1 Introduction

1.1 Degrees of Freedom

The *number of degrees of freedom* (DOF) of a system is equal to the number of independent parameters (measurements) that are needed to uniquely define its position in space at any instant of time. The number of DOF is defined with respect to a reference frame.

Figure 1.1 shows a rigid body (RB) lying in a plane. The rigid body is assumed to be incapable of deformation, and the distance between two particles on the rigid body is constant at any time. If this rigid body always remains in the plane, three parameters (three DOF) are required to completely define its position: two linear coordinates (x, y) to define the position of any one point on the rigid body, and one angular coordinate θ to define the angle of the body with respect to the axes. The minimum number of measurements needed to define its position are shown in the figure as x, y, and θ. A rigid body in a plane then has three degrees of freedom. Note that the particular parameters chosen to define its position are not unique. Any alternative set of three parameters could be used. There is an infinity of sets of parameters possible, but in this case there must always be three parameters per set, such as two lengths and an angle, to define the position because a rigid body in plane motion has three DOF.

Six parameters are needed to define the position of a free rigid body in a three-dimensional (3-D) space. One possible set of parameters that could be used are three lengths (x, y, z), plus three angles $(\theta_x, \theta_y, \theta_z)$. Any free rigid body in three-dimensional space has six degrees of freedom.

1.2 Motion

A rigid body free to move in a reference frame will, in the general case, have complex motion, which is simultaneously a combination of rotation and

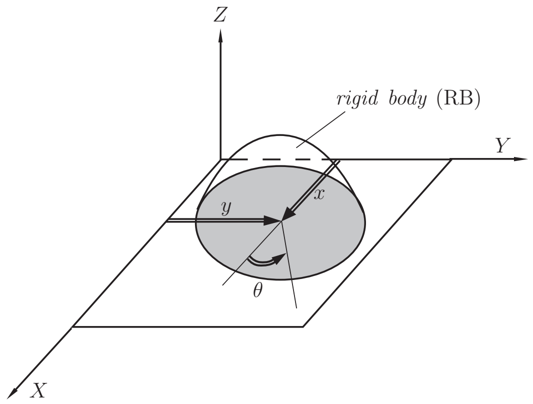

Figure 1.1. RB in planar motion with three DOF: translation along the x axis, translation along the y axis, and rotation, θ, about the z.

translation. For simplicity, only the two-dimensional (2-D) or planar case will be presented. For planar motion the following terms will be defined (see Fig. 1.2).

- Pure rotation is that in which the body possesses one point (center of rotation) that has no motion with respect to a "fixed" reference frame; see Fig. 1.2(a). All other points on the body describe arcs about that center.
- Pure translation is that in which all points on the body describe parallel paths; see Fig. 1.2(b).
- Complex motion is that which exhibits a simultaneous combination of rotation and translation; see Fig. 1.2(c). With general plane motion, points on the body will travel nonparallel paths, and there will be, at every instant, a center of rotation, which will continuously change location.

Translation and rotation represent independent motions of the body. Each can exist without the other. For a 2-D coordinate system, as shown in Fig. 1.1, the x and y terms represent the translation components of motion, and the θ term represents the rotation component.

1.3 Links and Joints

Linkages are basic elements of all mechanisms. Linkages are made up of links and joints. A *link*, sometimes known as an *element* or a *member*, is an (assumed) rigid body that possesses nodes. *Nodes* are defined as points at which links can be attached. A link connected to its neighboring elements by s nodes is an element of *degree* s. A link of degree 1 is also called unary, as in Fig. 1.3(a); that of degree 2, binary, as in Fig. 1.3(b); that of degree 3, ternary, as in Fig. 1.3(c); and so on.

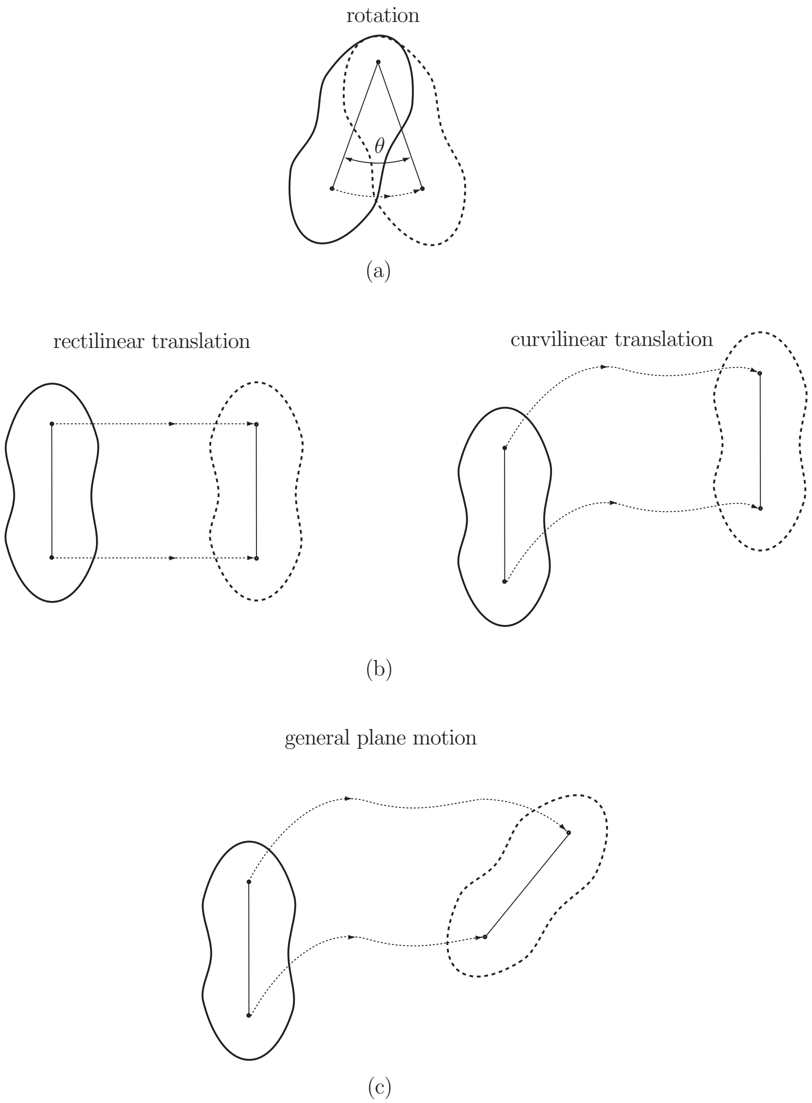

Figure 1.2. RB in motion: (a) pure rotation, (b) pure translation, and (c) general motion.

A *joint* is a connection between two or more links (at their nodes). A joint allows some relative motion between the connected links. Joints are also called *kinematic pairs*.

The number of independent coordinates that uniquely determine the relative position of two constrained links is termed the *degree of freedom* of a given joint. Alternatively the term *joint class* is introduced. A kinematic pair is of the jth class if it diminishes the relative motion of linked bodies by j

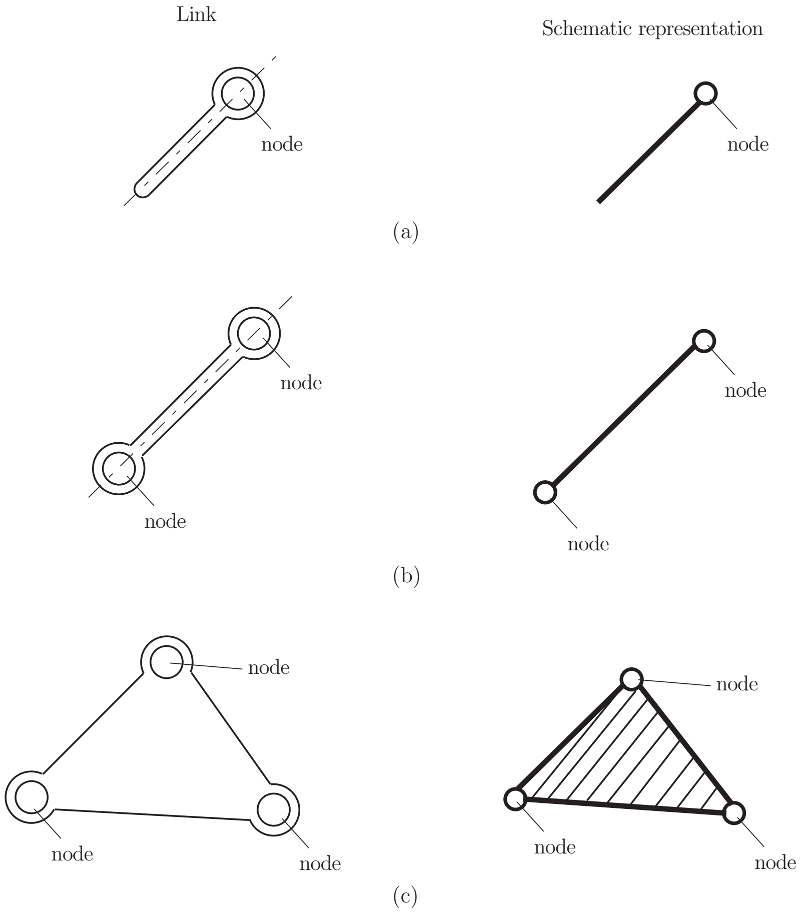

Figure 1.3. Types of links: (a) unary, (b) binary, and (c) ternary elements.

degrees of freedom; that is, j scalar constraint conditions correspond to the given kinematic pair. It follows that such a joint has $(6j)$ independent coordinates. The number of degrees of freedom is the fundamental characteristic quantity of joints. One of the links of a system is usually considered to be the reference link, and the position of other RBs is determined in relation to this reference body. If the reference link is stationary, the term *frame* or *ground* is used.

The coordinates in the definition of degree of freedom can be linear or angular. Also, the coordinates used can be absolute (measured with regard to the frame) or relative. Figures 1.4–1.9 show examples of joints commonly found in mechanisms. Figures 1.4(a) and 1.4(b) show two forms of a planar, one degree of freedom joint, namely a rotating pin joint and a translating slider

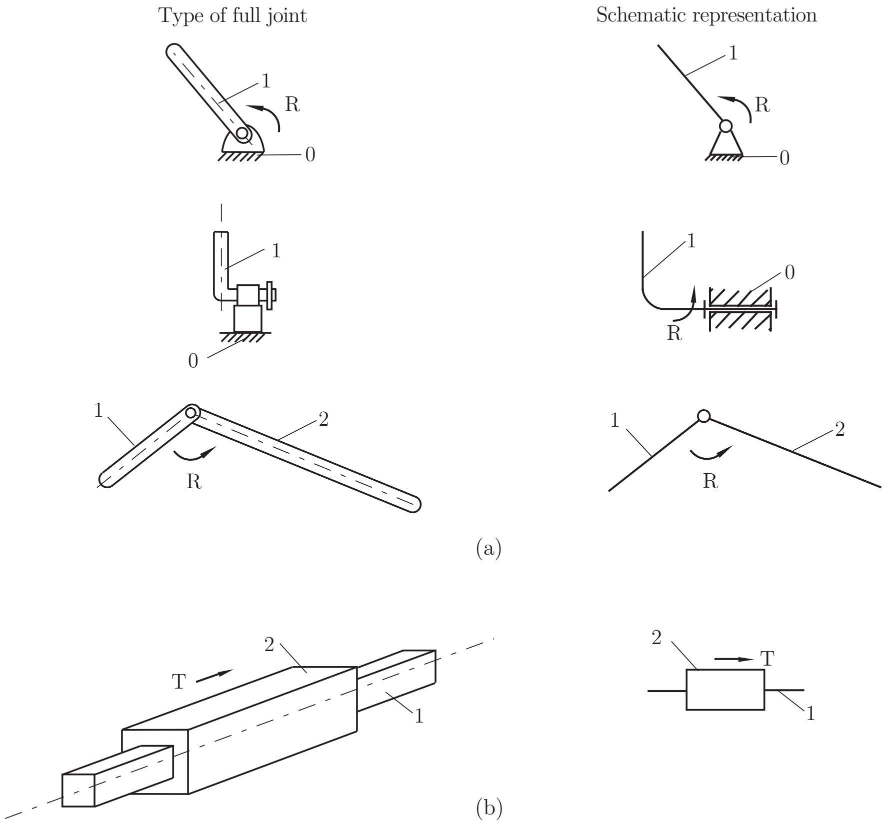

Figure 1.4. One degree of freedom joint, full joint (fifth class): (a) pin joint and (b) slider joint.

joint. These are both typically referred to as *full joints* and are of the fifth class. The pin joint allows one rotational (R) DOF, and the slider joint allows one translational (T) DOF between the joined links. These are both special cases of another common, one degree of freedom joint, the screw and nut, shown in Fig. 1.5(a). Motion of either the nut or the screw relative to the other results in helical motion. If the helix angle is made zero, Fig. 1.5(b), the nut rotates without advancing and it becomes a pin joint. If the helix angle is made 90°, the nut will translate along the axis of the screw, and it becomes a slider joint.

Figure 1.6 shows examples of two degrees of freedom joints, which simultaneously allow two independent, relative motions, namely translation (T) and rotation (R), between the joined links. A two degrees of freedom joint is usually referred to as a *half-joint* and is of the fourth class. A half-joint is sometimes also called a roll–slide joint because it allows both rotation (rolling) and translation (sliding).

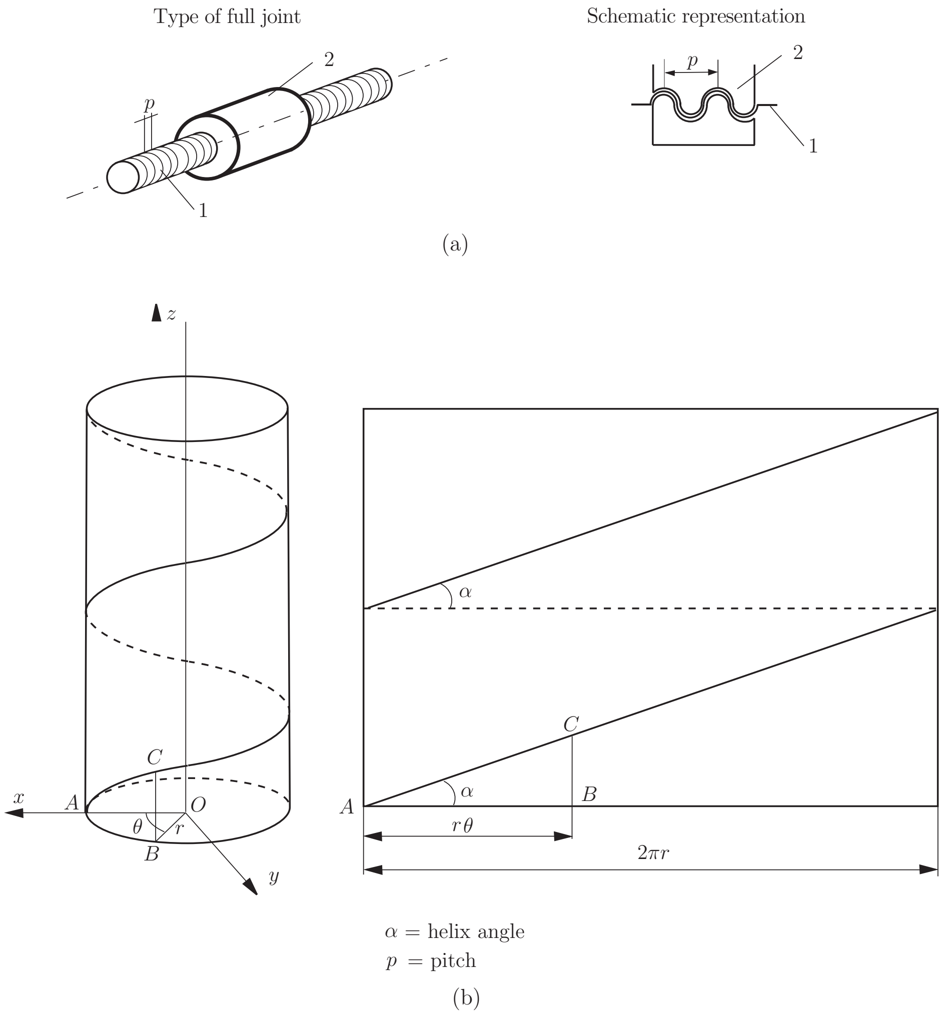

Figure 1.5. (a) Screw and nut joint; (b) helical motion.

A joystick, ball-and-socket joint, or sphere joint, shown in Fig. 1.7(a), is an example of a three degrees of freedom joint (third class), which allows three independent angular motions between the two links that are joined. This ball joint would typically be used in a 3-D mechanism, one example being the ball joints used in automotive suspension systems. A plane joint, Fig. 1.7(b), is also an example of a three degrees of freedom joint, which allows two translations and one rotation.

Note that in order to visualize the degree of freedom of a joint in a mechanism, it is helpful to "mentally disconnect" the two links that create the joint from the rest of the mechanism. It is easier to see how many degrees of

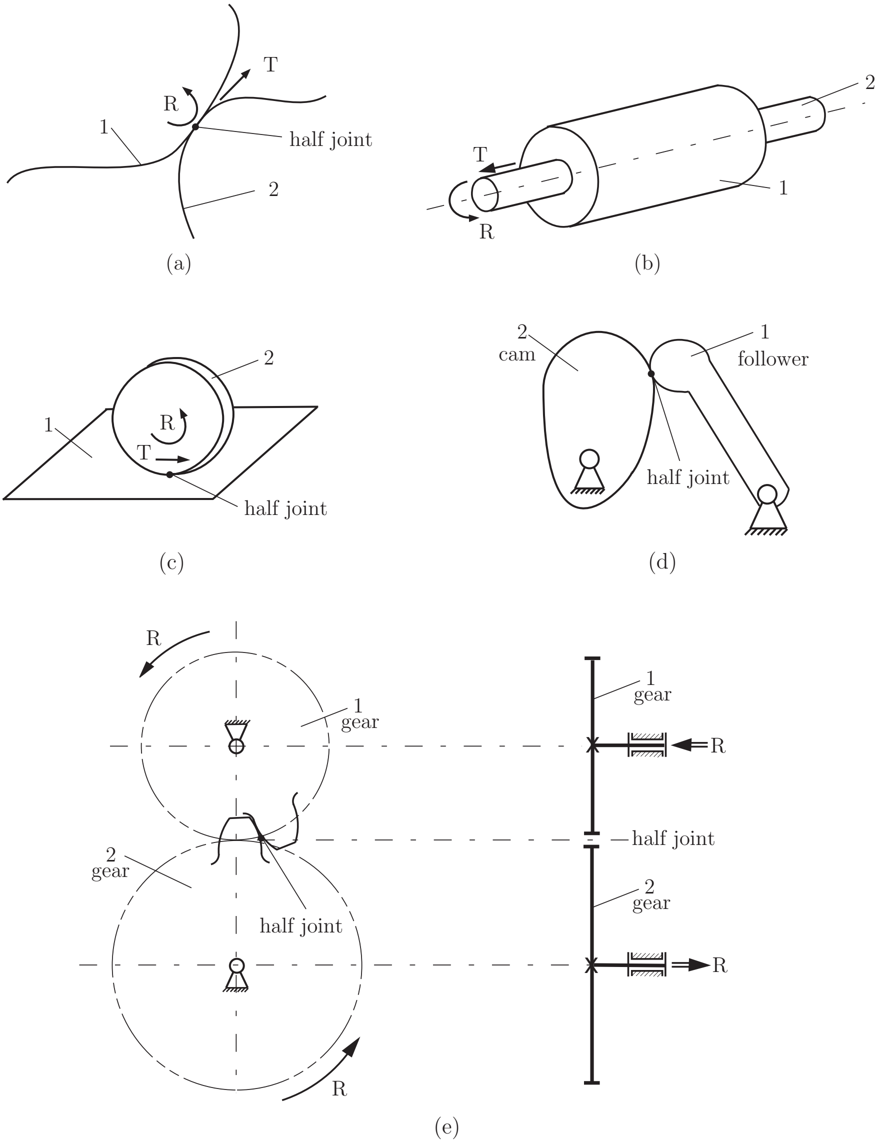

Figure 1.6. Two degrees of freedom joint, half-joint (fourth class): (a) general joint, (b) cylinder joint, (c) roll–slide disk, (d) cam-follower joint, and (e) gear joint.

freedoms the two joined links have with respect to one another. Figure 1.8 shows an example of a second class joint (cylinder on plane), and Fig. 1.9 represents a first class joint (sphere on plane).

The type of contact between the elements can be point (P), curve (C), or surface (S). The term *lower joint* was coined by Reuleaux to describe joints with

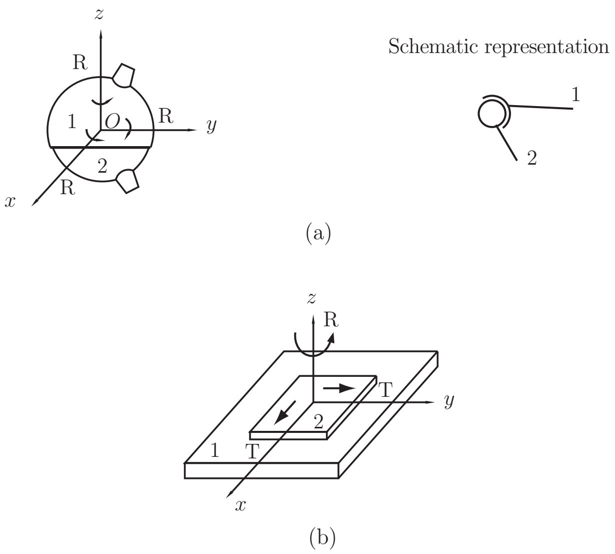

Figure 1.7. Three degrees of freedom joint (third class): (a) ball and socket joint, and (b) plane joint.

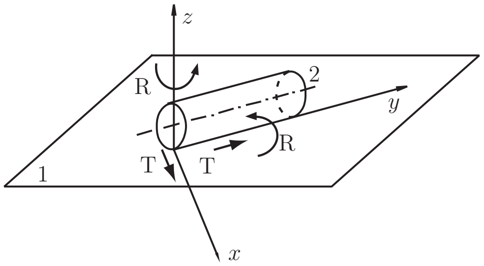

Figure 1.8. Four degrees of freedom joint (second class) cylinder on a plane.

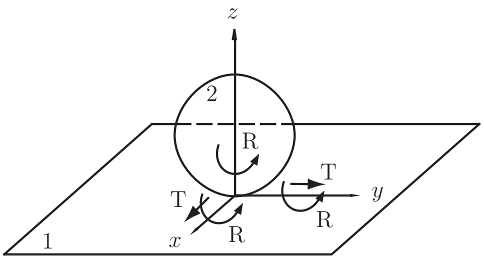

Figure 1.9. Five degrees of freedom joint (first class) sphere on a plane.

surface contact. He used the term *higher joint* to describe joints with point or curve contact. The main practical advantage of lower joints over higher joints is their better ability to trap lubricant between their enveloping surfaces. This is especially true for the rotating pin joint.

A *closed joint* is a joint that is kept together or closed by its geometry. A pin in a hole and a slider in a two-sided slot are forms of closed joints. A *force closed joint*, such as a pin in a half-bearing or a slider on a surface, requires some external force to keep it together or closed. This force could be supplied by gravity, by a spring, or by some external means. In linkages, closed joints are usually preferred, and they are easy to accomplish. For cam-follower systems, force closure is often preferred.

The *order of a joint* is defined as the number of links joined minus one. The simplest joint combination of two links has order one and it is a single joint, shown in Fig. 1.10(a). As additional links are placed on the same joint, the order is increased on a one for one basis, as shown in Fig. 1.10(b). Joint order

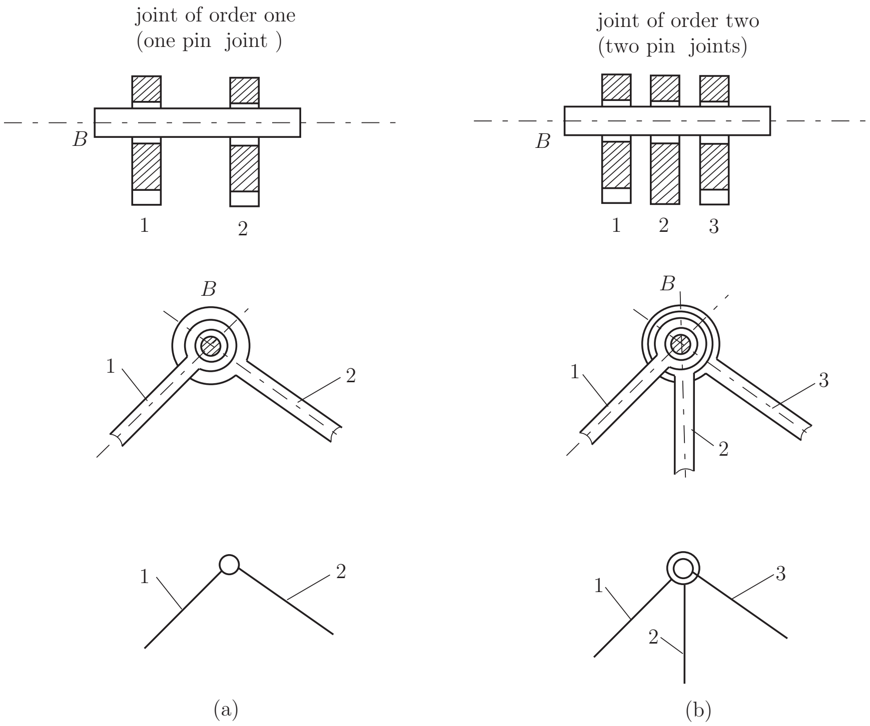

Figure 1.10. Order of a joint: (a) joint of order one, and (b) joint of order two (multiple joints).

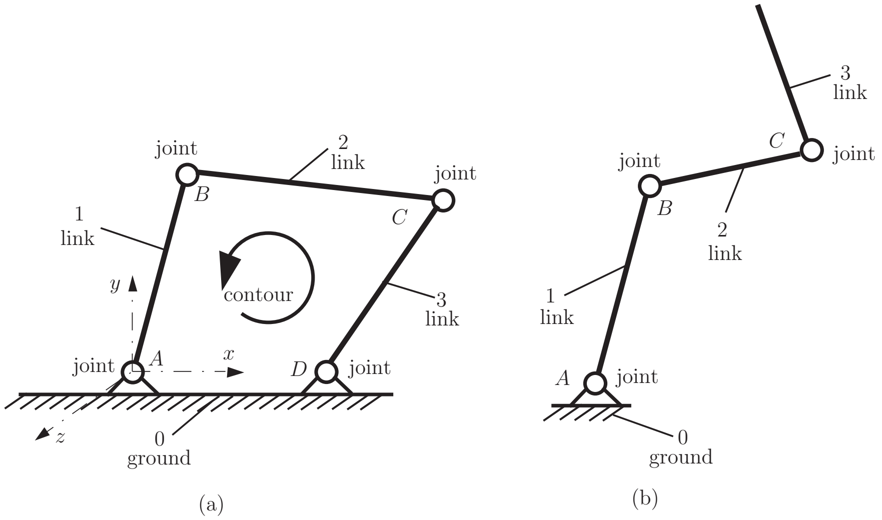

Figure 1.11. Kinematic chains: (a) closed and (b) open kinematic chains.

has significance in the proper determination of overall degrees of freedom for an assembly.

Bodies linked by joints form a *kinematic chain*. Simple kinematic chains are shown in Fig. 1.11.

A *contour* or *loop* is a configuration described by a polygon, as shown in Fig. 1.11(a).

The presence of loops in a mechanical structure can be used to define the following types of chains.

- *Closed kinematic chains* have one or more loops so that each link and each joint is contained in at least one of the loops, as shown in Fig. 1.11(a). A closed kinematic chain has no open attachment point.
- *Open kinematic chains* contain no loops, as shown in Fig. 1.11(b). A common example of an open kinematic chain is an industrial robot.
- *Mixed kinematic chains* are a combination of closed and open kinematic chains.

Another classification is also useful.

- *Simple chains* contain only binary elements.
- *Complex chains* contain at least one element of degree 3 or higher.

A *mechanism* is defined as a kinematic chain in which at least one link has been "grounded" or attached to the frame, as shown in Figs. 1.11(a) and 1.12. According to Reuleaux's definition, a *machine* is a collection of mechanisms