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THE PHYSICS OF SYNCHROTRON RADIATION

This book explains the underlying physics of synchrotron radiation and derives its main properties. It is divided into four parts. The first covers the general case of the electromagnetic fields created by an accelerated relativistic charge. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, deriving its distribution in angle, frequency, and polarization modes. The third part looks at undulator radiation. Starting from the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation, the author then discusses strong undulators, emitting more complicated radiation and containing higher harmonics. More general undulators are also considered, with a non-planar (helical) electron trajectory or non-harmonic field. The final part deals with applications and investigates the optics of synchrotron radiation dominated by diffraction due to the small opening angle. It also includes a description of electron-storage rings as radiation sources and the effect of the emitted radiation on the electron beam.

This book provides a valuable reference for scientists and engineers in the field of accelerators, and for all users of synchrotron radiation.

ALBERT HOFMANN received his doctorate in physics from the ETH (Swiss Federal Institute of Technology) in Zürich in 1964. From 1966 to 1972 he was a Research Fellow at the Cambridge Electron Accelerator, a joint laboratory of Harvard University and MIT. He then spent the next ten years working as Senior Physicist at CERN, Geneva. In 1983 he became a professor at Stanford University, working on the Stanford Linear Collider (SLC) and on optimizing the storage rings SPEAR and PEP for synchrotron-radiation use. He spent two years as head of the SLAC beam-dynamics group. He then returned to CERN, in 1987, and was jointly responsible for the commissioning of the Large Electron–Positron ring (LEP). After its completion, he worked on accelerator-physics problems with this machine until his retirement from CERN in 1998.

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Formerly CERN, Geneva



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To my wife Elisabeth
for her support

Contents

<i>Preface</i>	<i>page</i> xvii
<i>Acknowledgments</i>	xix
<i>Notation</i>	xx
Part I Introduction	1
1 A qualitative treatment of synchrotron radiation	3
1.1 Introduction	3
1.2 The opening angle	3
1.3 The spectrum emitted in a long magnet	4
1.4 The spectrum emitted in a short weak magnet	5
1.5 The wave front of synchrotron radiation	6
1.6 The polarization	8
2 Fields of a moving charge	9
2.1 Introduction	9
2.2 The particle motion relevant to the retarded potentials	9
2.3 The retarded electromagnetic potentials	11
2.4 The fields of a moving charge	14
2.5 A discussion of the field equations	18
2.6 Examples	20
2.6.1 The field of a charge moving with constant velocity	20
2.6.2 The field of a non-relativistic oscillating charge	27
2.7 The near field and the far field	34
2.8 The Fourier transform of the radiation field	35
2.8.1 The Fourier integral of the field	35
2.8.2 The periodic motion	37
2.8.3 The motion with a periodic velocity	38
3 The emitted radiation field and power	40
3.1 Introduction	40
3.2 The emitted and received powers	41
3.3 Transverse and longitudinal acceleration	42
3.3.1 The transverse acceleration	42
3.3.2 The longitudinal acceleration	45

3.4	The ultra-relativistic case for transverse acceleration	48
3.5	The angular spectral energy and power density	51
Part II	Synchrotron radiation	55
4	Synchrotron radiation: basic physics	57
4.1	Introduction	57
4.2	The geometry and approximations	58
4.2.1	The particle motion	58
4.2.2	The dipole approximation	59
4.2.3	The relevant motion	61
4.2.4	The ultra-relativistic approximation	62
4.3	The continuous spectrum radiated on a circular arc	65
4.3.1	The Fourier-transformed field	65
4.3.2	The spectral power density of the radiation	67
4.4	The radiation emitted on a circular arc in the time domain	68
4.4.1	The radiation field in the time domain	68
4.4.2	The radiated energy and power in the time domain	71
4.4.3	The radiation field in the time and frequency domains	72
4.5	The line spectrum radiated on closed circles	73
4.5.1	The relevant motion	73
4.5.2	The line spectrum of the electric field	74
4.5.3	The power of the line spectrum	77
4.5.4	The relation between the continuous and the line spectra	79
5	Synchrotron radiation: properties	81
5.1	Introduction	81
5.2	The total radiated power and energy	81
5.3	The angular spectral distribution	83
5.3.1	The general distribution	83
5.3.2	The distribution at low frequencies	85
5.3.3	The distribution at high frequencies	89
5.4	The spectral distribution	89
5.4.1	The general spectrum	89
5.4.2	The spectrum at low frequencies	92
5.4.3	The spectrum at high frequencies	92
5.4.4	The spectrum integrated up to a given frequency	92
5.4.5	The integral over all frequencies	93
5.5	The angular distribution	94
5.5.1	The angular distribution as a function of frequency	94
5.5.2	The frequency-integrated angular distribution	96
5.6	The polarization	98
5.6.1	The description of linear and circular polarization	98
5.6.2	The linear polarization	102

	<i>Contents</i>	xi
5.6.3	The elliptical polarization	105
5.7	The photon distribution	110
Part III	Undulator radiation	115
6	A qualitative treatment	117
6.1	Introduction	117
6.2	The interference	118
6.3	The undulator radiation as a wave front	120
6.4	The modulation of the emitted field	121
6.5	The weak undulator in the laboratory and moving frames	121
6.6	The strong undulator in the laboratory and moving frames	123
6.7	The helical undulator	124
6.8	Undulators and related devices	124
7	The plane weak undulator	126
7.1	The trajectory	126
7.1.1	The equation of motion	126
7.1.2	The approximation for a weak undulator	128
7.1.3	The observation from a large distance	129
7.1.4	The ultra-relativistic approximation	130
7.1.5	The particle motion in the moving system	131
7.2	The radiation field	131
7.2.1	The field calculated from the Liénard–Wiechert equation	131
7.2.2	The undulator field as Lorentz-transformed dipole radiation	132
7.2.3	The undulator radiation in the frequency domain	135
7.2.4	A discussion of the weak-undulator radiation field	136
7.3	Properties of weak-undulator radiation	138
7.3.1	The energy and power radiated in an undulator	138
7.3.2	The angular spectral power distribution	139
7.3.3	The angular power distribution	141
7.3.4	The spectral power distribution	146
7.4	The photon distribution	148
7.4.1	The number and energy of photons	148
7.4.2	The photon spectrum	151
7.4.3	The angular spectral photon distribution	151
7.4.4	The undulator radiation on the axis	152
8	The plane strong undulator	154
8.1	The trajectory	154
8.1.1	The trajectory in the laboratory frame	154
8.1.2	The trajectory in the moving frame	157
8.1.3	The relevant motion in a strong undulator	159
8.2	The radiation from a plane strong undulator	162

xii	<i>Contents</i>	
	8.2.1 The radiation field	162
	8.3 Properties of strong-undulator radiation	167
	8.3.1 The angular spectral power distribution	167
	8.3.2 The angular power distribution	168
	8.3.3 The spectral density of the radiation	171
	8.3.4 The power contained in each harmonic	171
	8.3.5 The properties of the radiation on the axis	173
	8.3.6 The development with respect to K_u^*	177
9	The helical undulator	181
	9.1 The trajectory	181
	9.2 The radiation emitted in a helical weak undulator	185
	9.2.1 The radiation obtained with the Liénard–Wiechert formula	185
	9.3 Properties of weak-helical-undulator radiation	187
	9.3.1 The total power	187
	9.3.2 The angular spectral power distribution	187
	9.3.3 The angular power distribution	188
	9.3.4 The spectral power distribution	189
	9.3.5 The total radiation	190
	9.3.6 The degree of circular polarization	190
	9.3.7 The on-axis radiation	192
	9.4 The radiation field from a strong helical undulator	193
	9.5 Properties of strong-helical-undulator radiation	197
	9.5.1 The total power	197
	9.5.2 The angular spectral power distribution	197
	9.5.3 The angular power distribution	198
	9.5.4 The spectral density of helical-undulator radiation	198
	9.5.5 The on-axis radiation	200
	9.5.6 The development with respect to K_{uh}^*	202
10	Wiggler magnets	206
	10.1 Introduction	206
	10.2 The wavelength shifter	206
	10.3 The multipole wiggler	207
11	Weak magnets – a generalized weak undulator	209
	11.1 Properties of weak-magnet radiation	209
	11.1.1 Introduction	209
	11.1.2 The trajectory	210
	11.1.3 The radiation from weak magnets	210
	11.2 Short magnets	213
	11.2.1 Introduction	213
	11.2.2 Qualitative properties of the short-magnet radiation	213

	<i>Contents</i>	xiii
11.3	The modulated undulator radiation	215
11.3.1	Introduction	215
11.3.2	The undulator of finite length	216
11.3.3	The undulator radiation with amplitude modulation	219
11.3.4	The undulator radiation with Lorentzian modulation	221
11.4	The Compton back scattering and quantum correction	224
Part IV	Applications	227
12	Optics of SR – imaging	229
12.1	Imaging with SR – a qualitative treatment	229
12.1.1	The limitation on resolution caused by diffraction and the depth-of-field effect	229
12.1.2	Diffraction and the depth-of-field effect for SR from long magnets	230
12.1.3	Diffraction and the depth-of-field effect for undulator radiation	231
12.1.4	Diffraction and the depth-of-field effect for short-magnet radiation	231
12.1.5	Discussion	232
12.2	Imaging with SR – a quantitative treatment	232
12.2.1	The Fraunhofer diffraction	232
12.2.2	The emittance of a photon beam	235
12.2.3	The diffraction of synchrotron radiation emitted in long magnets	236
12.2.4	The diffraction of undulator radiation	239
12.2.5	The diffraction for the undulator with a Lorentzian profile	242
12.2.6	A comparison of the properties of beams from various sources	243
13	Electron-storage rings	244
13.1	Introduction	244
13.1.1	Lattice magnets	245
13.2	The transverse particle dynamics in a storage ring	248
13.2.1	The particle dynamics over many revolutions	248
13.2.2	The beam with many particles	256
13.2.3	The dispersion	258
13.2.4	The chromatic aberrations and their correction with sextupoles	259
13.2.5	Coupling and vertical dispersion	261
13.2.6	An example: The FODO lattice	261
13.3	The longitudinal particle dynamics	264

13.3.1	Introduction	264
13.3.2	The longitudinal focusing – small amplitudes	266
13.3.3	The longitudinal focusing – large amplitudes	268
14	Effects of radiation on the electron beam	271
14.1	The energy loss	271
14.2	The radiation damping	272
14.2.1	Introduction	272
14.2.2	The damping of synchrotron oscillations	274
14.2.3	The damping of vertical betatron oscillations	274
14.2.4	The damping of horizontal betatron oscillations	276
14.2.5	The sum of the damping rates	278
14.3	The quantum excitation of oscillations	279
14.3.1	Introduction	279
14.3.2	The energy spread	280
14.3.3	The horizontal emittance	280
14.3.4	The vertical emittance	281
14.4	A summary of the effects of radiation on the electron beam	282
14.5	Changing effects of radiation with wiggler magnets	284
15	Radiation emitted by many particles	286
15.1	Effects of the electron distribution on the radiation	286
15.1.1	Introduction	286
15.1.2	The radiation geometry in the case of a large electron emittance	286
15.1.3	The electron and natural photon emittances are of the same magnitude	288
15.2	The spatial coherence	288
15.2.1	The diffraction limit	288
15.2.2	Small-emittance rings	289
15.3	The temporal coherence	290
15.4	Flux and brightness	295
15.5	The synchrotron radiation emitted by protons and ions	296
15.5.1	Introduction	296
15.5.2	The radiation from protons	296
15.5.3	The radiation from ions	297
A	Airy functions	300
A.1	Definitions and developments	300
A.2	Integrals involving Airy functions	301
B	Bessel functions	308
B.1	General relations	308
B.2	The approximation for large order and arguments	309

Cambridge University Press
978-0-521-03753-2 - The Physics of Synchrotron Radiation
Albert Hofmann
Frontmatter
[More information](#)

<i>Contents</i>	xv
B.3 Sums over squares of Bessel functions	310
B.4 Series of Bessel functions	312
C Developments of strong-undulator radiation	313
C.1 The plane-undulator radiation	313
C.2 The helical-undulator radiation	314
<i>References</i>	316
<i>Index</i>	321

Preface

Under the rubric of synchrotron radiation we understand the electromagnetic waves emitted by a charge moving with relativistic velocity and undergoing a transverse acceleration. It is characterized by a small opening angle and a high frequency caused by the velocity of the charge being close to that of light. Owing to the relatively simple motion of the charge, the radiation has clear polarization properties. Ordinary synchrotron radiation is emitted by a charge moving on a circular arc determined by a deflecting magnetic field. It has a broad spectrum, a typical frequency being γ^3 times higher than the Larmor frequency of the charge. This spectrum can be modified by varying the curvature of the trajectory $1/\rho$ within a distance smaller than the formation length of the radiation, as is realized in undulators.

Synchrotron radiation has been investigated theoretically for over a century and experimentally for about half this time. Thanks to its unique properties, this radiation has become a research tool for many fields of science and electron-storage rings serving as radiation sources are spread over the whole globe.

This book tries to explain synchrotron radiation from basic principles and to derive its main properties. It is divided into four parts. First the general case of the electromagnetic fields created by an accelerated relativistic charge is investigated. This gives the angular distribution with the small opening angle of the emitted radiation and distinguishes between the ‘near’ (Coulomb) and the ‘far’ (radiation) field. The second part concentrates on the radiation emitted by a charge moving on a circular trajectory, which we usually call synchrotron radiation. Its distributions in angle, frequency, and polarization modes are derived. Undulator radiation is treated in the next part. We start with the simple case of a plane weak undulator with a spatially periodic field that emits quasi-monochromatic radiation. A strong undulator emits radiation that is more complicated and contains higher harmonics. There are more general undulators having a non-planar (helical) electron trajectory or a non-harmonic field. The last part deals with applications and investigates first the optics of synchrotron radiation, which is dominated by diffraction due to the small opening angle. This is followed by a description of electron-storage rings serving as radiation sources and the effect of the emitted radiation on the electron beam.

There are some technical remarks to be made. Throughout the book MKSA units are used. With very few exceptions the radiation field refers to a single positive elementary charge e as a source. For convenience sometimes the radiation emitted by a current I is

also given and, in the last chapter, the temporal coherence of the radiation from different particles is considered. As a basis for the properties of the radiation we give first the total emitted power or energy. In the case of ordinary synchrotron radiation we denote by P_s the power radiated by the electron *while* it is going through the magnet and by U_s the energy radiated during one revolution. For undulators we denote by P_u the power radiated in the undulator but averaged over one period and by U_s the energy emitted during one traversal through the undulator. These powers and energies can also be expressed in terms of the photon number or photon flux. Distributions in terms of angle and frequency are then given with these total values as a factor that makes it easy to express them in terms of power, energy, photon-number or photon-flux distributions or in other units. Vectors are printed in bold. They are also written as an array with three components between square brackets, like $\mathbf{E} = [E_x, E_y, E_z]$. For radiation fields the z -component can often be neglected. The remaining two-component vector is written as $E_\perp = [E_x, E_y]$. These field components give the polarization of the radiated power. To mark the contributions of the horizontal or vertical polarization to the power, which is of course a scalar, we write it as a sum $P = P_\sigma + P_\pi$. The calculation of synchrotron radiation leads to some integrals that can be expressed in terms of modified Bessel functions or Airy functions. Here the second type is chosen, but the important results are given in both. Some properties, integrals, and sums of Airy and Bessel functions are given in the appendices, partly for convenience and partly because they are not so easy to find. However, this is not meant to provide rigorous mathematical derivations but rather to provide some insight into how some results are obtained.

There are lots of publications on synchrotron radiation and related topics. Apart from well-known books and journals they appear often in laboratory reports and proceedings of workshops. The bibliography to this volume is by no means complete and refers mostly to the topics covered and the methods used to investigate them.

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Notation

A	vector potential
$\text{Ai}(x), \text{Ai}'(x)$	Airy function and its derivative
B	magnetic-field vector
$\tilde{\mathbf{B}}(\omega)$	Fourier transformed B -field of radiation
B_0	amplitude of magnetic undulator field
$\tilde{B}_y(k_g)$	weak-magnet Fourier component at k_g
c	speed of light
C_q	quantum excitation factor
D_x, D'_x	particle-beam-optics dispersion
e	elementary charge
E	electric-field vector
$\tilde{\mathbf{E}}(\omega)$	Fourier-transformed E -field of radiation
$E_e = m_0 c^2 \gamma$	particle energy
$E_\gamma = \hbar \omega$	photon energy
$F_s(\psi, \omega/\omega_c)$	normalized angular spectral density of SR
$F_u(\theta, \phi)$	normalized angular power density of UR
$h, \hbar = h/(2\pi)$	Planck's constant
\mathcal{H}	emittance function
$I_{s2}, I_{s3}, I_{s4}, I_{s5}$	synchrotron-radiation integrals
$J_n(x)$	Bessel function of order n
J_ϵ, J_x, J_y	longitudinal and transverse damping partitions
$K_{1/3}, K_{2/3}$	modified Bessel function of order $1/3, 2/3$
K_f	quadrupole focusing parameter
k_g	wave number of general weak magnet
$k_u = 2\pi/\lambda_u$	undulator period wave number
$K_u = eB_0/(m_0 c k_u)$	undulator parameter
K_u^*	reduced plane undulator parameter
K_{uh}^*	reduced helical undulator parameter
$L_u = N_u \lambda_u$	undulator length

m_0	rest mass of a particle
$\mathbf{n} = \mathbf{r}/r$	unit vector in \mathbf{r} -direction
$n_B = -\rho^2 K_f$	field index
n_s, \dot{n}_s	photons per revolution, photon flux
n_u, \dot{n}_u	photon number per traversal, photon flux
N_u	undulator period number
P_s	instantaneous radiated power of SR
P_u	period-averaged total power of plane UR
$\mathbf{r}(t')$	distance from source to observer
$\mathbf{R}(t')$	vector from origin to particle
\mathcal{R}, Φ	polar coordinates in image plane
$r_0 = e^2/(4\pi\epsilon_0 m_0 c^2)$	classical electron radius
\mathbf{r}_p	vector from origin to observer
$\mathbf{S} = [\mathbf{E} \times \mathbf{B}]/\mu_0$	Poynting vector
$S_{hm}(\omega_m)$	normalized spectral power of helical UR
$S_s(\omega/\omega_c)$	normalized spectral power densities of SR
$t = t' + r/c$	observation time
$t_p = t - r_p/c$	reduced observation time
$T_0 = 2\pi/\omega_0$	revolution time without straight sections
$T_{rev} = 2\pi/\omega_{rev}$	revolution time with straight sections
t'	emission time
U_s	energy radiated per turn of SR
$\mathbf{v}(t') = d\mathbf{R}/dt'$	particle velocity
V	scalar potential
\hat{V}	peak voltage of RF system
w	transverse coordinate, x or y
X, Y	rectangular coordinates in image plane
α_c	momentum compaction
$\alpha_\epsilon, \alpha_h, \alpha_v$	longitudinal and transverse damping rates
$\alpha_f = e^2/(2\epsilon_0 ch)$	fine structure constant
$\alpha_w = -\beta'_w/2$	particle-optics functions
$\beta = v/c$	normalized velocity
$\boldsymbol{\beta} = \mathbf{v}/c$	normalized velocity vector
β_w	particle-optics functions
β^*	normalized drift velocity in plane undulator
β_h^*	normalized drift velocity in helical undulators
$\gamma = 1/\sqrt{1 - \beta^2}$	Lorentz factor
γ_w	particle-optics functions
γ^*	Lorentz factor of drift velocity
γ_h^*	drift Lorentz factor in helical undulators
ϵ_0	vacuum permittivity
ϵ_x, ϵ_y	horizontal and vertical particle-beam emittance

xxii	Notation
$\epsilon_{\gamma x}, \epsilon_{\gamma y}$	horizontal and vertical photon-beam emittance
$\boldsymbol{\eta}_x, \boldsymbol{\eta}_y$	unit vectors in x - and y -directions
$\lambda_{\text{Comp}} = h/(m_0 c)$	Compton wavelength
λ_u	undulator period length
μ_0	vacuum permeability
ρ	bending radius
σ_x, σ'_x	RMS electron-beam size and angular spread
φ_B	bending angle in a dipole magnet
$\varphi_w(s)$	betatron phase within one turn
φ_s	synchrotron phase angle in RF acceleration
ψ	angle between median plane and \mathbf{r}_p
$\omega_0 = \beta c/\rho = 2\pi/T_0$	angular velocity, Larmor frequency
ω_1	fundamental UR frequency off axis
ω_{10}	fundamental UR frequency on axis
$\omega_c = 3\omega_0\gamma^3/2$	critical frequency
ω_m	m th harmonic UR frequency off axis
ω_{m0}	m th harmonic UR frequency on axis
$\omega_{\text{rev}} = 2\pi/T_{\text{rev}}$	revolution frequency with straight sections
$\Omega_u = \beta c k_u$	particle-motion frequency in undulator
$dP/d\Omega$	power radiated per unit solid angle
$d^2P/(d\Omega d\omega)$	angular spectral radiated power density
$(\)_\sigma, (\)_\pi$	horizontal, vertical linear polarization
$(\)_+, (\)_-$	positive, negative helicity circular polarization
$\{ \ }_{\text{ret}}$	parenthesis evaluated at emission time t'