The Scalar–Tensor Theory of Gravitation

The scalar–tensor theory of gravitation is one of the most popular alternatives to Einstein's theory of gravitation. This book provides a clear and concise introduction to the theoretical ideas and developments, exploring scalar fields and placing them in context with a discussion of Brans–Dicke theory. Topics covered include the cosmological constant problem, time-variability of coupling constants, higher-dimensional space-time, branes, and conformal transformations. The authors emphasize the physical applications of the scalar–tensor theory and thus provide a pedagogical overview of the subject, keeping more mathematically detailed sections for the appendices.

This book is suitable as a textbook for graduate courses in cosmology, gravitation, and relativity. It will also provide a valuable reference for researchers.

YASUNORI FUJII received his PhD, on the analogy between the strong interaction and the electromagnetic interaction, from Nagoya University in 1959. Between 1963 and 1992 he did research on the theory of particle physics and gravity, including pioneering work on the idea of non-Newtonian gravity, at the Institute of Physics, University of Tokyo. During this period, he also spent two years at Stanford University and a year at Purdue University. He is currently emeritus professor at the University of Tokyo and continues to pursue his research interests at the Nihon Fukushi University. Professor Fujii is the author of several books in Japanese, both for expert physicists and for general readers. He has also published numerous articles and papers in leading journals.

KEI-ICHI MAEDA received his PhD from Kyoto University in 1980. He and his contemporaries created a new research group in Kyoto, which was at the root of numerical relativity research in Japan. In 1983 he became a postdoctoral student at SISSA, Trieste, working under Dennis Sciama. He moved to the Meudon Observatory in Paris in 1987 and worked on black hole solutions in string theory. In 1989 Professor Maeda became affiliated with the Department of Physics at Waseda University. Since 1998, he has been the associate editor of the *Journal of General Relativity and Gravitation*, and also the vice-chief editor of the *Journal of the Physical Society of Japan* since 2001. In addition to many important journal articles, Professor Maeda has written several popular science books in Japanese.

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YASUNORI FUJII
Nihon Fukushi University

KEI-ICHI MAEDA
Waseda University, Tokyo

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Preface

During the last few decades of the twentieth century, we saw an almost triumphant success in establishing that Einstein's general relativity is correct, both experimentally and theoretically. We find nevertheless considerable efforts still being made in terms of "alternative theories." This trend may be justified insofar as the scalar–tensor theory is concerned, as will be argued, not to mention one's hidden desire to see nature's simplest imaginable phenomenon, a scalar field, be a major player.

The success on the theoretical front prompted researchers to study theories with the aim of unifying gravitation and microscopic physics. Among them string theory appears to be the most promising. According to this theory, the graviton corresponding to the metric tensor has a scalar companion, called the dilaton. The interaction between these two fields is surprisingly similar to what Jordan foresaw nearly half a century ago, without sharing ideas that characterize the contemporary unification program. There seems to be, however, a crucial point that might constrain the original proposal through the value of the parameter ω, whose inverse measures the strength of the coupling of the scalar field.

More specifically, string theory predicts that $\omega = -1$, which goes against the widely accepted constraint from observation, namely $\omega \gtrsim 10^3 \gg 1$. Although many more details have yet to be worked out in order for string theory to be compared with the real world, we point out that expecting the dilaton to be close to the limit of total decoupling is by no means obvious or natural. One may even suspect that the scalar force has a finite force-range, rendering the solar-system experiments from which a large value of ω derives irrelevant.

Also related is the fact that string theory, like other unification theories, allows coupling to matter at the level of the Lagrangian, thus inevitably violating the weak equivalence principle (WEP), in contrast to what Brans and Dicke proposed as a modification of Jordan's model.

Both of these considerations, together with our own study of the cosmological equations, suggest a possible way out by revisiting the idea of non-Newtonian gravity, which might be present somewhat below the constraints obtained so far, as a manifestation of the scalar field.

There are other aspects of the scalar–tensor theory requiring more careful understanding, including such issues as how the physical "conformal frame" is singled out, and how much time-variability of the gravitational constant there could be. There is also the question arising from the sign of the energy of the scalar field in the original conformal frame, placing string theory further away from the near-complete decoupling.

One of the purposes of this book is to provide detailed accounts of subtleties that might have escaped attention, which are based on naive questions but need to be treated with sufficient caution. We find that this theory, with appropriate modifications, seems to provide a small window through which we can look into what the expected unification theory is.

We then ask whether there are any observational signals that make such a departure from the standard theory urgent. We may consider a modern version of the problem of the cosmological constant as well as observational searches for time-variability of the coupling constants. From this point of view, we apply the scalar–tensor theory to a cosmology with Λ in accordance with the recent discovery from type-Ia supernovae. There has been an expectation, which is sound but nonetheless somewhat vague, that a light scalar field with a universal coupling should play a role. We offer what we hope is a realistic implementation.

We discuss another more phenomenological approach, which is based on a scalar field now widely known as "quintessence." Also included is a brief introduction to "brane cosmology," which might provide a new breed of cosmological model descending from higher-dimensional space-time.

Toward the end of the book we attempt to relate the problem of the cosmological constant to the possible time-dependence of the fine-structure constant. Even though it is still provisional, this argument illustrates how the scalar–tensor theory, a highly constrained theory, has the capability of linking two otherwise disparate phenomena.

Since we focus on a limited range of subjects, we have not attempted to make the book encyclopedic.

We have dealt with some of the technical complications in 14 appendices so that readers can gain a good overview of the underlying flow of our story without being deterred by these complexities. They belong basically to the chapters as shown:

Chapter 1: A, B

Chapter 2: C, D, E, F

Chapter 3: G, H

Chapter 4: J, K

Chapter 5: L

Chapter 6: M, N, O.

Some of them might be skipped entirely. We hope, on the other hand, that some could serve as problems or exercises. Appendix D, for example, might be an answer to the following problem: "Show that the matter energy–momentum tensor is covariantly conserved if $\mathcal{L}_{\rm matter}$ is independent of ϕ." Appendix L was prepared particularly for readers unfamiliar with "brane" geometry. Appendix C is truly fundamental throughout the book. Appendix A might be used as a basis of part of section 5.3, as might Appendix L.

The book is a consequence of discussions with and critical comments from many of our friends and colleagues. Among them we wish to express our thanks particularly to Yuichi Chikashige, Yon-Min Cho, Ephraim Fischbach, Shoichi Ichinose, Takashi Ikegami, Satoru Ikeuchi, Susumu Kamefuchi, Mitsuhiro Kato, Shinsaku Kitakado, Kazuaki Kuroda, Shuntaro Mizuno, Masahiro Morikawa, Wei-Tou Ni, Janis Niedra, Tsuyoshi Nishioka, Nobuyoshi Ohta, Minoru Omote, Takeshi Saito, Misao Sasaki, Tetsuya Shiromizu, Naoshi Sugiyama, Akira Tomimatsu, and Tamiaki Yoneya for their invaluable help in shaping our basic attitude toward the subject.

Our thanks are also due to Humitaka Sato for his having suggested that one of us (Y. F.) begin writing this book. Our gratitude goes also to Yasushi Takahashi who compiled a series including a book in Japanese, whose title translates as *Gravitation and Scalar Field*, by Y. F., which was published by Kodan-sha in 1997, an outgrowth of which constituted a major part of the present book.

For the writing of this book, we consulted Jordan's book to ascertain how it differed from the ensuing paper by Brans and Dicke. A consequence was our adding a short passage in Chapter 1. We leave more stories to a forthcoming publication by Carl Brans, to whom we express our gratitude for his comments and having shown us his thesis.

We thank Owen Parkes, and Bonnie and Patrick Ion for helping us by correcting our English during the earlier periods of our work.

Y. Fujii
K. Maeda

Yokohama
March 2002

Conventions and notation

Greek indices run from 0 through 3, where $x^0 = ct$. The Minkowskian metric is diagonal $(-1, +1, +1, +1)$. Indices with overbars are those for D dimensions. Some of the quantities in higher dimensions are attached as a superscript (D) to the left, but not strictly all the time, as long as no confusion is expected to ensue.

We use the reduced Planckian system of units by choosing $c = \hbar = M_{\rm P} \,(= [8\pi G/(c\hbar)]^{-1/2}) = 1$, yielding the units of length, time, and energy given, respectively, by

$$8.07 \times 10^{-33}\,\mathrm{cm}, \quad 2.71 \times 10^{-43}\,\mathrm{s}, \quad 2.44 \times 10^{18}\,\mathrm{GeV}.$$

In this unit of time, the present age of the universe $(1.1\text{–}1.4) \times 10^{10}$ years is expressed as $10^{60.11}$–$10^{60.21}$.

The Christoffel symbol is defined as usual:

$$\Gamma^{\lambda}{}_{\mu\nu} = \tfrac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}).$$

The same definition applies to higher dimensions as well, but simply with overbars in the indices.

The Riemann curvature tensor is defined by

$$R^{\rho}{}_{\sigma,\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\sigma\nu} + \Gamma^{\rho}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\sigma\nu} - (\text{terms with } \mu \leftrightarrow \nu).$$

The Ricci tensor and scalar curvature are derived as

$$R_{\mu\nu} = R^{\rho}{}_{\mu,\rho\nu} \quad \text{and} \quad R = R^{\mu}{}_{\mu},$$

respectively. $R > 0$ for a sphere.

Symbols used multiply

We followed the usual usage of symbols as much as we could. We could not avoid, however, using some of them for two or more different purposes. Some of them might be worth listing.

- σ Originally the fluctuating part of the scalar field ϕ in the scalar–tensor theory (Chapter 2), but also the scalar field in the E frame, and further used to denote a scalar field in the quintessence models.
- Φ The dilaton field in string theory (Chapter 1), but used also for the scalar field as a simplified representative of (nongravitational) matter fields (Chapters 4–6).
- χ Decomposition of the metric (Chapter 2), also the second scalar field in the two-scalar model (Chapter 5).
- q The exponent of the power of the scalar field ϕ which multiplies Λ in Chapter 4. The same symbol is used also as a negative exponent of σ in the inverse-power potential of the quintessence field in Chapter 5.

Other special symbols

- ξ Related to the original notation ω by $\xi = |\omega^{-1}|/4$.
- ϵ The sign of ω, agreeing also with the sign of the kinetic energy of the scalar field ϕ in the J frame.
- A The radius of internal space in Appendix A for the Kaluza–Klein theory, also with $\mathcal{A} = A^2$. Similar usage is found in Appendix B as well. On the other hand, the scale factor of the universe is denoted by a.
- D The dimensionality of space-time. We also use $d = D/2$.