THE EXPANDING UNIVERSE

by

SIR ARTHUR EDDINGTON

The right of the University of Cambridge to print and sell all manner of books was granted by Henry VIII in 1534. The University has printed and published continuously since 1584.

CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York New Rochelle Melbourne Sydney

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
32 East 57th Street, New York, NY 10022, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1933
Reprinted 1933, 1944, 1946, 1952
Reissued in the Cambridge Science Classics series 1987

Library of Congress Cataloging-in-Publication Data

Eddington, Arthur Stanley, Sir, 1882–1944.

The expanding universe.

(Cambridge science classics)
Reprint. Originally published:
Cambridge University Press, 1933.
Includes index.
1. Expanding universe. I. Title II. Series.
QB991.E94E33 1987 523.1'8 87-23918

ISBN 0-521-34976-1 paperback

British Library Cataloging in Publication Data applied for.

Transferred to digital printing 2002

CONTENTS

THE EXPANDING UNIVERSE

Chapter I

THE RECESSION OF THE GALAXIES

Pricked out with less and greater lights, between the poles of the universe, the Milky Way so gleameth white as to set very sages questioning.
Dante, *Paradiso*

I

THE first hint of an "expanding universe" is contained in a paper published in November 1917 by Prof. W. de Sitter. Einstein's general theory of relativity had been published two years before, but it had not yet attained notoriety; it was not until the eclipse expeditions of 1919 obtained confirmation of its prediction of the bending of light that public interest was aroused. Meanwhile many investigators had been examining the various consequences of the new theory. Prominent among them was de Sitter who was interested especially in the astronomical consequences. In the course of a highly technical discussion he found that the relativity theory led to an expectation that *the most remote celestial objects would be moving away from us*, or at least that they would deceive the observer into thinking that they were moving away.

De Sitter was perhaps a tipster rather than a prophet. He would not promise anything definitely; but he suggested that we ought to keep a look out for the recession as a rather likely phenomenon. Theory was at the cross-roads, and desired guidance from observation as to which of two possible courses should be pursued. If astronomers were to find a general motion of recession of the most distant objects visible, it would be a strong indication that the road rather fancied by de Sitter was the one to follow. If not, the inference was more doubtful; it might mean that the other road should be followed, or it might only mean that our astronomical survey had not yet been extended to sufficient distance.

Subsequent researches in the field opened up by de Sitter's pioneer investigation have developed and modified his theory. A new point of view has been discovered which renders the results less paradoxical than they appeared originally. We are still led to expect a recession of remote objects, though the recession now predicted is not the original de Sitter effect, which has turned out to be of minor importance. It varies with the distance according to a different law. Moreover, it is a genuine receding motion of remote objects, whereas the phenomenon predicted by de Sitter might be regarded as an imitation recession, and generally was so regarded.

We shall put aside theory for the present, and consider first what astronomical observation tells us. Practically all that I have to relate has been discovered since de Sitter's forecast, much of it within the last four years. These observational results are in some ways so disturbing that there is a natural hesitation in accepting them at their face value. But they have not come upon us like a bolt from the blue, since theorists for the last fifteen years have been half expecting that a study of the most remote objects of the universe might yield a rather sensational development.

The spiral nebulae are the most remote objects known. Rough measurements of their distances have been made, and we place them from 1 million to 150 million light years away; they doubtless extend far beyond the latter distance, but at present it is the limit of our survey. The name "nebula" is applied to different classes of astronomical objects which have nothing in common except a cloudy appearance. There are *gaseous nebulae*, shown by their spectrum to be extremely rarefied gas, either attached to and controlled by a single star or spreading irregularly through a region containing many stars; these are not particularly remote. The *spiral nebulae* on the other hand are extra-galactic objects; that is to say, they lie beyond the limits of the Milky Way aggregation of stars which is the system to which our sun belongs, and are separated from it by wide gulfs of empty space. When we have taken together the sun and all the naked-eye stars and many hundreds of millions of telescopic stars, we have not reached the end of things; we have explored only one island—one oasis in the desert of space. Other islands lie beyond. It is possible with the naked eye to make out a hazy patch of light in the constellation Andromeda which is one of the other islands. A telescope shows many more—an archipelago of island galaxies stretching away one behind another until our sight fails. It is these island galaxies which appear to us as spiral nebulae.

Each island system is believed to be an aggregation of thousands of millions of stars with a general resemblance to our own Milky Way system. As in our own system there may be along with the stars great tracts of nebulosity, sometimes luminous, sometimes dark and obscuring. Many of the nearest systems are seen to have a beautiful double-spiral form (see Frontispiece); and it is believed that the coils of the Milky Way would give the same spiral appearance to our own system if it were viewed from outside. The term "spiral nebula" is, however, to be regarded as a name rather than a description, for it is generally applied to all external galaxies whether they show traces of spiral structure or not.

The island systems are exceedingly numerous. From sample counts it is estimated that more than a million of them are within reach of our present telescopes. If the theory treated in this book is to be trusted, the total number of them must be of the order 100,000,000,000.

In order to fix in our minds the vastness of the system that we shall have to consider, I will give you a "celestial multiplication table". We start with a star as the unit most familiar to us, a globe comparable to the sun. Then—

> A hundred thousand million Stars make one Galaxy;
>
> A hundred thousand million Galaxies make one Universe.

These figures may not be very trustworthy, but I think they give a correct impression.

The lesson of humility has so often been brought home to us in astronomy that we almost automatically adopt the view that our own galaxy is not specially distinguished—not more important in the scheme of nature than the millions of other island galaxies. But astronomical observation scarcely seems to bear this out. According to the present measurements the spiral nebulae, though bearing a general resemblance to our Milky Way system, are distinctly smaller. It has been said that if the spiral nebulae are islands, our own galaxy is a continent. I suppose that my humility has become a middle-class pride, for I rather dislike the imputation that we belong to the aristocracy of the universe. The earth is a middle-class planet, not a giant like Jupiter, nor yet one of the smaller vermin like the minor planets. The sun is a middling sort of star, not a giant like Capella but well above the lowest classes. So it seems wrong that we should happen to belong to an altogether exceptional galaxy. Frankly I do not believe it; it would be too much of a coincidence. I think that this relation of the Milky Way to the other galaxies is a subject on which more light will be thrown by further observational research, and that ultimately we shall find that there are many galaxies of a size equal to and surpassing our own. Meanwhile the question does not much affect the present discussion. If we are in a privileged position, we shall not presume upon it.

I promised to leave aside theory for the present, but I must revert to it for a moment to try to focus our conception of this super-system of galaxies. It is a vista not only of space but of time. A faint cluster of nebulae in Gemini, which at present marks the limit of our soundings of space, takes us back 150 million years into the past—to the time when the light now reaching us started on its journey across the gulf of space. Thus we can scarcely isolate the thought of vast extension from the thought of time and change; and the problem of form and organisation becomes merged in the problem of origin and development. We must, I suppose, imagine the island galaxies to have been formed by gradual condensation of primordial matter. Perhaps in the first stage only the rudiments of matter existed—protons and electrons traversing the void—and the evolution of the elements has progressed simultaneously with the evolution of worlds. Slight condensations occurring here and there by accident would by their gravitating power draw more particles to themselves. Some would quickly disperse again, but some would become firmly established—

> Champions fierce,
> Strive here for mastery, and to battle bring
> Their embryon atoms....To whom these most adhere,
> He rules a moment: Chaos umpire sits,
> And by decision more embroils the fray
> By which he reigns: next him, high arbiter,
> Chance governs all.*

By such conflict the matter of the universe would slowly be collected into islands, leaving comparatively empty spaces from which it had been drained away. We think that one of these original islands has become our Milky Way system, having subdivided again and again into millions of stars. The other islands similarly developed into galaxies, which we see to-day shining as spiral nebulae. It is to these prime units of subdivision of the material universe that our discussion here will relate.

* *Paradise Lost*, Book II.

II

If a spiral nebula is not too faint it is possible to measure its radial velocity in the line of sight by measuring the shift of the lines in its spectrum. A valuable early series of such determinations was made by Prof. V. M. Slipher at the Lowell Observatory.

More recently the distances of some of the spiral nebulae have been determined by a fairly trustworthy method. In the nearest spirals it is possible to make out some of the individual stars; but only the most luminous stars, some hundreds or thousands of times brighter than the sun, can be seen at so great a distance. Fortunately among the very brightest of the stars there is a particularly useful class called the Cepheid variables. They vary periodically in brightness owing to an actual pulsation or physical change of the star, the period being anything from a few hours to a few weeks. It has been ascertained from observational study that Cepheids which have the same period are nearly alike in their other properties—luminosity, radius, spectral type, etc. The period is thus a badge, easily recognisable at a distance, which labels the star as having a particular luminosity. For example, if the star is seen to have a period of 10 days, we immediately recognise it as a star of luminosity 950 times greater than the sun. Knowing then its real brightness we put the question, How far off must it be situated so as to be reduced to the faint point of light which we see? The answer gives the distance of the star and of the galaxy in which it lies. This method uses the Cepheid variables as standard candles. If you see a standard candle anywhere and note how bright it appears to you, you can calculate how far off it is; in the same way an astronomer observes his "standard candle" in the midst of a nebula, notes its apparent brightness or magnitude, and deduces the distance of the nebula.

Dr E. P. Hubble at Mount Wilson Observatory was able to discover Cepheid variables in two or three of the nearest spiral nebulae, and so obtained the first real measurement of their distances. Unfortunately this method is not available for the more distant galaxies, and he has had to use more indirect devices for extending the survey. I think that, apart from those distances actually determined by the Cepheid method, we must regard the distances assigned to the spiral nebulae as rather risky estimates; but there is reason to believe that they cannot be entirely misleading, and we shall provisionally accept them here.

When the collected data as to radial velocities and distances are examined a very interesting feature is revealed. The velocities are large, generally very much larger than ordinary stellar velocities. The more distant nebulae have the bigger velocities; the results seem to agree very well with a linear law of increase, the velocity being simply proportional to the distance. The most striking feature is that the galaxies are almost unanimously running away from us.

Let us consider especially the last result and state the observational evidence in more detail. The light of the spiral nebulae, being compounded of the light of a great variety of stars, does not give a good spectrum for measurement. For this reason and because of its faintness the deduced velocities are inaccurate as judged by ordinary standards; but, except for the nearest nebulae, the velocities are themselves so enormous that the error of measurement is comparatively unimportant. Taking the results as published, the present position is that line-of-sight velocities of about 90 galaxies have been measured, and of these only five are moving towards us. At first sight it may seem wrong to pass over the minority as insignificant. But the five exceptions are confined to the very nearest of the nebulae, and their approaching velocities are not large. Since the phenomenon is one which depends on distance (the effect increasing with distance), it is natural that we should have to go out to a fair distance before we find it strong enough to prevail over all other effects (including observational error) so as to display itself uniformly. The five approaching velocities are at least partly attributable to the use of an inappropriate standard of reference. Line-of-sight velocities as published are relative to the sun; but it would be more satisfactory to discuss the velocities relative to our Milky Way system as a whole. It has been found that the sun is pursuing an orbit round the centre of the Milky Way system and has an orbital speed from 200 to 300 kilometres per second. When we correct for this so as to obtain the velocities referred to our galaxy as a whole, the approaching velocities are reduced or disappear. I think it will turn out ultimately that, after all corrections are applied, these nearest nebulae have small receding velocities; for the existence of even one genuine exception would be difficult to explain.

In saying that the speeds of the nebulae are large, the velocities of ordinary stars are our standard of comparison. For stars in our neighbourhood the individual speed averages 10 to 50 km. per sec. If the speed exceeds 100 km. per sec. the star is described as a "runaway". (We do not here include the abovementioned orbital motion about the centre of the galaxy which is shared by all stars in the neighbourhood of the sun.) Slipher's first determination of the radial velocities of 40 nebulae included a dozen with velocities from 800–1800 km. per sec. The survey has since been extended to fainter and more distant nebulae by M. L. Humason at Mount Wilson Observatory, and much higher velocities have been found. The speed record is continually being broken. The present holder of the trophy is a nebula forming one of a faint cluster in the constellation Gemini, which is receding with a velocity of 25,000 km. per sec. (15,000 miles per second). This is about the speed of an Alpha particle. Its distance is estimated at 150,000,000 light-years. Doubtless a faster and more distant nebula will have been announced by the time these words are in print.

The simple proportionality of speed to distance was first found by Hubble in 1929. This law is also predicted by relativity theory. According to the original investigation of de Sitter a velocity proportional to the square of the distance would have been expected; but the theory had become better understood since then, and it was already known (though perhaps only to a few*) that simple proportionality to the distance was the correct theoretical result.

* I was not myself aware of it in 1929. For the nature of the change, see p. 49.

According to Hubble's most recent determination, the speed of recession amounts to 550 km. per sec. per megaparsec.* That is to say, a nebula at 1 megaparsec distance should have a speed 550 km. per sec.; at 10 megaparsecs distance, 5500 km. per sec.; and so on. It has been claimed that this determination is accurate to 20 per cent., but I do not think many astronomers take so optimistic a view. The uncertainty lies almost entirely in the scale of nebular distances; there are weak links in the long chain of connection between these vast distances and our terrestrial standard metre. Corrections which have been suggested mostly tend to increase the result; and perhaps the fairest statement is that the velocity of recession is probably between 500 and 1000 km. per sec. per mp.

Specimens of the spectra from which these radial velocities are obtained are shown in Plate II. In the lower four photographs the spectra of the nebulae are the torpedo-shaped black patches; they have terrestrial comparison spectra above and below, which are used to place them in correct vertical alignment. Practically the only recognisable features in the nebular spectra are the *H* and *K* lines—two interruptions in the tail of the torpedo where it is fading away. It will be seen that these interruptions move to the right, i.e. to the red end of the spectrum, as we go down the plate. It is this displacement which is measured and gives the receding velocities stated at the foot of the plate.

* 1 megaparsec = 3·26 million light-years.

III

We can exclude the spiral nebulae which are more or less hesitating as to whether they shall leave us by drawing a sphere of rather more than a million light-years radius round our galaxy. *In the region beyond, more than* 80 *have been observed to be moving outwards, and not one has been found coming in to take their place.*

The inference is that in the course of time all the spiral nebulae will withdraw to a greater distance, evacuating the part of space that we now survey. Ultimately they will be out of reach of our telescopes unless telescopic power is increased to correspond. I find that the observer of nebulae will have to double the aperture of his telescope every 1300 million years merely to keep up with their recession. If we have been thinking that the human race has still billions of years before it in which to find out all that can be found out about the universe, we must count the problem of the spiral nebulae as one of urgency. Let us make haste to study them before they disappear into the distance!

The unanimity with which the galaxies are running away looks almost as though they had a pointed aversion to us. We wonder why we should be shunned as though our system were a plague spot in the universe. But that is too hasty an inference, and there is really no reason to think that the animus is especially directed against our galaxy. If this lecture-room were to expand to twice its present size, the seats all separating from each other in proportion, you would notice that everyone had moved away from you. Your neighbour who was 2 feet away is now 4 feet away; the man over yonder who was 40 feet away is now 80 feet away. It is not *you* they are avoiding; everyone is having the same experience. In a general dispersal or expansion every individual observes every other individual to be moving away from him. The law of a general uniform expansion is that each individual recedes from you at a rate proportional to his distance from you—precisely the law which we observe in the receding motions of the spiral nebulae.*

We shall therefore no longer regard the phenomenon as a movement away from our galaxy. It is a general scattering apart, having no particular centre of dispersal.

I do not wish to insist on these observational facts dogmatically. It is granted that there is a possibility of error and misinterpretation. The survey is just beginning, and things may appear in a different light as it proceeds. But if you ask what is the picture of the universe now in the minds of those who have been engaged in practical exploration of its large-scale features—men not likely to be moved overmuch by ideas of bending of space or the gauge-invariance of the Riemann-Christoffel tensor—I have given you their answer. Their picture is the picture of an *expanding universe*. The super-system of the galaxies is dispersing as a puff of smoke disperses. Sometimes I wonder whether there may not be a greater scale of existence of things, in which it *is* no more than a puff of smoke.

* Our observations determine the *relative* velocity of recession of a nebula, i.e. the rate at which its distance from us is increasing. They do not indicate whether the nebula is moving away from us or we are moving away from the nebula.

For the present I make no reference to any "expansion of space". I am speaking of nothing more recondite than the expansion or dispersal of a material system. Except for the large scale of the phenomenon the expansion of the universe is as commonplace as the expansion of a gas. But nevertheless it gives very serious food for thought. It is perhaps in keeping with the universal change we see around us that time should set a term even to the greatest system of all; but what is startling is the rate at which it is found to be melting away. We do not look for immutability, but we had certainly expected to find a permanence greater than that of terrestrial conditions. But it would almost seem that the earth alters less rapidly than the heavens. The galaxies separate to double their original distances in 1300 million years. That is only of the order of geological time; it is approximately the age assigned to the older rocks in the earth's crust. This is a rude awakening from our dream of leisured evolution through billions of years.*

Such a conclusion is not to be accepted lightly; and those who have cast about for some other interpretation of what seems to have been observed have displayed no more than a proper caution. If the apparent recession of the spiral nebulae is treated as an isolated discovery it is too slender a thread on which to hang far-reaching conclusions; we can only state the bare results of observation, contemplate without much conviction the amazing possibility they suggest, and await further information on the subject.

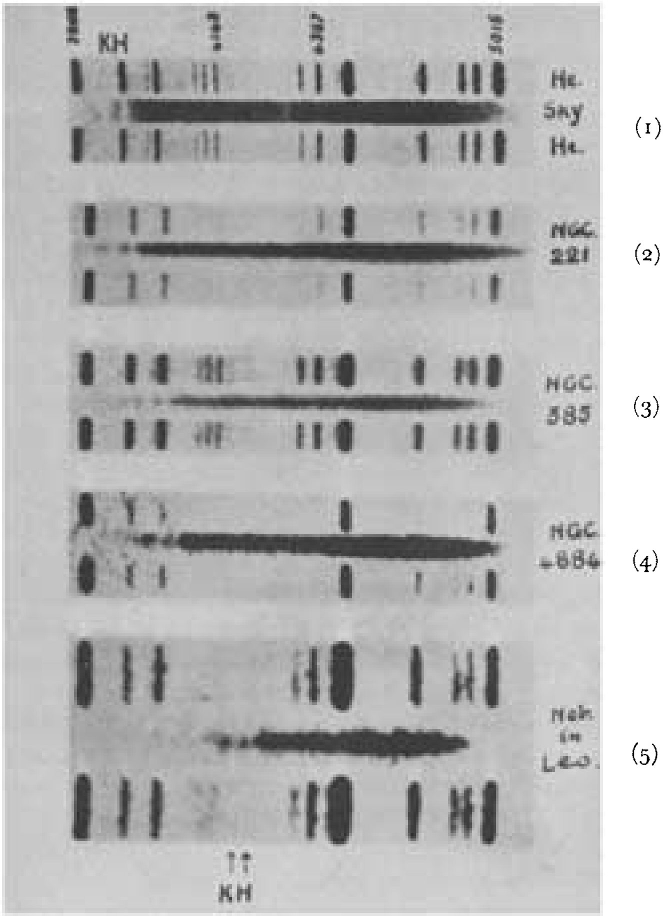
* I may remind American readers that the English billion is a million million.

If that is not my own attitude, it is because the motion of the remote nebulae does not present itself to me as an isolated discovery. Following de Sitter, I have for fifteen years been awaiting these observational results to see how far they would fall into line with and help to develop the physical theory, which though at first merely suggestive has become much more cogent in the intervening years. After Prof. Weyl's famous extension of the relativity theory I became convinced that the scale of structure of atoms and electrons is determined by the same physical agent that was concerned in de Sitter's prediction. So that hope of progress of a really fundamental kind in our understanding of electrons, protons and quanta is bound up with this investigation of the motions of remote galaxies. Therefore when Dr Hubble hands over a key which he has picked up in intergalactic space, I am not among those who are turning it over and over unable to decide from the look of it whether it is good metal or base metal. The question for me is, Will it unlock the door?

If the observed radial velocities are accepted as genuine, there is no evading the conclusion that the nebulae are rapidly dispersing. The velocities are direct evidence of a hustle which (according to the usual ideas of the rate of evolutionary change) is out of keeping with the character of our staid old universe. Thus the only way of avoiding a great upset of ideas would be to explain away these radial velocities as spurious. What is actually observed is a shifting of the spectrum of the nebula towards the red. Such a shift is commonly caused by the Doppler effect of a receding velocity, in the same way that the pitch of a receding whistle is lowered; but other causes are imaginable. The reddening signifies lower frequency of the light-waves and (in accordance with quantum theory) lower energy; so that if for any cause a light-quantum loses some of its energy in travelling to reach us, the reddening is accounted for without assuming any velocity of the source. For example, the light coming to us from an atom on the sun uses up some of its energy in escaping from the sun's gravitational attraction, and consequently becomes slightly reddened as compared with the light of a terrestrial atom which does not suffer this loss; this is the well-known red shift predicted by Einstein.

In one respect this hypothesis of the loss of energy of nebular light is attractive. If the loss occurs during the passage of the light from the nebula to the observer, we should expect it to be proportional to the distance; thus the red-shift, misinterpreted as a velocity, should be proportional to the distance—which is the law actually found. But on the other hand there is nothing in the existing theory of light (wave theory or quantum theory) which justifies the assumption of such a loss. We cannot without undue dogmatism exclude the possibility of modifications of the existing theory. Light is a queer thing—queerer than we imagined twenty years ago—but I should be surprised if it is as queer as all that.

A theory put forward by Dr Zwicky, that light, by its gravitational effects, parts with its energy to the material particles thinly strewn in intergalactic space which it passes on its way, at one time attracted atten-



Humason

SPECTRA OF NEBULAE

showing lines shifted to the red (to the right), interpreted as velocity of recession. (See p. 11.)

(1) Sky; velocity, nil. (2) N.G.C. 221; velocity, – 185 km. per sec. (3) N.G.C. 385; velocity, +4900 km. per sec. (4) N.G.C. 4884; velocity, +6700 km. per sec. (5) Nebula in Leo; velocity, +19,700 km. per sec.

tion. But the numerical accordance alleged to support his theory turned out to be fallacious, and the suggestion seems definitely untenable.

I think then we have no excuse for doubting the genuineness of the observed velocities—except in so far as they share the general uncertainty that surrounds all our attempts to probe into the secrets of nature.

IV

Now let us turn to theory.

A scientist commonly professes to base his beliefs on observations, not theories. Theories, it is said, are useful in suggesting new ideas and new lines of investigation for the experimenter; but "hard facts" are the only proper ground for conclusion. I have never come across anyone who carries this profession into practice—certainly not the hard-headed experimentalist, who is the more swayed by his theories because he is less accustomed to scrutinise them. Observation is not sufficient. We do not believe our eyes unless we are first convinced that what they appear to tell us is credible.

It is better to admit frankly that theory has, and is entitled to have, an important share in determining belief. For the reader resolved to eschew theory and admit only definite observational facts, *all* astronomical books are banned. *There are no purely observational facts about the heavenly bodies.* Astronomical measurements are, without exception, measurements of phenomena occurring in a terrestrial observatory or station; it is only by theory that they are translated into knowledge of a universe outside.

When an observer reports that he has discovered a new star in a certain position, he is probably unaware that he is going beyond the simple facts of observation. But he does not intend his announcement to be taken as a description of certain phenomena that have occurred in his observatory; he means that he has located a celestial body in a definite direction in interstellar space. He looks on the location as an observational fact—on a surer footing therefore than theoretical inferences such as have been deduced from Einstein's theory. We must break it to him that his supposed "fact", far from being purely observational, is actually an inference based on Einstein's theory—unless, indeed, he has based it on some earlier theory which is even more divorced from observational facts. The observer has given a theoretical interpretation to his measurements by assuming for theoretical reasons that light travels through interstellar space approximately in a straight line. Perhaps he will reply that, in assuming the rectilinear propagation of light, he is not concerned with any theory but is using a fact established by direct experiment. That begs the question how far an experiment under terrestrial conditions can be extrapolated to apply to interstellar space. Surely a reasoned theory is preferable to blind extrapolation. But indeed the observer is utterly mistaken in supposing that the straightness of rays of light assumed in astronomy has been verified by terrestrial experiment. If the rays in interstellar space were no straighter than they are on the earth,* the direction in which a star is seen would be no guide to its actual

* They are deflected by the earth's gravitational field.

position. Light would in fact curl round and come back again before traversing the distance to the nearest star.

Our warrant for concluding that the celestial body is nearly in the direction in which it is seen, is Einstein's theory, which determines the deviation of light from a straight line. Coupled with other theoretical deductions as to the density of matter in interstellar space, it allows us to conclude that the deviation in this case is inappreciable. So if we are willing to use both fact and theory as a basis for belief, we can accept the observer's announcement; but it is not a "hard fact of observation". Although it is a minor point, we may also insist that the theory concerned is Einstein's theory. There was an earlier theory according to which light in empty space travels in straight lines in all circumstances; but since this has been found experimentally to be untrue, it can scarcely be the basis of our observer's conclusion. Perhaps, however, the observer is one of those who do not credit the eclipse observations of the deflection of light, or who deem them insufficient ground for quitting the old theory. If so, he illustrates my dictum that with the hard-headed experimentalist the basis of belief is often theory rather than observation.

My point is that in astronomy it is not a question of whether we are to rely on observation or on theory. The so-called facts are in any case theoretical interpretations of the observations. The only question is, Shall we for this interpretation use the fullest resources of modern theory? For my own part I can see no more reason for preferring the theories of fifty years ago than for preferring the observational data of fifty years ago.

In turning now to the more theoretical side of the problem of the expanding universe, I do not think that we should feel that we are stepping from solid ground into insecurity. Perhaps we are a little safer, for we no longer depend on the interpretation of one type of observation; and our theory comes from the welding together of different lines of physical research. I do not, however, promise security. An explorer is jealous of his reputation for proper caution, but he can never aspire to the quintessence of caution displayed by the man who entrenches himself at home.

V

In 1915 Einstein had by his general theory of relativity brought a large section of the domain of physics into good order. The theory covered *field-physics*, which includes the treatment of matter, electricity, radiation, energy, etc., on the ordinary macroscopic scale perceptible to our senses, but not the phenomena arising from the minute subdivision into atoms, electrons, quanta. For the study of microscopic structure another great theory was being developed—the quantum theory. At that time it lagged far behind, and even now it has not reached the clearness and logical perfection of the relativity theory. It is recognised that the two theories will meet, and that they must ultimately coalesce into one comprehensive theory. The first bridge between them was made by Prof. P. A. M. Dirac in 1928 by his relativity wave-equation of an electron. I hope to show in the last chapter that the