CHAPTER 1

Introduction and overview

William A. Barnett, David F. Hendry, Svend Hylleberg, Timo Teräsvirta, Dag Tjøstheim, & Allan Würtz

1 Introduction

The theme of the $(EC)^2$ conference held at the University of Aarhus in December 1995 was "Nonlinear modeling in economics". This theme was topical, as the University had just established a new economic and econometric research centre called the "Centre for Nonlinear Modeling in Economics", which is funded by the Danish Social Science Research Council and the Research Foundation of the University of Aarhus.

Economic theory is often nonlinear. Examples of nonlinearities in economics include economic processes with thresholds, capacity constraints restricting production, persistent disequilibria due to rationing, institutional restrictions such as tax brackets, multiple equilibria, and asymmetries of various kinds such as asymmetries in cyclical fluctuations of employment or unemployment due to asymmetric hiring and firing costs. The latter case is an example of the situation where the nonlinear theory is microeconomic theory, and it is not obvious what its implications are on the aggregated level. On the other hand, the literature contains examples of nonlinear macroeconomic models of employment based on or at least inspired by the microeconomic theory of asymmetric adjustment costs. Finally, it may be mentioned that nonlinear economic theory based on the mathematical theory of deterministic processes, called chaos, has also been a topic of discussion in the economics literature.

If the nonlinear economic theory is to be tested with data, the equations to be estimated may be expected to be nonlinear as well. However, a vast majority of econometric equations actually estimated in economics have been linear, most often because the relevant equations in question have been replaced by linear approximations – an approach which has been considered quite successful in practice. Another reason for using linear equations is the desire to avoid "incredible" (Sims 1980) theory and carry out the modeling with as few theory-based assumptions as possible. This has led to growing application of linear vector autoregressive models that have become a very important tool for macroeconomic modelers having to do with stationary and nonstationary series. Besides, statistical theory for linear models is well developed, and in addition it has been possible to develop consistent modeling strategies based on linear models.

The application of the linear model has recently encountered several problems. Tests often reject parameter constancy, indicating so-called structural breaks, defined as changes in the parameters of a linear model. The solution has often been to dismiss the model or to pad with dummy variables in order to repair the deficiencies. As an economy, a market, a firm, or a household is much too complicated to be fully and adequately described by a few linear difference equations, one must expect a linear model to break down from time to time. Even so, frequent breakdowns imply a lack of credibility. However, just to apply a set of shift variables, called dummies, whenever a break is observed is an ad hoc and unsatisfactory solution. A much more satisfactory answer is to apply a specification which allows for nonlinearities.

The problems encountered for the linear models is one reason for the upsurge in popularity of nonlinear econometric models. A second reason is the advances in nonlinear time series analysis. New nonlinear models have been introduced, sometimes in parallel with econometric work, and some of these models have successfully been applied to economic series.

A third reason for the increasing interest in nonlinear models is the enormous growth in computational power available at a relatively low cost to an ordinary researcher. Nonlinear approaches such as nonparametric and semiparametric modeling have gained in popularity just because many methods have only recently become computationally feasible and because new computational possibilities have spurred the development of new statistical methods in the area. In a way, theory-free and flexible nonparametric models may be seen as a nonlinear counterpart of vector autoregressive models, except that the data requirements in nonparametric modeling are even greater than they are when vector autoregressive models are applied to economic series. Thus nonparametric models are likely to play an important role in financial econometrics while continuing to offer more limited possibilities to macroeconomic modelers. But whenever sufficient data is available, nonparametric analyses may also be a useful tool of preliminary analysis preceding the construction of parametric, possibly nonlinear models. See Yatchew (1998) and Tjøstheim (1999) for a review of nonparametric regression techniques.

From the economic theory point of view the choice between a linear or a nonlinear specification is clear: linear models should be used if the theory is linear or may be easily linearized without losing essential elements of it. From the practical point of view, the availability of data is important: it is hardly realistic to fit a nonlinear model to a data set consisting of, say, 20 annual observations. From the econometric point of view the choice may also be based on statistical considerations. In cases where there is a linear model nested in a nonlinear specification, it would be wise and in some cases necessary to test linearity first before considering the maintained nonlinear model. The choice between these two alternatives may also be made afterwards. It can be made by applying misspecification tests and other diagnostic devices and by comparing the out-of-sample forecasting performance of linear and nonlinear models. It should be noted, however, that any choice based on out-of-sample forecasting is a function of the forecasting period, which may or may not contain all the important (dynamic) characteristics of the estimation period. Nevertheless, the risk of overfitting is greater in nonlinear than in linear models, which underlines the importance of forecast comparisons.

Finally, although the extension of the econometrician's toolbox by nonlinear models certainly increases the possibilities for adequate and efficient modeling, criticism can be raised against any specific nonlinear form as well as any specific linear model. Firstly, the basis in theory is often vague, as a specific functional form of the estimating equation is the result of a choice of a specific functional form of the criterion function such as the utility function. The form of the criterion function is almost always chosen for analytical convenience and not because it can be justified by strong theoretical and/or empirical arguments. Secondly, the nonlinear model is also by nature a simplifying construction, which must be expected to break down from time to time. As is the case with linear models, a breakdown of the nonlinear model should lead to a total rethinking of the whole model, and not just to an ad hoc padding up of the observed deficiencies by adding variables and by even further complicating the nonlinear features of the model.

For a more elaborate discussion of nonlinear models in econometrics see the books by Granger and Teräsvirta (1993) and Tong (1990) for an introduction, and Gallant (1987) for a presentation of the statistical problems.

The articles of this volume relate in different ways to the above rather general, but by no means complete, outline of issues, problems, and solutions in nonlinear time series econometrics. A marriage of modern theoretical macroeconomics with a microeconomic foundation and cointegration analysis is suggested in the chapter by William A. Barnett, Barry E. Jones, and Travis D. Nesmith entitled "Time series cointegration tests and nonlinearity". It is argued that much economic theory implies that agents behave according to nonlinear decision rules, but that most cointegration analysis has not explored that avenue of possible nonlinear relations between macroeconomic variables. The choice of variables and the actual definitions of the monetary aggregates applied in the empirical analysis are based on the work on aggregation and index number theory by Barnett and others. This implies the use of the Törnqvist–Theil discrete time approximation to the continuous time Divisia index. In particular, the Törnqvist–Theil quantity variance is applied as a correction for the aggregation error.

The number of cointegrating vectors in a system of variables such as monetary services, the dual cost user index, the monetary service quantity variance, industrial production, and the consumer price index for two levels of aggregation is found by now standard methods based on the work of Johansen and Juselius (1990). The data are constructed using monthly seasonally adjusted data from Thornton and Yue (1992) for the period 1960:1 to 1992:12. The results indicate two cointegrating relations if the quantity variance is included and only one if the quantity variance is excluded from the set of variables in the VAR. To test whether the cointegrating relation is a linear process the frequency domain bispectrum test suggested by Hinich is applied; see Hinich (1982).

In their study "Risk-related asymmetries in foreign exchange markets" of the forward rate as an unbiased predictor of the future spot rate in foreign exchange markets, Giampiero M. Gallo and Barbara Pacini suggest a new procedure to deal with a time-varying risk-related premium. The risk premium is included in the analysis of foreign exchange markets together with interest rate parity. In its simplest form, an unbiasedness hypothesis says that the expected difference between the spot rate and the forward rate is zero. Also, the correlation over time and its variance are important for the efficient market hypothesis. Most studies have found, however, that this is not a good description of data. Instead, a time-varying risk premium can be included, without violating the efficiency market hypothesis.

The first problem is to find a measure of the time-varying risk-related term. There is no general increasing relationship between conditional variance and risk premium. In view of the lack of theory on a measure of risk, Gallo and Pacini adopt a nonparametric measure of risk. This measure is calculated by a latent variable approach, and as a result an instrumental variable is necessary. The conditional expectations are estimated by the Nadaraya–Watson kernel regression estimator. The input is obtained from a consistent parametric estimation of the residuals without instruments. Gallo and Pacini compare their approach with two competing estimators. They find that the unbiasedness hypothesis does not hold, despite the inclusion of a general time-varying risk-related term. The results do confirm, however, that the time-varying risk-related term is important in explaining the exchange rate movements, even more so when trading signals from technical analysis are inserted in the model.

As emphasized in most of the chapters, considerable care needs to be taken when departing from a linear model into classes of nonlinear models. This point is nicely illustrated by Gary Koop and Simon Potter, who address the problem of choosing among linear models and three different alternatives to linearity in their chapter entitled "Nonlinearity, structural breaks, or outliers in economic time series?". The three alternatives to linear models are motivated by empirical macroeconomics in that they allow for different effects of shocks on the dynamics of the model. If the dynamics change in a predictable way over the business cycle, the model is called nonlinear, while a model where the dynamics change in an unpredictable way is called an outlier model. An example of the first class is the threshold AR model, where the change of regime is a function of the lagged endogenous variable. If, instead, the dynamics of the model change in an unpredictable way, two kinds of models are considered, depending on whether a large shock has a permanent or a temporary effect on the dynamics. A temporary effect of an unpredictably large shock is characterized in an outlier model, whereas a structural break model is appropriate if the effect is permanent. In their simplest forms, the outlier model eliminates a few outliers, whereas the structural break model divides the sample period into subsample periods, each with a separate model.

Problems in classical econometrics of unidentified nuisance parameters under the null hypothesis in nonlinear models and selection among competing models are avoided by Koop and Potter by applying a Bayesian approach. To make the estimation tractable, they use prior distributions for which the posterior distribution can be derived analytically. As an example, Koop and Potter compare linear, nonlinear, structural breaks, and outlier models for the growth in US GDP and for the growth in the British industrial production. For each model, they also estimate a version which allows for heteroskedasticity. For the GDP series they find the structural break models to fit considerably better than the other three types of models, whereas there is evidence that the industrial production series is best described by a nonlinear threshold AR model.

Estimation of threshold models in a Bayesian context usually involves calculating high dimensional integrals using, for instance, the Gibbs sampler. In the paper by Michel Lubrano entitled "Bayesian analysis of nonlinear time series models with a threshold", he shows how to specify a threshold model such that only a low dimensional integral needs to be calculated by deterministic integration. The class of threshold models considered by Lubrano includes switching regime models where each regime is characterized by a linear index. The switching function is either a step or a smooth function of time, exogenous variables, or lagged endogenous variables. The same variables are included in all the regimes, and none of these variables enter in the switching function. The key assumption is the choice of prior on the parameters in the linear index. They could be noninformative or natural conjugate prior densities. Then the marginal posterior densities of the parameters of the linear index is calculated as a two dimensional integral, or a three dimensional integral in the case where heteroskedasticity is allowed.

The problem of unidentified parameters under the null hypothesis of a linear model is solved by assuming a particular prior on the parameters in the linear index. Lubrano proves that a noninformative prior leads to a posterior density which is infinite when there is no switching. Instead, this problem is avoided if a partially informative normal prior is imposed. The important property of the partially informative normal prior is that it depends on the parameter characterizing the degree of smoothness in the switching function, namely, the parameter which causes the nonidentification when it equals zero. In addition, to get an integrable posterior density for this smoothness parameter, a convenient prior is the truncated Cauchy density. Using these assumptions and results, Lubrano investigates the French consumption function and US real GNP and industrial production index by applying different types of threshold models.

The consistency and asymptotic normality of the nonlinear least squares estimator of a nonlinear dynamic model is derived by Santiago Mira and Alvaro Escribano, in the chapter "Nonlinear time series models: Consistency and asymptotic normality of NLS under new conditions". The conditions are new and easier to check than the conventional ones. Their results cover parametric nonlinear models which are used in practice, for example, state-dependent and smooth transition autoregressive models.

The time series are allowed to be nonstationary, though excluding unit root processes. The main assumption is that the series are strongly mixing. This assumption replaces the common assumption of stationary ergodicity or geometric ergodicity. The strong mixing assumption allows for some degree of heterogeneity. Both assumptions are hard to test in practice, but geometrically ergodic series are contained in the class of strongly mixing series.

The chapter reviews the basic assumptions as given in for instance Gallant and White (1988), before providing new assumptions which are easier to verify. Together with the assumption on strong mixing, the new conditions on the series are mainly moment conditions. In addition, the regression function must be differentiable and bounded by a linear function. As an application, the new conditions are verified for a smooth transition autoregressive model.

To interpret a cointegrated VAR model, often nonlinear functions of the parameters are of interest, for example, impulse responses. The asymptotic distribution of nonlinear functions of the parameters of a cointegrated VAR model is derived by Pentti Saikkonen and Helmut Lütkepohl in their chapter "Asymptotic inference on nonlinear functions of coefficients of infinite order cointegrated VAR processes". With suitable normalizations, the distribution of the nonlinear functions of the parameters is standard normal. They assume that the cointegrated VAR system is of infinite order, but only a finite order VAR is estimated. To derive the asymptotic distribution of the nonlinear functions of the parameters, the finite lag length of the estimated VAR model increases as a function of the sample size. Hence, the approach can be considered nonparametric.

Using the asymptotic distribution of the nonlinear functions, tests can be derived on hypotheses concerning the nonlinear functions. This is done for a Wald type test. Since the approach involves infinitely many lags, it is possible to test hypotheses with infinitely many restrictions. This is particularly interesting when an impulse in one variable has an impact on other variables for any lead time. A special case of that test is for a finite number of restrictions, for instance, the total effect of an impulse. In both cases, the Wald test statistic has a chi-square distribution. Although the emphasis is on the asymptotic distribution theory, the article also contains applications with different types of impulse responses.

Market frictions, transactions cost, and heterogeneity among traders can lead to nonlinear adjustments toward a long-run equilibrium path. In the standard model with cointegrated variables, the error-correction representation shows that adjustments to the equilibrium path are linear. In the chapter entitled "Nonlinear error-correction models for interest rates in the Netherlands" Dick van Dijk and Philip Hans Franses investigate different models where the adjustment to the equilibrium path is nonlinear. In particular, they consider smooth transition autoregressive adjustments. The strategy for identifying an appropriate specification is firstly to estimate a linear error-correcting model and the number of cointegration relationships. Since the linear error-correction model is misspecified in case of nonlinear adjustments, the consequences for estimating the cointegration relationships are investigated in finite samples using a Monte Carlo study. The Monte Carlo study mainly confirms the asymptotic results that the tests for cointegration and the estimates of the cointegration relationship are not affected by nonlinearity of the error-correcting term.

After finding the cointegrating relationships, the time series representing the error or equilibrium correcting relationship is tested for nonlinearity. While the cointegrating relationship is always assumed linear, the strength of the adjustment is assumed nonlinear under the alternative. The authors use an LM-type linearity test that has power against smooth transition regression. When the test is applied to a pair of Dutch interest rates, the results support the idea of a nonlinear equilibrium correction. One problem often encountered when estimating nonlinear models is that the nonlinearity mainly captures potential outliers. Therefore, the LM tests for nonlinearity are modified by estimating the equations using a robust method. This test suggests that the nonlinearity detected by the original LM test could be caused by a few outliers. Disaggregating the data, however, again provides more evidence for nonlinearity – a finding which accords with previous findings in the literature.

The editors believe that this selection of articles is a useful addition to the econometric literature on nonlinearities and nonlinear models and will lead to further investigations in this rapidly growing field.

REFERENCES

Gallant, A. R. (1987), *Nonlinear Statistical Models*. New York: Wiley.

Gallant, A. R., and H. White (1988), *A Unified Theory of Estimation and Inference for Nonlinear Dynamic Models*. New York: Blackwell.

Granger, C. W. J., and T. Teräsvirta (1993), *Modelling Nonlinear Economic Relationships.* Oxford: Oxford University Press.

Hinich, M. J. (1982), "Testing for Gaussianity and Linearity of a Stationary Time Series", *Journal of Time Series Analysis* 3: 169–76.

Johansen, S., and K. Juselius (1990), "Maximum Likelihood Estimation and Inference on Cointegration – with an Application to the Demand for Money", *Oxford Bulletin of Economics and Statistics* 52: 169–210.

Sims, C. A. (1980), "Macroeconomics and Reality", *Econometrica* 48: 1–49.

Thornton, D. L., and P. Yue (1992), "An Extended Series of Divisia Monetary Aggregates", *Federal Reserve Bank of St. Louis Review*, November/December, 35–52.

Tjøstheim, D. (1999), "Nonparametric Specification Procedures for Time Series", in S. Ghosh (ed.), *Asymptotics, Nonparametrics and Time Series*. New York: Marcel Dekker.

Tong, H. (1990), *Non-linear Time Series. A Dynamical System Approach*. Oxford: Clarendon Press.

Yatchew, A. (1998), "Nonparametric Regression Techniques in Economics", *Journal of Economic Literature* 36: 669–721.

CHAPTER 2

Time series cointegration tests and non-linearity

William A. Barnett, Barry E. Jones, & Travis D. Nesmith

1 Introduction

Modern macroeconomic theory emphasizes the interactions among representative agents (households and firms) who are, in general, assumed to behave according non-linear decision rules that are obtained as optimal solutions to dynamic optimization problems. Consequently, it is reasonable to posit the existence of non-linear relationships among macroeconomic variables.

During the last decade, as theoretical macroeconomics has been concerned with microeconomic foundations, cointegration has become one of the most important characterizations of macroeconomic time series. For example, real business cycle research now commonly assumes balanced growth between output, consumption, and investment, and stable long run money demand with unit income elasticity; see for example King and Watson (1996). These assumptions imply the existence of cointegration relations among the key business cycle variables. Cointegration studies have, however, rarely explored the possibility of non-linear relationships among macroeconomic variables. $I(1)$ cointegration analysis focuses on non-stationary economic variables that are integrated of order one, meaning that their first differences are stationary. The existence of

William Barnett is a professor of economics at Washington University in St. Louis. Barry E. Jones and Travis D. Nesmith were Ph.D. candidates at Washington University in St. Louis and visiting scholars at the Research Department of the Federal Reserve Bank of St. Louis when this paper was written. Barry E. Jones and Travis Nesmith wish to thank the Federal Reserve Bank of St. Louis for partially supporting this research. Any views expressed in this article are those of the authors and not necessarily those of the Federal Reserve Bank of St. Louis or the Federal Reserve System. William Barnett acknowledges partial support from National Science Foundation grant SES9223557. The authors wish to thank Melvin Hinich for providing them with the code for his bispectrum test and for his helpful technical advice. They wish to thank Richard Anderson, John Keating, Lisbeth la Cour, Yi Liu, Houston Stokes, and Haiyang Xu for their helpful comments and suggestions. Any remaining errors are the authors'.

cointegration implies that some linear combination of these integrated variables is stationary. The integrated variables may be non-linear stochastic processes – a hypothesis that is seldom entertained either as a feature of the data-generating process or as a convenient statistical description of the data in the cointegration literature. This paper begins to address this gap between the implications of modern dynamic macroeconomic theory and the cointegration literature by applying tests for non-linearity to the stationary linear combinations produced from cointegration.

A number of studies have tested for the existence of non-linearity in macroeconomic data (Hinich and Patterson 1985, Barnett and Chen 1988, Barnett and Hinich 1992, Brock and Sayers 1988); however, most of the existing non-linearity tests are univariate, and some of the available non-linearity tests are not invariant to prior linear filtering of the data.[1] In this chapter, we investigate the application of univariate non-linearity tests to stationary linear combinations of non-stationary (and possibly non-linear) macroeconomic time series, which have been identified through cointegration analysis. Thus, rather than testing the first differences of individual economic time series for non-linearity, we test the long run relationships between those series.

The remainder of this paper is organized as follows: in Section 2, we review the relevant aggregation theory and indexes number theory; in Section 3, we present the results of the cointegration analysis; in Section 4, we present the results of the non-linearity tests; and Section 5 concludes.

2 Aggregation and indexes number theory

In this section, we briefly review the monetary aggregation theory motivating the choice of variables in our empirical analysis; for more extensive reviews, see Barnett (1987, 1990), Barnett, Fisher, and Serletis (1992), and Anderson, Jones, and Nesmith (1997a), in which the most general conditions under which monetary aggregates exist are discussed.

Arrow and Hahn (1971) showed that if monetary assets are valued in general equilibrium, there exists a derived utility function containing monetary assets. If we assume that a representative agent exists and that current period monetary assets are blockwise weakly separable in that agent's utility function, a conditional second stage monetary services allocation decision exists. In that second stage, the representative agent can be viewed as solving a current period

[1] Barnett, Gallant, Hinich, Jungeilges, Kaplan, and Jensen (1994, 1995) study the power of several competing univariate non-linearity tests with artificial and monetary data respectively. Hinich and Wilson (1992) analyzes the cross bispectrum, which can detect multivariate non-linear relationships.