This book deals with the effect of crystal symmetry in determining the tensor properties of crystals. Although this is a well-established subject, the author provides a new approach using group theory and, in particular, the method of symmetry coordinates, which has not been used in any previous book.

Using this approach, all tensors of a given rank and type can be handled together, even when they involve very different physical phenomena. Applications to technologically important phenomena as diverse as the electro-optic, piezoelectric, photoelastic, piezomagnetic, and piezoresistance effects, as well as magnetothermoelectric power and third-order elastic constants are presented. Attention is also given to 'special magnetic properties', that is those that require the concepts of time reversal and magnetic symmetry, an important subject not always covered in other books in this area.

This book will be of interest to researchers in solid-state physics and materials science, and will also be suitable as a text for graduate students in physics and engineering taking courses in solid-state physics.

CRYSTAL PROPERTIES VIA GROUP THEORY

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To Joan,
the source of my inspiration
and to symmetry,
a source of beauty in our lives

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Preface

The study of the anisotropic properties of crystals, often called 'Crystal Physics', is the oldest branch of solid-state physics, dating back to the turn of the twentieth century and the treatises of W. Voigt. It deals with the 'matter tensors' that describe such anisotropic properties, and the way that these tensors are simplified as a result of the existence of crystal symmetry. In recent years, there have been many textbooks on this subject. Most widely known is that by J. F. Nye (*Physical Properties of Crystals*, Oxford University Press, 1957), who introduced matrices and tensors to create a more compact notation than that used earlier, but did not use group theory.

Group theory provides the ideal mathematical tools for dealing with these problems elegantly and compactly. These methods have been used by various authors, notably Fumi, Bhagavantum and Juretshke. However, the usefulness of group theory was not always recognized. In fact Nye (page 122 of his book), commenting on work using group theory, states: 'group theory . . . does not reveal which moduli are independent but only the total number of independent ones'. The present book is dedicated to showing, not only that this statement is untrue, but that the use of group theory lends elegance and beauty to what would otherwise be dull calculations. In this book we utilize the method of symmetry coordinates, very much as is used in the study of molecular vibrations (e.g. as described in the book by Wilson, Decius and Cross). This method has not been employed in other books on the present subject, even those that utilize group theory.

The plan of the book is as follows. In the first two chapters we introduce the full range of properties to be studied (electrical, magnetic, mechanical, dielectric, optical, transport, etc.), including the concept of matter tensors. Here we distinguish between equilibrium properties (Chapter 1) and transport properties (Chapter 2), each of which has a different theoretical basis. The tensors introduced are classified by their ranks and intrinsic symmetries. This scheme, whereby the properties are handled separately from the questions of crystal symmetry, is unique to the present book. Next, in Chapter 3, we introduce the concepts of crystal symmetry and of group theory. The reader who has had previous exposure to group theory may treat this chapter as a review, but a reader with no previous background may study it as an introduction to the subject. Indeed he/she will find such a background useful in studying other branches of solid-state physics as well.

In Chapter 4, we show how the method of symmetry coordinates can be applied to the study of matter tensors in crystals. A unique feature is the presentation of the 'Symmetry-Coordinate Transformation tables' for all 32 point groups. These include the requirement of similarity of orientation for those symmetry coordinates that belong to two- or three-dimensional irreducible representations. (To my knowledge, such tables are not given in any other book on group theory.)

The subsequent chapters then deal with the various tensors by rank, rather than by their physical nature. Thus we show that the simplification of matter tensors due to crystal symmetry is a mathematical problem, and may occur in exactly the same way for widely diverse physical properties that have the same tensor rank and intrinsic symmetry. These chapters deal, in ascending complexity, with tensors up to sixth rank (so as to cover the 'third-order elastic constants'). Along the way, 'special magnetic properties', namely, those that require consideration of time reversal and of magnetic point groups in order to analyze them correctly, are handled separately (in Chapters 5 and 8).

Although the subject of the book is a mathematical one, practical examples of materials that show such diverse and technologically important phenomena as the piezoelectric effect, elasticity, the electro-optic effect, photoelasticity, piezoresistance, magnetoresistance and third-order elasticity are discussed. Properties of materials of special interest, such as quartz, the YBCO high-temperature superconductor and lithium niobate are used for illustration, where appropriate.

The material in Chapters 1–8 can comfortably constitute a one-semester course. Problems are given at the end of each chapter, and a list of general references for further reading, which is combined with the specific references mentioned in the text, is given at the end of the book.

I am particularly grateful to the late William R. Heller, with whom I first carried out research in crystal properties and in defect relaxation phenomena using the present method of symmetry coordinates. He introduced me to this method and was a wonderful collaborator. I am also indebted to my colleague Daniel N. Beshers and to several former graduate students, particularly Wing-Kit Lee, Shiun Ling, Diane S. Richter and Tracey Scherban, all of whom gave critical comments which contributed to improving the manuscript. Finally, I wish to thank A. Continenzà for the quartz-crystal model that appears on the front cover.

A. S. Nowick