

Cambridge University Press

978-0-521-05092-0 - The Distribution of the Galaxies: Gravitational Clustering in Cosmology

William C. Saslaw

Frontmatter

[More information](#)

## THE DISTRIBUTION OF THE GALAXIES

This topical volume examines one of the leading problems in astronomy today – how galaxies cluster in our Universe. Many observational surveys and theoretical projects are currently underway to understand the distribution of galaxies. This is the first book to describe gravitational theory, computer simulations, and observations related to galaxy distribution functions (a general method for measuring the clustering and velocities of galaxies). It embeds distribution functions in a broader astronomical context, including other exciting contemporary topics such as correlation functions, fractals, bound clusters, topology, percolation, and minimal spanning trees.

Key results are derived and the necessary gravitational physics provided to ensure the book is self-contained. And throughout the book, theory, computer simulation, and observation are carefully interwoven and critically compared. The book also shows how future observations can test the theoretical models for the evolution of galaxy clustering at earlier times in our Universe.

This clear and authoritative volume is written at a level suitable for graduate students and will be of key interest to astronomers, cosmologists, physicists, and applied statisticians.

William C. Saslaw is professor of astronomy at the University of Virginia, Charlottesville and also does research at the National Radio Astronomy Observatory and at the University of Cambridge. He received his Ph.D. in applied mathematics and theoretical physics from Cambridge. He is also the author of *Gravitational Physics of Stellar and Galactic Systems*.

Cambridge University Press

978-0-521-05092-0 - The Distribution of the Galaxies: Gravitational Clustering in Cosmology

William C. Saslaw

Frontmatter

[More information](#)

# THE DISTRIBUTION OF THE GALAXIES

## Gravitational Clustering in Cosmology

WILLIAM C. SASLAW

*University of Cambridge*

*University of Virginia*

*National Radio Astronomy Observatory, U.S.A.*



**CAMBRIDGE**  
**UNIVERSITY PRESS**

Cambridge University Press  
978-0-521-05092-0 - The Distribution of the Galaxies: Gravitational Clustering in Cosmology  
William C. Saslaw  
Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521394260](http://www.cambridge.org/9780521394260)

© Cambridge University Press 2000

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 2000  
This digitally printed version 2008

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Saslaw, William C.  
The distribution of the galaxies : gravitational clustering in  
cosmology / William C. Saslaw.  
p. cm.

Includes bibliographical references.  
ISBN 0-521-39426-0

1. Galaxies – Clusters. 2. Gravitation. I. Title.  
QB858.7.S28 1999

523.1'12 – dc21 99-14191  
CIP

ISBN 978-0-521-39426-0 hardback  
ISBN 978-0-521-05092-0 paperback

Contents

<i>Prologue</i>	<i>page ix</i>
<b>Part I: Historical</b>	<b>1</b>
1 Cosmogony Myths and Primitive Notions	3
2 First Qualitative Physics: The Newton–Bentley Exchange	11
3 Glimpses of Structure	17
4 Number Counts and Distributions	26
5 Seeds of Grand Creation	33
6 Clusters versus Correlations	41
7 The Expanding Search for Homogeneity	48
<b>Part II: Descriptions of Clustering</b>	<b>55</b>
8 Patterns and Illusions	57
9 Percolation	69
10 Minimal Spanning Trees	80
11 Topology	85
12 Fractals	91
13 Bound Clusters	100
13.1 Identification of Bound Clusters, the Virial Theorem, and Dark Matter	100
13.2 Some Observed Properties of Groups and Clusters	107
13.3 Physical Processes in Bound Clusters	112
14 Correlation Functions	121
14.1 Definitions	121
14.2 Properties	128
14.3 Measuring $\xi(r)$ and $W(\theta)$	133
14.4 Origin and Evolution of Correlation Functions	136
15 Distribution Functions	141
15.1 Definitions	141
15.2 Theoretical Results	143

vi	<i>Contents</i>	
15.3	Numerical Simulations	154
15.4	Observed Galaxy Spatial Distribution Functions	160
15.5	Observed Galaxy Velocity Distribution Functions	163
<b>Part III: Gravity and Correlation Functions</b>		169
16	The Growth of Correlations: I. A Fluid Description	170
16.1	Introduction	170
16.2	The Cosmological Background	173
16.3	Linear Fluid Perturbations and Correlations	179
16.4	Other Types of Linear Fluid Analyses	185
17	The Growth of Correlations: II. A Particle Description	195
17.1	Introduction	195
17.2	Liouville's Equation and Entropy	196
17.3	The BBGKY Hierarchy	205
17.4	Gravitational Graininess Initiates Clustering	207
17.5	Growth of the Two-Galaxy Correlation Function	211
18	General Correlation Properties	216
18.1	Scaling	216
18.2	Real Space and Redshift Space	219
18.3	Bias	221
18.4	A Relation Among the Amplitude, Range, and Slope of $\xi$	223
19	Computer Simulations	227
19.1	Direct Methods	227
19.2	Other Methods	230
20	Simulations and Observations of Two-Particle Correlations	234
20.1	Simulations	234
20.2	Observations	242
<b>Part IV: Gravity and Distribution Functions</b>		247
21	General Properties of Distribution Functions	249
21.1	Discrete and Continuous Distributions	249
21.2	Expectations, Moments, and Cumulants	251
21.3	Generating and Characteristic Functions	254
21.4	Convolutions, Combinations, and Compounding	258
21.5	Infinite Divisibility	261
21.6	Relation to Correlation Functions	262
22	Dynamics of Distribution Functions	264
22.1	Introduction	264
22.2	The Cosmic Energy Equation	268
22.3	Dynamical Implications of the Cosmic Energy Equation	273

	<i>Contents</i>	vii
23	Short Review of Basic Thermodynamics	278
23.1	Concepts	278
23.2	Interrelations	282
23.3	Connections with Kinetic Theory and Statistical Mechanics	286
23.4	The Three Laws of Thermodynamics	290
23.5	Fluctuations and Ensembles	294
23.6	Phase Transitions	298
24	Thermodynamics and Gravity	301
25	Thermodynamic Formulation of the Cosmological Many-Body Problem	306
25.1	Expansion Removes the Mean Gravitational Field from Local Dynamics	306
25.2	Extensivity and Gravity	308
25.3	The Energy Equation of State	309
25.4	The Pressure Equation of State	311
25.5	Dynamical Derivation of the Equations of State	315
25.6	Physical Conditions for Quasi-Equilibrium	317
26	The Functional Form of $b(\bar{n}, T)$	319
26.1	$b(\bar{n}, T) = b(\bar{n}T^{-3})$	319
26.2	The Specific Function $b(\bar{n}T^{-3})$	320
26.3	Minimal Clustering	325
27	Derivation of the Spatial Distribution Function	327
27.1	Entropy and Chemical Potential	327
27.2	The Gravitational Quasi-Equilibrium Distribution $f(N)$	329
28	Properties of the Spatial Gravitational Quasi-Equilibrium Distribution	333
28.1	Physical Limiting Cases and Self-Organized Criticality	333
28.2	Normalizations, Underdense Regions, and the Shape of the GQED	335
28.3	Specific Heats, Compressibility, and Instability	343
28.4	Fluctuations	345
28.5	Projection	346
28.6	Random Selection	346
28.7	Recovery of $\xi(r)$ from $b_V$	348
28.8	The GQED Generating Function	348
28.9	Scaling and Moments	353
28.10	Bias and Selection	355
28.11	The Multiplicity Function and Related Interpretations	361
28.12	Relation to Multifractals	364
29	The Velocity Distribution Function	366
30	Evolution of the GQED	372

viii	<i>Contents</i>	
30.1	Evolution of $b(t)$	372
30.2	Evolution of Energy, Entropy, and Specific Heat	377
30.3	Evolution of Correlations	381
	<b>Part V: Computer Experiments for Distribution Functions</b>	387
31	Spatial Distribution Functions	389
31.1	Poisson Initial Conditions	389
31.2	Scale Dependence of $b$	403
31.3	Evolution of $b$ and Effects of $\Omega_0$	408
31.4	Non-Poisson Initial Conditions	411
31.5	Models with Dark Matter	413
32	Velocity Distribution Functions	416
	<b>Part VI: Observations of Distribution Functions</b>	427
33	Observed Spatial Distribution Functions	429
33.1	Basic Questions	429
33.2	Catalogs: A Brief Sketch	430
33.3	Partial Answers to Basic Questions	435
34	Observed Peculiar Velocity Distribution Functions	452
35	Observed Evolution of Distribution Functions	459
	<b>Part VII: Future Unfoldings</b>	463
36	Galaxy Merging	465
37	Dark Matter Again	473
38	Initial States	477
39	Ultimate Fates	479
40	Epilogue	484
	<i>Bibliography</i>	485
	<i>Index</i>	503

## Prologue

Despite appearances, it is not the Epilogue, but the Prologue that is often left for last. Only after seeing what is done, can one acknowledge and apologize. My main acknowledgments are to many students and collaborators, for they have taught me much. My apologies are to those colleagues who may not find enough of their own results in the pages still ahead. For them I can only echo Longfellow that “Art is long and Time is fleeting.” The subject of large-scale structure in the universe, of which the distribution of the galaxies represents only a part, has burgeoned beyond all previous bounds as the new millennium approaches. Driven as much by the scope and depth of its questions as by new streams of data from the depths of time, there is an increasing excitement that fundamental answers are almost in reach. And there will be no stopping until they are found.

On the timescales of the physical processes we are about to consider, millennia count for very little. But on the timescale of our own understanding, years, decades, and certainly centuries have changed the whole conceptual structure surrounding our views. This may happen again when the role of dark matter becomes more transparent.

Meanwhile, this monograph is really no more than an extended essay on aspects of galaxy clustering that I’ve found especially interesting. It emphasizes galaxy distribution and correlation functions – but mostly distribution functions because correlations have already been discussed widely elsewhere. Besides, distribution functions contain more convenient descriptions of more information.

Both these statistics are embedded here in a broader context, but their main virtue is that, of all descriptions so far, correlations and distributions can be related most directly to physical theories of gravitational clustering. Even for gravitational clustering, I have emphasized the simplest and most fundamental problem: how gravitating point masses cluster self-consistently within the background of an expanding universe. Three general approaches – statistical thermodynamics, computer  $N$ -body experiments, and astronomical observations – all give consistent results, and all are discussed here in detail. The observational agreement suggests that this cosmological many-body problem will remain a useful and basic aspect for understanding galaxy clustering, whatever detailed scenario produced it. And we really need to understand clustering by the known gravitational forces of the galaxies alone, before adding more speculative contributions.



The cosmological many-body problem, moreover, is endlessly fascinating in its own right. Much has been learned about it since Newton's first qualitative discussion, but much more remains for future discovery. Part of this fascination comes from its inherent and essential nonlinearity. Its statistical thermodynamics hints at a deeper theory. Computer simulations can check these theories and prevent them from going too far astray. We have just begun to explore the theory of systems with ranges of masses and different initial conditions. Important new problems are easy to find.

To begin, I have sketched the historical background, from Babylonian myths until 1970. Our subject started slowly and for centuries lay well outside the mainstreams of astronomy. Nevertheless it made quiet progress, and I've selected some milestones with the hindsight of history. These lead, in Part II, to a brief general review of the main descriptions of galaxy clustering; each has its weakness and all have some virtue. Thus the first third of the book provides an overall introduction.

Next, the general theme of gravity takes over. Part III discusses its relation to correlation functions in the context of the cosmological many-body problem and reviews several topics of recent interest along with a sketch of computer simulation techniques and results. This ends with a brief description of observations. Naturally there is some repetition of the earlier introduction. Although most discussions are self-contained, my earlier book *Gravitational Physics of Stellar and Galactic Systems* (GPSGS) sometimes extends them.

In the book's second half, I discuss distribution functions for galaxy positions and peculiar velocities and how they evolve. These are the generalizations, for the cosmological many-body system, of the Poisson and Maxwell–Boltzmann distributions in a perfect gas.

Distribution functions may be less familiar than correlations and other descriptions of clustering. So I've started by summarizing their mathematical properties that are especially useful for our exploration. Then the cosmic energy equation provides a dynamical link to the cosmological distributions. Like most complex dynamics, this link is easier to follow in the linear regime of fairly weak clustering. To examine the observed range of nonlinear clustering, it helps to develop the statistical thermodynamic theory. After reviewing thermodynamics, we apply it to derive the spatial and velocity distribution functions of the cosmological many-body problem. Then we follow their quasi-equilibrium evolution as the universe expands. There are no free parameters in this theory – after all it's just gravity.

Initially the applicability of gravitational thermodynamics to the cosmological many-body problem was rather surprising. To paraphrase Mark Twain, it gratified some astrophysicists and astonished the rest. This apparent impasse arose because thermodynamics is essentially an equilibrium theory, whereas gravitational clustering is a manifestly nonequilibrium phenomenon. What this seeming contradiction failed to appreciate, however, is that under a wide range of conditions, cosmological many-body clustering can evolve through a sequence of equilibrium states. This quasi-equilibrium evolution enables thermodynamics to provide a very good approximation.

Cambridge University Press

978-0-521-05092-0 - The Distribution of the Galaxies: Gravitational Clustering in Cosmology

William C. Saslaw

Frontmatter

[More information](#)*Prologue*

xi

Computer  $N$ -body experiments, which directly integrate the mutual orbits of many galaxies in an expanding universe, show that gravitational thermodynamics does indeed apply for a variety of initial conditions in different universes. Part V describes these tests. They also determine the conditions under which gravitational thermodynamics fails to give a good description of clustering and the reasons for this failure. We still need more diverse computer experiments to explore the whole range of the theory.

Naturally, the grandest experiment of all is the analog computer in the sky. In our own Universe, the initial conditions and detailed evolution of galaxy clustering remain controversial and highly uncertain. Here the cosmological many-body problem is perhaps the simplest model of this process. It assumes, in this application, that when the galaxies clustered most of the dark matter that affected their orbits was associated with the individual galaxies, either inside them or in halos around them. Thus galaxies acting effectively as point masses dominated the clustering. Other types of models are dominated by vast quantities of unobserved dark matter whose growing large-scale inhomogeneities determine the clustering, and galaxies merely go along for the ride. More than thirty years of increasingly ingenious and sensitive searching have failed to reveal the specific forms of dark matter these other models require.

As a model for galaxy clustering, the cosmological many-body theory describes the observed galaxy distribution functions remarkably well, as Part VI discusses. This suggests that the clustering effects of intergalactic dark matter are small, or that they have contrived to mimic many-body clustering. Consistency between models and observations is a good sign, but it is not proof. History has shown that our Universe is complex, and so a final decision here will have to await further developments.

Part VII introduces some aspects of clustering that may unfold in the future. These generally involve more complicated modeling than simple gravitational clustering, but such models are necessary to understand many detailed astrophysical consequences, including galaxy formation itself. No one doubts a connection between galaxy formation and clustering, but the nature and strength of this link is still so uncertain that at present I think it wise – or at least expedient – to consider clustering as a separate problem. The formation of galaxies (not a particularly well-defined process) sets the initial conditions for their clustering, which I assume are subsumed by those studied here. If not, we will have to modify them. Optimistically there is hope that eventually the fluctuations of the cosmic microwave background will lead to a clear solution of the origins of clustering.

The work of determining galaxy distribution functions from observations, numerical simulations, and gravitational theory in which I've participated has benefitted greatly from discussions with dozens of astronomers, physicists, and mathematicians. Most of the results have come from many collaborations over the years and over the world. Although some collaborators were officially called students, they quickly outgrew any secondary role and became equal participants. In a young subject, everyone rapidly reaches frontiers. It is adventurous and all great fun. For these

Cambridge University Press

978-0-521-05092-0 - The Distribution of the Galaxies: Gravitational Clustering in Cosmology

William C. Saslaw

Frontmatter

[More information](#)

collaborations, I especially thank Sverre Aarseth, H. M. Antia, Kumar Chitre, Paul Coleman, Phil Crane, Naresh Dadhich, James Edgar, Fan Fang, Andrew Hamilton, Shirin Haque-Copliah, Shogo Inagaki, Makoto Itoh, Sanjay Jain, Leo Krzewina, Ofer Lahav, Sunil Maharaj, Hojung Mo, Somak Raychaudhury, Yoel Rephaeli, Ravi Sheth, Trinh Thuan, and David Valls-Gabaud. In addition, I am happy to thank Ian Du Quesnay for helpful advice on the Greek philosophers; Arthur Haberman, E. R. Harrison, Ofer Lahav, Morton Roberts, and Mark Whittle for their detailed comments on substantial parts of the manuscript; and especially David Valls-Gabaud for reading and commenting on the entire manuscript. In the University of Cambridge I am very grateful to Pat Cassidy at Jesus College and to Judith Moss at the Institute of Astronomy for their exceptionally fine work, which converted the manuscript with its occasionally complicated equations into a form directly suitable for the printers. And both Jesus College and the Institute of Astronomy provided, as they have for many years, conditions especially conducive to creative research. I am glad to thank George Kessler at NRAO for producing many diagrams with an ideal blend of modern electronics and old-fashioned skill. Richard Sword's enthusiastic and interactive artistry designed the book's cover. It has also been a pleasure, as I expected it would be, to work with Simon Mitton, Adam Black, and the staff of the Cambridge University Press.