THE PHYSICS OF SYNCHROTRON RADIATION

This book explains the underlying physics of synchrotron radiation and derives its main properties. It is divided into four parts. The first covers the general case of the electromagnetic fields created by an accelerated relativistic charge. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, deriving its distribution in angle, frequency, and polarization modes. The third part looks at undulator radiation. Starting from the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation, the author then discusses strong undulators, emitting more complicated radiation and containing higher harmonics. More general undulators are also considered, with a non-planar (helical) electron trajectory or non-harmonic field. The final part deals with applications and investigates the optics of synchrotron radiation dominated by diffraction due to the small opening angle. It also includes a description of electron-storage rings as radiation sources and the effect of the emitted radiation on the electron beam.

This book provides a valuable reference for scientists and engineers in the field of accelerators, and for all users of synchrotron radiation.

ALBERT HOFMANN received his doctorate in physics from the ETH (Swiss Federal Institute of Technology) in Zürich in 1964. From 1966 to 1972 he was a Research Fellow at the Cambridge Electron Accelerator, a joint laboratory of Harvard University and MIT. He then spent the next ten years working as Senior Physicist at CERN, Geneva. In 1983 he became a professor at Stanford University, working on the Stanford Linear Collider (SLC) and on optimizing the storage rings SPEAR and PEP for synchrotron-radiation use. He spent two years as head of the SLAC beam-dynamics group. He then returned to CERN, in 1987, and was jointly responsible for the commissioning of the Large Electron–Positron ring (LEP). After its completion, he worked on accelerator-physics problems with this machine until his retirement from CERN in 1998.

Over the years Albert Hofmann has done consulting work for other machines, such as the European Synchrotron Radiation Facility (ESRF), the Synchrotron Radiation Research Center (SRRC) in Taiwan, and the Swiss Light Source (SLS). He has taught in over 25 short-term schools on accelerator physics and synchrotron radiation, and has published numerous papers. In 1992 he was elected to become a fellow of the American Physical Society, and in 1996 he received the Robert Wilson Prize from this Society. In 2001 he obtained the degree Doctor honoris causa from the University of Geneva.

CAMBRIDGE MONOGRAPHS ON
PARTICLE PHYSICS
NUCLEAR PHYSICS AND COSMOLOGY
20

1. K. Winter (ed.): *Neutrino Physics*
2. J. F. Donoghue, E. Golowich and B. R. Holstein: *Dynamics of the Standard Model*
3. E. Leader and E. Predazzi: *An Introduction to Gauge Theories and Modern Particle Physics, Volume I: Electroweak Interactions, the 'New Particles' and the Parton Model*
4. E. Leader and E. Predazzi: *An Introduction to Gauge Theories and Modern Particle Physics, Volume 2: CP-Violation, QCD and Hard Processes*
5. C. Grupen: *Particle Detectors*
6. H. Grosse and A. Martin: *Particle Physics and the Schrödinger Equation*
7. B. Andersson: *The Lund Model*
8. R. K. Ellis, W. J. Stirling and B. R. Webber: *QCD and Collider Physics*
9. I. I. Bigi and A. I. Sanda: *CP Violation*
10. A. V. Manohar and M. B. Wise: *Heavy Quark Physics*
11. R. K. Bock, H. Grote, R. Frühwirth and M. Regler: *Data Analysis Techniques for High-Energy Physics, Second edition*
12. D. Green: *The Physics of Particle Detectors*
13. V. N. Gribov and J. Nyiri: *Quantum Electrodynamics*
14. K. Winter (ed.): *Neutrino Physics, Second edition*
15. E. Leader: *Spin in Particle Physics*
16. J. D. Walecka: *Electron Scattering for Nuclear and Nucleon Structure*
17. S. Narison: *QCD as a Theory of Hadrons*
18. J. F. Letessier and J. Rafelski: *Hadrons and Quark–Gluon Plasma*
19. A. Donnachie, H. G. Dosch, P. V. Landshoff and O. Nachtmann: *Pomeron Physics and QCD*
20. A. Hofmann: *The Physics of Synchrotron Radiation*

THE PHYSICS OF SYNCHROTRON RADIATION

ALBERT HOFMANN

Formerly CERN, Geneva

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521308267

First published 2004
This digitally printed version 2007

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Hofmann, Albert, 1933–
The physics of synchrotron radiation / Albert Hofmann.
p. cm. – (Cambridge monographs on particle physics, nuclear physics, and cosmology; 20)
Includes bibliographical references and index.
ISBN 0 521 30826 7
1. Synchrotron radiation. I. Title. II. Series.
QC793.5.E627H64 2003
539.7′35 – dc21 2003043939

ISBN 978-0-521-30826-7 hardback
ISBN 978-0-521-03753-2 paperback

Cambridge University Press
978-0-521-03753-2 - The Physics of Synchrotron Radiation
Albert Hofmann
Frontmatter
More information

To my wife Elisabeth
for her support

Contents

Preface

Under the rubric of synchrotron radiation we understand the electromagnetic waves emitted by a charge moving with relativistic velocity and undergoing a transverse acceleration. It is characterized by a small opening angle and a high frequency caused by the velocity of the charge being close to that of light. Owing to the relatively simple motion of the charge, the radiation has clear polarization properties. Ordinary synchrotron radiation is emitted by a charge moving on a circular arc determined by a deflecting magnetic field. It has a broad spectrum, a typical frequency being γ^3 times higher than the Larmor frequency of the charge. This spectrum can be modified by varying the curvature of the trajectory $1/\rho$ within a distance smaller than the formation length of the radiation, as is realized in undulators.

Synchrotron radiation has been investigated theoretically for over a century and experimentally for about half this time. Thanks to its unique properties, this radiation has become a research tool for many fields of science and electron-storage rings serving as radiation sources are spread over the whole globe.

This book tries to explain synchrotron radiation from basic principles and to derive its main properties. It is divided into four parts. First the general case of the electromagnetic fields created by an accelerated relativistic charge is investigated. This gives the angular distribution with the small opening angle of the emitted radiation and distinguishes between the 'near' (Coulomb) and the 'far' (radiation) field. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, which we usually call synchrotron radiation. Its distributions in angle, frequency, and polarization modes are derived. Undulator radiation is treated in the next part. We start with the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation. A strong undulator emits radiation that is more complicated and contains higher harmonics. There are more general undulators having a non-planar (helical) electron trajectory or a non-harmonic field. The last part deals with applications and investigates first the optics of synchrotron radiation, which is dominated by diffraction due to the small opening angle. This is followed by a description of electron-storage rings serving as radiation sources and the effect of the emitted radiation on the electron beam.

There are some technical remarks to be made. Throughout the book MKSA units are used. With very few exceptions the radiation field refers to a single positive elementary charge e as a source. For convenience sometimes the radiation emitted by a current I is also given and, in the last chapter, the temporal coherence of the radiation from different particles is considered. As a basis for the properties of the radiation we give first the total emitted power or energy. In the case of ordinary synchrotron radiation we denote by P_{s} the power radiated by the electron *while* it is going through the magnet and by U_{s} the energy radiated during one revolution. For undulators we denote by P_{u} the power radiated in the undulator but averaged over one period and by U_{s} the energy emitted during one traversal through the undulator. These powers and energies can also be expressed in terms of the photon number or photon flux. Distributions in terms of angle and frequency are then given with these total values as a factor that makes it easy to express them in terms of power, energy, photon-number or photon-flux distributions or in other units. Vectors are printed in bold. They are also written as an array with three components between square brackets, like $\mathbf{E} = [E_x,\ E_y,\ E_z]$. For radiation fields the z-component can often be neglected. The remaining two-component vector is written as $E_\perp = [E_x,\ E_y]$. These field components give the polarization of the radiated power. To mark the contributions of the horizontal or vertical polarization to the power, which is of course a scalar, we write it as a sum $P = P_\sigma + P_\pi$. The calculation of synchrotron radiation leads to some integrals that can be expressed in terms of modified Bessel functions or Airy functions. Here the second type is chosen, but the important results are given in both. Some properties, integrals, and sums of Airy and Bessel functions are given in the appendices, partly for convenience and partly because they are not so easy to find. However, this is not meant to provide rigorous mathematical derivations but rather to provide some insight into how some results are obtained.

There are lots of publications on synchrotron radiation and related topics. Apart from well-known books and journals they appear often in laboratory reports and proceedings of workshops. The bibliography to this volume is by no means complete and refers mostly to the topics covered and the methods used to investigate them.

Acknowledgments

I received much help from many people while writing this book. I owe many thanks to my colleague and friend Bruno Zotter from CERN. He not only answered many questions concerning the mathematics I had to use, but also read the whole book and made many suggestions, corrections, and significant improvements. Jim Murphy from Brookhaven National Laboratory read and corrected part of the book and clarified questions concerning mainly coherent radiation. On many occasions I sought advice from my friend and colleague Hermann Winick from Stanford University. Thanks to his insight and experience, he could answer many questions on synchrotron radiation and explained difficult topics to me. I also profited from discussions with many experts in the field: R. Coisson, K. J. Kim, B. M. Kincaid, S. Krinsky, F. Méot, M. Sands, and H. Wiedemann. I also thank the staffs of the laboratories where I had the opportunity to work on and learn about synchrotron radiation: CEA, Cambridge Electron Accelerator, Harvard University – MIT, Cambridge, MA, U.S.A.; CERN, European Laboratory of Particle Physics, Geneva, Switzerland; SLAC, Stanford Linear Accelerator Center, SSRL division, Stanford, CA, U.S.A.; and LNLS, Laboratório Nacional de Luz Síncrotron, Campinas, Brazil.

Notation

$\mathbf{A}$	vector potential
$\mathrm{Ai}(x),\ \mathrm{Ai}'(x)$	Airy function and its derivative
$\mathbf{B}$	magnetic-field vector
$\tilde{\mathbf{B}}(\omega)$	Fourier transformed B-field of radiation
B_0	amplitude of magnetic undulator field
$\tilde{B}_y(k_g)$	weak-magnet Fourier component at k_g
c	speed of light
C_q	quantum excitation factor
$D_x,\ D'_x$	particle-beam-optics dispersion
e	elementary charge
$\mathbf{E}$	electric-field vector
$\tilde{\mathbf{E}}(\omega)$	Fourier-transformed E-field of radiation
$E_e = m_0c^2\gamma$	particle energy
$E_\gamma = \hbar\omega$	photon energy
$F_s(\psi, \omega/\omega_c)$	normalized angular spectral density of SR
$F_u(\theta, \phi)$	normalized angular power density of UR
$h,\ \hbar = h/(2\pi)$	Planck's constant
$\mathcal{H}$	emittance function
$I_{s2},\ I_{s3},\ I_{s4},\ I_{s5}$	synchrotron-radiation integrals
$J_n(x)$	Bessel function of order n
$J_\epsilon,\ J_x,\ J_y$	longitudinal and transverse damping partitions
$K_{1/3},\ K_{2/3}$	modified Bessel function of order 1/3, 2/3
K_f	quadrupole focusing parameter
k_g	wave number of general weak magnet
$k_u = 2\pi/\lambda_u$	undulator period wave number
$K_u = eB_0/(m_0ck_u)$	undulator parameter
K_u^*	reduced plane undulator parameter
K_{uh}^*	reduced helical undulator parameter
$L_u = N_u\lambda_u$	undulator length

m_0	rest mass of a particle
$\mathbf{n} = \mathbf{r}/r$	unit vector in **r**-direction
$n_B = -\rho^2 K_f$	field index
$n_s,\ \dot{n}_s$	photons per revolution, photon flux
$n_u,\ \dot{n}_u$	photon number per traversal, photon flux
N_u	undulator period number
P_s	instantaneous radiated power of SR
P_u	period-averaged total power of plane UR
$\mathbf{r}(t')$	distance from source to observer
$\mathbf{R}(t')$	vector from origin to particle
$\mathcal{R}$, Φ	polar coordinates in image plane
$r_0 = e^2/(4\pi\epsilon_0 m_0 c^2)$	classical electron radius
$\mathbf{r}_p$	vector from origin to observer
$\mathbf{S} = [\mathbf{E} \times \mathbf{B}]/\mu_0$	Poynting vector
$S_{hm}(\omega_m)$	normalized spectral power of helical UR
$S_s(\omega/\omega_c)$	normalized spectral power densities of SR
$t = t' + r/c$	observation time
$t_p = t - r_p/c$	reduced observation time
$T_0 = 2\pi/\omega_0$	revolution time without straight sections
$T_{rev} = 2\pi/\omega_{rev}$	revolution time with straight sections
t'	emission time
U_s	energy radiated per turn of SR
$\mathbf{v}(t') = \mathrm{d}\mathbf{R}/\mathrm{d}t'$	particle velocity
V	scalar potential
$\hat{V}$	peak voltage of RF system
w	transverse coordinate, x or y
$X,\ Y$	rectangular coordinates in image plane
α_c	momentum compaction
$\alpha_\epsilon,\ \alpha_h,\ \alpha_v$	longitudinal and transverse damping rates
$\alpha_f = e^2/(2\epsilon_0 ch)$	fine structure constant
$\alpha_w = -\beta'_w/2$	particle-optics functions
$\beta = v/c$	normalized velocity
$\boldsymbol{\beta} = \mathbf{v}/c$	normalized velocity vector
β_w	particle-optics functions
β^*	normalized drift velocity in plane undulator
β^*_h	normalized drift velocity in helical undulators
$\gamma = 1/\sqrt{1-\beta^2}$	Lorentz factor
γ_w	particle-optics functions
γ^*	Lorentz factor of drift velocity
γ^*_h	drift Lorentz factor in helical undulators
ϵ_0	vacuum permittivity
$\epsilon_x,\ \epsilon_y$	horizontal and vertical particle-beam emittance

$\epsilon_{\gamma x},\ \epsilon_{\gamma y}$	horizontal and vertical photon-beam emittance
$\boldsymbol{\eta}_x,\ \boldsymbol{\eta}_y$	unit vectors in x- and y-directions
$\lambda_{\mathrm{Comp}} = h/(m_0 c)$	Compton wavelength
λ_{u}	undulator period length
μ_0	vacuum permeability
ρ	bending radius
$\sigma_x,\ \sigma'_x$	RMS electron-beam size and angular spread
φ_{B}	bending angle in a dipole magnet
$\varphi_w(s)$	betatron phase within one turn
φ_{s}	synchrotron phase angle in RF acceleration
ψ	angle between median plane and $\mathbf{r}_{\mathrm{p}}$
$\omega_0 = \beta c/\rho = 2\pi/T_0$	angular velocity, Larmor frequency
ω_1	fundamental UR frequency off axis
ω_{10}	fundamental UR frequency on axis
$\omega_c = 3\omega_0\gamma^3/2$	critical frequency
ω_m	mth harmonic UR frequency off axis
ω_{m0}	mth harmonic UR frequency on axis
$\omega_{\mathrm{rev}} = 2\pi/T_{\mathrm{rev}}$	revolution frequency with straight sections
$\Omega_{\mathrm{u}} = \beta c k_{\mathrm{u}}$	particle-motion frequency in undulator
$\mathrm{d}P/\mathrm{d}\Omega$	power radiated per unit solid angle
$\mathrm{d}^2 P/(\mathrm{d}\Omega\,\mathrm{d}\omega)$	angular spectral radiated power density
$(\ )_\sigma,\ (\ )_\pi$	horizontal, vertical linear polarization
$(\ )_+,\ (\ )_-$	positive, negative helicity circular polarization
$\{\ \ \}_{\mathrm{ret}}$	parenthesis evaluated at emission time t'