; S is some star, and its polar distance or co-declination (90°—declination) is SOP; A is some fixed place on Earth and ZA is the direction of local gravity. The colatitude (90°—latitude) of A is defined by the angle ZOP between the celestial pole and the zenith. The left figure illustrates precession. The celestial pole has shifted away from Polaris. This changes the declination but not the latitude. The right figure illustrates wobble. The pole of reference M has shifted to the left of the pole of rotation R. The declination is unchanged, but now the latitude is altered. Hence precession is determined from declination; wobble is determined from latitude. A brief account of the instruments and methods actually used is given in ch. 7.

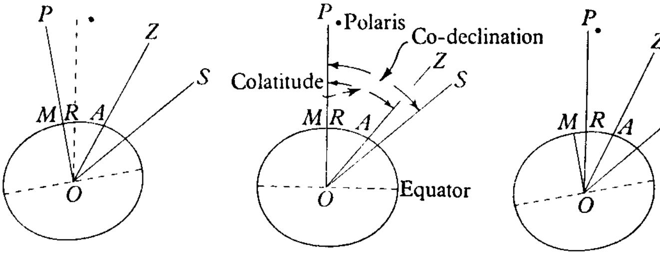

Fig. 2.1. Precession (*left*) and wobble (*right*). The undisturbed position is shown at the center. The cut is at right angles with the equatorial plane and through the Earth's center O. S is some star. A is some fixed place and Z its zenith. P is the north celestial pole. M the pole of reference and R the pole of rotation.

2. Causes of precession and forced nutation

Changes of the Earth's rotation axis in space are caused largely by the pull of the Moon and Sun on the Earth's equatorial bulge. There would be no such effect if the Earth were spherical; nor if the equatorial plane were coincident with the plane of the Sun's orbit (the ecliptic plane) and the plane of the Moon's orbit. Actually the equatorial plane is inclined by 23½° to the ecliptic, and the inclination to the Moon's orbit is not far different. If it were not for the Earth's rotation, the action of Sun and Moon would be to bring these planes into coincidence. But because of the gyroscopic effect of rotation the action is at right-angles to what one would otherwise expect. The obliquity of the ecliptic remains near 23½°, and the celestial pole describes a circle about the pole of the ecliptic in 26,000 years, a motion known as the precession of the equinox. From the observed precession and the mass of the Moon one can evaluate the precessional constant

$$H = \frac{C - A}{C} = 0{\cdot}00327293 \pm 0{\cdot}00000075, \qquad (2.2.1)$$

where A, A, C are the moments of inertia about the Earth's principal axes, C being taken about the axis of greatest moment (Jeffreys, 1952, p. 145).

There is a slight additional precession due to other planets. The complex interplay of the Sun's and Moon's orbits is associated with oscillations of shorter periods, among them the 19-yearly lunar nutation. The shorter periods involve chiefly a motion of the celestial pole to and from the ecliptic pole, a nodding which gives the motion its name, *nutation*.*

We shall not consider precession and forced nutation any further. They are treated in many textbooks (for example, Routh, 1905, ch. 11). But we do not wish to imply that there are no remaining problems of geophysical interest. For example, a genuine discrepancy by one part in 600 between the observed amplitude of the 19-yearly nutation and that computed for a rigid Earth has been explained by allowing for the fluidity of the core together with certain modifications in the elastic properties of the mantle (Jeffreys and Vicente, 1957*a*, *b*).

3. Wobble and length of day

Rotation can be represented by a vector drawn parallel to the rotation axis and proportional in length to the rate of rotation. Relative to a reference system suitably attached to the Earth (§ 3.2),

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* The 'forced nutations' are not to be confused with the Chandler wobble (ch. 10) which is often referred to as 'free nutation' or 'Eulerian nutation'.

the x_1, x_2-components then designate the wobble, and the x_3-component (drawn roughly parallel to the reference axis) is associated with the length of the sidereal day. Wobble can be expressed either as an angular displacement of the rotation axis relative to the axis of reference, or as a linear displacement of the pole of rotation relative to the pole of reference. For comparison,

$$0''.0100 \quad \text{and} \quad 1{\cdot}01 \text{ ft} \tag{2.3.1}$$

are equivalent, a relationship that makes it convenient to use centiseconds of arc and feet of displacement as interchangeable units.

The sidereal day is defined as the time-interval between successive transits of a star across the celestial meridian (a great circle through the celestial poles and the zenith). Measurements of the l.o.d. require a good clock in addition to a proper telescope. Measurements of wobble require only the determination of an angle. In principle, measurements of l.o.d. and wobble are quite independent, but in practice the segregation is not so clear-cut (7.4.8, 8.3.5).

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CHAPTER 3

DYNAMICS*

1. Fundamental equations

The Eulerian equations of motion in a coordinate system x_i ($i = 1, 2, 3$), rotating with angular velocity ω_i relative to coordinates X_i fixed in space and coinciding with x_i for the moment, are

$$L_i = \frac{\mathrm{d}H_i}{\mathrm{d}t} + \varepsilon_{ijk}\omega_j H_k. \tag{3.1.1}$$

L_i are the components of torque, H_i those of angular momentum. ε_{ijk} is the 'alternating' tensor, defined by the following properties:

$$\left.\begin{aligned} \varepsilon_{ijk} &= 0 \text{ if any two subscripts are equal,} & & i = j,\ i = k,\ j = k, \\ &= +1 \text{ if subscripts are in even order,} & & 1, 2, 3, 1, 2\ .\ .\ ., \\ &= -1 \text{ if subscripts are in odd order,} & & 1, 3, 2, 1, 3\ .\ .\ .\ . \end{aligned}\right\} \tag{3.1.2}$$

By the usual summation convention, in any expression containing a repeated suffix, that suffix is to be given all possible values and the results then added.

Equations (3.1.1) are quite general. They refer, for example, to a system of particles moving among themselves. It is convenient to separate angular momentum into two parts:

$$H_i = C_{ij}(t)\omega_j + h_i(t), \tag{3.1.3}$$

where

$$C_{ij} = \int_V \rho(x_k x_k \delta_{ij} - x_i x_j)\,\mathrm{d}V \tag{3.1.4}$$

is the (variable) tensor of inertia for matter contained in a volume V, and δ_{ij} is the Kronecker delta (or substitution tensor), with the properties $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ if $i \neq j$. The second part of (3.1.3) designates a relative angular momentum

$$h_i = \int_V \rho\varepsilon_{ijk} x_j u_k\,\mathrm{d}V \tag{3.1.5}$$

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* For a more complete treatment the reader is referred to the classic works of v. Oppolzer (1886), Tisserand (1891), and Klein and Sommerfeld (1903). A treatment by Woolard (1953) has been found helpful.

due to motion u_i relative to the x_i-system. Substituting (3.1.3) into (3.1.1) leads to

$$L_i = \frac{\mathrm{d}}{\mathrm{d}t}(C_{ij}\omega_j + h_i) + \varepsilon_{ijk}\omega_j(C_{kl}\omega_l + h_k). \qquad (3.1.6)$$

All subsequent inquiries concerning irregularities in rotation take the form of special solutions to (3.1.6). This equation was given by Liouville in 1858 (Routh, 1905: § 22), and it will be referred to as the Liouville equation.

2. Frames of reference

Routh (1905, § 22) and other textbooks distinguish between the rotating reference-axes in the Eulerian equations (3.1.1) and the 'body axes' from which changes in the body can be described (3.1.4, 5). The two rotating axes can be combined without loss of generality, as we have done, at the expense (possibly) of not making the best use of the symmetry of the situation.

The choice of the rotating x_i-system is altogether arbitrary. It could, for example, rotate at some rate in a sense opposite to the Earth's rotation. For convenience the coordinate axes must now be attached to the Earth in some way. In most papers on the subject the coordinate system is said to rotate 'with the Earth'. If the Earth were rigid, there would be no further difficulty. Winds, ocean currents, and the fluid core introduce complications. To get around this difficulty the axes can be attached to the 'solid' Earth. However, there are tidal distortions and, for processes on a geologic time-scale, convective motion in the mantle has been postulated; in all events we are faced with relative motion of different parts of the crust. Such motion is known to take place along geologic faults and has been postulated by Wegener on a grandiose scale as a drift of continents with respect to one another.

Clearly we require a set of rigid axes that are kinematically defined so as not to impose any restraints on the deforming Earth. There are a number of possible choices.

(1) *Tisserand's mean axes of body* are defined so that $h_i = 0$. Thus if winds, ocean currents, and all other relative motion ceased, these axes would rotate with the resulting frozen body. An alternate