

Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

---

Like the Earth and planets, stars rotate. Understanding how stars rotate is central to modeling their structure, formation, and evolution and how they interact with their environment and companion stars. This authoritative volume provides a lucid introduction to stellar rotation and the definitive reference to the subject. It combines theory and observation in a comprehensive survey of how the rotation of stars affects the structure and evolution of the Sun, single stars, and close binaries.

This timely book will be of primary interest to graduate students and researchers studying solar and stellar rotation and close binary systems. It will also appeal to those with a more general interest in solar and stellar physics, star formation, binary stars, and the hydrodynamics of rotating fluids – including geophysicists, planetary scientists, and plasma physicists.

Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

STELLAR ROTATION

Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

*Cambridge astrophysics series*

*Series editors*

Andrew King, Douglas Lin, Stephen Maran, Jim Pringle and Martin Ward

**Titles available in this series**

- 7. Spectroscopy of Astrophysical Plasmas  
*by A. Dalgarno and D. Layzer*
- 10. Quasar Astronomy  
*by D. W. Weedman*
- 17. Molecular Collisions in the Interstellar Medium  
*by D. Flower*
- 18. Plasma Loops in the Solar Corona  
*by R. J. Bray, L. E. Cram, C. J. Durrant and R. E. Loughhead*
- 19. Beams and Jets in Astrophysics  
*edited by P. A. Hughes*
- 20. The Observation and Analysis of Stellar Photospheres  
*by David F. Gray*
- 21. Accretion Power in Astrophysics 2nd Edition  
*by J. Frank, A. R. King and D. J. Raine*
- 22. Gamma-ray Astronomy 2nd Edition  
*by P. V. Ramana Murthy and A. W. Wolfendale*
- 23. The Solar Transition Region  
*by J. T. Mariska*
- 24. Solar and Stellar Activity Cycles  
*by Peter R. Wilson*
- 25. 3K: The Cosmic Microwave Background Radiation  
*by R. B. Partridge*
- 26. X-ray Binaries  
*by Walter H. G. Lewin, Jan van Paradijs and Edward P. J. van den Heuvel*
- 27. RR Lyrae Stars  
*by Horace A. Smith*
- 28. Cataclysmic Variable Stars  
*by Brian Warner*
- 29. The Magellanic Clouds  
*by Bengt E. Westerlund*
- 30. Globular Cluster Systems  
*by Keith M. Ashman and Stephen E. Zepf*
- 31. Pulsar Astronomy 2nd Edition  
*by Andrew G. Lyne and Francis Graham-Smith*
- 32. Accretion Processes in Star Formation  
*by Lee W. Hartmann*
- 33. The Origin and Evolution of Planetary Nebulae  
*by Sun Kwok*
- 34. Solar and Stellar Magnetic Activity  
*by Carolus J. Schrijver and Cornelis Zwaan*
- 35. The Galaxies of the Local Group  
*by Sidney van den Bergh*
- 36. Stellar Rotation  
*by Jean-Louis Tassoul*

Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

---

# STELLAR ROTATION

---

JEAN-LOUIS TASSOUL  
*Université de Montréal*



Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo  
  
Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK  
  
Published in the United States of America by Cambridge University Press, New York  
  
[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521772181](http://www.cambridge.org/9780521772181)

© Cambridge University Press 2000  
  
This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without the written  
permission of Cambridge University Press.

First published 2000  
This digitally printed version 2007

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*  
Tassoul, Jean Louis.

Stellar rotation / Jean-Louis Tassoul.  
p. cm. – (Cambridge astrophysics)  
ISBN 0-521-77218-4  
1. Stars – Rotation. I. Title. II. Series.  
QB810.T36 2000  
523.8 – dc21 99-16740  
CIP

ISBN 978-0-521-77218-1 hardback  
ISBN 978-0-521-03769-3 paperback

Cambridge University Press  
978-0-521-03769-3 - Stellar Rotation  
Jean-Louis Tassoul  
Frontmatter  
[More information](#)

---

To my wife  
  
Monique

There are epochs in the history of every great operation and in the course of every undertaking, to which the co-operations of successive generations of men have contributed (...), when it becomes desirable to pause for a while, and, as it were, to take stock; to review the progress made, and estimate the amount of work done: not so much for complacency, as for the purpose of forming a judgement of the efficiency of the methods resorted to, to do it; and to lead us to inquire how they may yet be improved, if such improvement be possible, to accelerate the furtherance of the object, or to ensure the ultimate perfection of its attainments. In scientific, no less than in material and social undertakings, such pauses and *résumés* are eminently useful, and are sometimes forced on our considerations by a conjuncture of circumstances which almost of necessity obliges us to take a *coup d'oeil* of the whole subject, and make up our minds, not only as to the validity of what is done, but of the manner in which it has been done, the methods employed, and the direction in which we are henceforth to proceed, and probability of further progress.\*

Sir John Herschel (1792–1871)

\* Quoted in Hatton Turnor, *Astra Castra – Experiments and Adventures in the Atmosphere*, p. v, London: Chapman and Hall, 1865.

Contents

|                                             |                  |
|---------------------------------------------|------------------|
| <i>Preface</i>                              | <i>page xiii</i> |
| <b>1 Observational basis</b>                | <b>1</b>         |
| 1.1 Historical development                  | 1                |
| 1.2 The Sun                                 | 5                |
| 1.3 Single stars                            | 11               |
| 1.4 Close binaries                          | 16               |
| 1.5 Bibliographical notes                   | 21               |
| <b>2 Rotating fluids</b>                    | <b>25</b>        |
| 2.1 Introduction                            | 25               |
| 2.2 The equations of fluid motion           | 25               |
| 2.3 The vorticity equation                  | 30               |
| 2.4 Reynolds stresses and eddy viscosities  | 33               |
| 2.5 Applications to the Earth’s atmosphere  | 36               |
| 2.6 The wind-driven oceanic circulation     | 43               |
| 2.7 Barotropic and baroclinic instabilities | 49               |
| 2.8 Self-gravitating fluid masses           | 55               |
| 2.9 Bibliographical notes                   | 62               |
| <b>3 Rotating stars</b>                     | <b>65</b>        |
| 3.1 Introduction                            | 65               |
| 3.2 Basic concepts                          | 66               |
| 3.3 Some tentative solutions                | 69               |
| 3.4 The dynamical instabilities             | 73               |
| 3.5 The thermal instabilities               | 82               |
| 3.6 The eddy–mean flow interaction          | 86               |
| 3.7 Bibliographical notes                   | 89               |
| <b>4 Meridional circulation</b>             | <b>93</b>        |
| 4.1 Introduction                            | 93               |
| 4.2 A frictionless solution                 | 94               |
| 4.3 A consistent first-order solution       | 101              |
| 4.4 A consistent second-order solution      | 113              |
|                                             | <b>xi</b>        |

|            |                                                    |     |
|------------|----------------------------------------------------|-----|
| <b>xii</b> | <b><i>Contents</i></b>                             |     |
| 4.5        | Meridional circulation in a cooling white dwarf    | 118 |
| 4.6        | Meridional circulation in a close-binary component | 120 |
| 4.7        | Meridional circulation in a magnetic star          | 126 |
| 4.8        | Discussion                                         | 133 |
| 4.9        | Bibliographical notes                              | 135 |
| <b>5</b>   | <b>Solar rotation</b>                              | 138 |
| 5.1        | Introduction                                       | 138 |
| 5.2        | Differential rotation in the convection zone       | 139 |
| 5.3        | Meridional circulation in the radiative core       | 145 |
| 5.4        | Spin-down of the solar interior                    | 151 |
| 5.5        | Discussion                                         | 158 |
| 5.6        | Bibliographical notes                              | 159 |
| <b>6</b>   | <b>The early-type stars</b>                        | 162 |
| 6.1        | Introduction                                       | 162 |
| 6.2        | Main-sequence models                               | 162 |
| 6.3        | Axial rotation along the upper main sequence       | 172 |
| 6.4        | Circulation, rotation, and diffusion               | 179 |
| 6.5        | Rotation of evolved stars                          | 182 |
| 6.6        | Bibliographical notes                              | 185 |
| <b>7</b>   | <b>The late-type stars</b>                         | 190 |
| 7.1        | Introduction                                       | 190 |
| 7.2        | Schatzman’s braking mechanism                      | 191 |
| 7.3        | Rotation of T Tauri and cluster stars              | 194 |
| 7.4        | Rotational evolution of low-mass stars             | 197 |
| 7.5        | Bibliographical notes                              | 204 |
| <b>8</b>   | <b>Tidal interaction</b>                           | 207 |
| 8.1        | Introduction                                       | 207 |
| 8.2        | The tidal-torque mechanism                         | 208 |
| 8.3        | The resonance mechanism                            | 214 |
| 8.4        | The hydrodynamical mechanism                       | 217 |
| 8.5        | Contact binaries: The astrophysical balance        | 230 |
| 8.6        | Discussion                                         | 237 |
| 8.7        | Bibliographical notes                              | 240 |
|            | <i>Epilogue</i>                                    | 245 |
|            | <i>Subject index</i>                               | 249 |
|            | <i>Author index</i>                                | 252 |

---

## Preface

---

When I wrote my first book – *Theory of Rotating Stars* (Princeton: Princeton University Press, 1978) – I was not aware of the fact that the 1970s were a period of transition and that major unexpected developments would take place in the field of stellar rotation during the 1980s.

In the mid-1970s, we had no direct information about the internal rotation of the Sun. Little was known about the rotation of main-sequence stars of spectral type G and later, although it was already well established that the surface rotation rate of these stars decayed as the inverse square root of their age. We certainly had much more information about axial rotation in the upper-main-sequence stars, but the actual distribution of specific angular momentum within these stars was still largely unknown. On the theoretical side, important progress in the study of rotating stars had been made by direct numerical integration of the partial differential equations of stellar structure. However, because there was no clear expectation for the actual rotation law in an early-type star, the angular momentum distribution always had to be specified in an ad hoc manner. The presence of large-scale meridional currents in a stellar radiative zone was also a serious problem: All solutions presented to date had unwanted mathematical singularities at the boundaries, and the back reaction of these currents on the rotational motion had never been properly taken into account. As far as I remember, there was only one bright spot that was emerging from this rather gloomy picture of stellar rotation: The observed degree of synchronism and orbital circularization in the short-period close binaries appeared to be in reasonable agreement with the (then current) theoretical views on tidal interaction in close binary systems. The year was 1977 and, as I said, we did not realize that the tide was turning fast.

Ten years later, helioseismology was already providing a wealth of detail about the internal rotation of the Sun through the inversion of  $p$ -mode frequency splittings. At the same time, spectroscopic rotational velocities for numerous lower-main-sequence stars and pre-main-sequence stars were derived on the basis of high signal-to-noise ratio data and Fourier analysis techniques. Modulation of starlight due to dark or bright areas on a rotating star was also currently used to obtain rotation periods for a number of low-mass main-sequence stars. Helioseismology has forced us to reconsider our views on the Sun's internal rotation. Similarly, the newly derived rotational velocities of stars belonging to open clusters have provided us with a general outline of the rotational history of solar-type stars. However, very little observational progress has been made in measuring the surface rotation rates of main-sequence stars more massive than the

## xiv *Preface*

Sun; and since asteroseismology is still in its infancy, we do not yet know their internal distribution of angular velocity. Unexpectedly, renewal of interest in the close binaries has led to the conclusion that synchronous rotators and circular orbits are observed in binaries with orbital periods substantially larger than previously thought possible. This is a most challenging result since it requires that we reconsider the currently held views on tidal interaction in close (and not so close) binaries.

Over the course of the past two decades, theoreticians have also made great progress in developing an understanding of the effects of rotation in stellar radiative zones. This progress has *not* resulted from the development of new observational techniques or faster supercomputers, however, but from the recognition that rotation generates meridional currents as well as a wide spectrum of small-scale, eddylike motions wherever radiative transfer prevails. The importance of these rotationally driven motions lies in the fact that, under certain conditions, they can produce chemical mixing in regions that remain unmixed in standard calculations of nonrotating stellar models. Meridional circulation and eddylike motions also explain in a natural way the correlation between slow rotation and abnormal spectrum in the Am and Ap stars. This new approach, which is based on the idea that eddylike motions are an ever-present feature of a stellar radiative zone, also resolves in a very simple manner the many contradictions and inconsistencies that have beset the theory of meridional streaming in rotating stars.

All these new developments provide sufficient justification for a new book on rotating stars that would summarize the basic concepts and present a concise picture of the recent important advances in the field. Unfortunately, because the subject has grown so much in breadth and in depth over the past twenty years, a complete coverage of all the topics discussed in my first book has become an almost impossible task for a single individual. This is the reason why I have tried to concentrate almost exclusively on topics dealing with main-sequence stars, making occasional incursions into their pre-main-sequence and post-main-sequence phases. Admittedly, although much attention has been paid in the book to the correspondence between theory and observation, the text is basically theoretical with greater emphasis on firm quantitative results rather than on quick heuristic arguments. The book's prime emphasis, therefore, is on problems of long standing rather than on more recent developments (such as rotationally induced mixing in stellar radiative zones) that are still in the process of rapid and diverse growth. The view adopted throughout the book is that the study of rotating stars is a multidisciplinary endeavor and that much can be learned from a parallel study of other rotating fluid systems, such as the Earth's atmosphere and the oceans.\*

The contents of the various chapters are as follows: Chapter 1 presents the main observational data on which the subsequent discussion is based. Chapters 2 and 3 provide the theoretical background necessary for the understanding of the structure and evolution of a rotating star. In particular, Sections 2.5–2.7 describe some important geophysical concepts that will find their application in subsequent chapters. Even though the reader may not wish to go through these two chapters, I recommend reading the whole of Section 3.6,

\* This is not the place to discuss the psychological impact that the new trends toward interdisciplinary modes of research may have on individual members of the scientific community. For pertinent comments, see Juan G. Roederer, "Tearing Down Disciplinary Barriers," *Astrophysics and Space Science*, **144**, 659, 1988.

## *Preface*

xv

however, because it summarizes several basic ideas and concepts that are recurring throughout the book. Chapter 4 describes the state of motion in a star that consists of a convective core surrounded by a radiative envelope, whereas Chapter 5 is concerned with the rotational deceleration of the Sun – a star that consists of a radiative core and an outer convection zone that is slowly but inexorably losing angular momentum to outer space. These twin chapters are purely theoretical in the sense that both of them attempt to develop a clear understanding of the many hydrodynamical phenomena that arise in the early-type and late-type stars as they slowly evolve on the main sequence. On the contrary, in the next two chapters I review the observational evidence for axial rotation in single stars and, as far as possible, I compare the theoretical models with observation. Chapter 6 is entirely devoted to stars more massive than the Sun, whereas Chapter 7 discusses the rotational history of solar-type stars. Finally, Chapter 8 is concerned with tidal interaction in close binary stars and contact binaries. Sections 8.4 and 8.5 present distinct applications of two well-known geophysical concepts, namely, Ekman pumping and geostrophy.

All chapters end with a short section entitled “Bibliographical notes,” where references have been listed for elaboration of the material discussed in the corresponding sections. No attempt at completeness has been made, however, because that would have involved far too many entries. In each chapter, then, I have tried to include a useful selection of significant research papers and reviews from which further references may be obtained. Particular attention has been paid to original credits and priorities. For any inadvertent omission I offer a sincere apology in advance.

I am indebted to Paul Charbonneau and Georges Michaud who kindly provided valuable comments on portions of the manuscript. I appreciate also the untiring efforts of my wife, Monique, who typed and converted the original draft into  $\text{\LaTeX}$  format, offered many helpful comments and corrections, and assisted with the proofreading and indexes. Their help is gratefully acknowledged, but of course they are in no way responsible for any errors of fact or judgment that the book may contain.

Montréal, Québec  
 December 1997