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## Tensor properties of crystals: equilibrium properties

A crystal is made up of a regular arrangement of atoms in a pattern that repeats itself in all three spatial dimensions. This structural feature has the practical result that most properties of crystals are anisotropic, that is, different values are obtained for the property when measured along the different directions of a crystal. This anisotropy distinguishes crystals from non-crystalline materials (glasses) or from random polycrystalline aggregates, both of which show isotropic properties.

The purpose of this book is to show how to analyze the anisotropic properties of crystals in terms of the (tensor) nature of the properties and of the symmetry of the crystals. In the first two chapters, we focus on the nature of the various properties with which we will be dealing. We will see how the tensor character of a property helps to define its variation with orientation. Questions of crystal symmetry will then be dealt with, starting from Chapter 3.

### 1–1 Definition of crystal properties

For a crystal, regarded as a thermodynamic system (i.e. in equilibrium with its surroundings), any physical property can be defined by a relation between two measurable quantities. For example, the property of crystal density is simply the ratio of mass to volume, both measurable quantities; similarly, elastic compliance is the ratio of mechanical strain to stress. Often, one of the measurable quantities can be regarded as a generalized ‘force’ and the other as a response to that force. Examples of such ‘forces’ are electric field, magnetic field or mechanical stress, while the corresponding responses are, respectively, electric displacement, magnetic induction and strain. A given response,  $Y_i$ , is said to be ‘conjugate’ to a force  $X_i$  if the product  $X_i\Delta Y_i$  that accompanies a change  $\Delta Y_i$  in the value

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of  $Y_i$  represents an element of work *per unit volume* done on the system. According to the first and second laws of thermodynamics:

$$dU = X_i dY_i + T dS \quad (1-1)$$

where  $U$  is the internal energy per unit volume,  $T$  the Kelvin temperature and  $S$  the entropy per unit volume. In the  $X_i dY_i$  term a summation over  $i$  is implied, in accordance with the Einstein summation convention that when a given suffix appears twice in the same term, summation with respect to the suffix is to be understood. (This convention will be used throughout this book.) In view of Eq. (1),\* it is also convenient to regard the temperature  $T$  to be a ‘force’ and entropy  $S$  as its conjugate response, in which case Eq. (1) may simply be written as

$$dU = X_i dY_i \quad (1-2)$$

Although the contributions to  $U$  come from conjugate pairs, it does not follow that a force  $X_i$  only gives rise to its conjugate response  $Y_i$ . In fact, in general,  $X_i$  must be considered to give rise to various  $Y_j$ . To a first-order approximation, it is often assumed that a given response  $Y_i$  is, in fact, *linearly* related to all the forces acting on the material. Thus, we have

$$Y_i = K_{ij} X_j \quad (1-3)$$

This equation assumes implicitly that our reference state is that for which  $Y_i$  and  $X_j$  are both zero and that all values  $Y_i$  and  $X_j$  remain small. More generally, we may write linear equations for changes of these variables from reference values  $Y_{i0}$  and  $X_{j0}$  as

$$Y_i - Y_{i0} = K_{ij}(X_j - X_{j0}) \quad (1-4)$$

(The form of (4) is particularly appropriate for the conjugate variables  $T$  and  $S$ , for which reference values are not conveniently taken as zero.)

Equation (3) or Eq. (4) represent nothing more than a Taylor expansion of  $Y_i$  in terms of all the  $X_j$  carried out to terms of first order. (Carrying the expansion further leads to higher-order effects which will be dealt with later in this chapter.) An equation such as Eq. (3) is called a ‘constitutive relation’, and the quantities  $K_{ij}$  represent the physical properties of the crystal. Thus, while the magnitude of a force can be chosen arbitrarily, in order to obtain any given response, the quantity  $K_{ij}$ , which is a response per unit force, is a property of the crystal, over which the experimenter has no control once the material is selected.

\* Throughout this book, reference to equations in the same chapter will be given without the chapter number, for example, Eq. (1) rather than (1-1); where equation numbers other than those from a current chapter are referred to, these will be preceded by the appropriate chapter number.

An important thermodynamic result is that the matrix of coefficients  $K_{ij}$  is symmetric, namely,

$$K_{ij} = K_{ji} \quad (1-5)$$

This result may be proved as follows. We define a thermodynamic potential,  $\Phi$ , by

$$\Phi \equiv U - X_i Y_i \quad (1-6)$$

Then by combining Eqs. (2) and (6) we obtain for the differential of  $\Phi$

$$d\Phi = -Y_i dX_i \quad (1-7)$$

Since, from thermodynamics, the internal energy function is a state function of all the  $X_i$  taken as independent variables, the function  $\Phi$  is also such a state function. Accordingly, it follows that

$$\partial Y_i / \partial X_j = \partial Y_j / \partial X_i = -(\partial^2 \Phi / \partial X_i \partial X_j) \quad (1-8)$$

When this result is compared with Eq. (3) or Eq. (4) it is clear that Eq. (5) follows.

The result of Eq. (5) may be called the *intrinsic symmetry* of the physical property matrix  $K_{ij}$ , since it follows from the nature of the property itself and does not require for its validity any symmetry on the part of the crystal under consideration. This intrinsic symmetry is to be contrasted to that imposed by crystalline symmetry, which will be the major subject of the later chapters of this book.

### 1-2 Physical quantities as tensors; tensor properties

Inherently, a crystal is an anisotropic medium. This means that the response of a crystal to an external 'force' depends not only on the magnitude of that force but also on its orientation relative to the crystal axes. In such cases, the measurable quantities (forces and responses) are components of vectors or tensors, and it is then advantageous to group these components together as appropriate tensor quantities. Thus, for example, the electric field  $\mathbf{E}$  is a vector with three components  $E_1$ ,  $E_2$  and  $E_3$  which lie parallel to three mutually perpendicular axes. The conjugate response, the electric displacement  $\mathbf{D}$ , is also a vector quantity.\* In the absence of any other fields, these two quantities are related by

$$\mathbf{D} = \kappa \mathbf{E} \quad (1-9)$$

\* One could equally well take the polarization  $\mathbf{P}$  as conjugate to  $\mathbf{E}$ , the relation between  $\mathbf{D}$  and  $\mathbf{P}$  being:  $\mathbf{D} = \kappa_0 \mathbf{E} + \mathbf{P}$ , where  $\kappa_0$  is the permittivity of vacuum.

where  $\kappa$  is a second-rank symmetric tensor, called the dielectric permittivity tensor, that relates the two vector quantities. (It is symmetric by virtue of Eq. (5).) Thus, just as the forces and responses have been combined in this case to form vectors  $\mathbf{E}$  and  $\mathbf{D}$ , the appropriate crystal properties can be grouped together in the form of a tensor quantity  $\kappa$ . We may then speak of such quantities as ‘tensor properties’ of crystals, or simply as ‘matter tensors’.

As background to this discussion, for the reader who will find it useful, Appendix A contains a brief review of the theory of tensors. It deals with such questions as the rank of a tensor and the difference between polar and axial tensors.

### 1–3 The basic linear relations

In order to introduce some important linear relations and the corresponding tensor properties, we consider the following four types of ‘forces’ and their conjugate ‘responses’:

Type	‘Force’	‘Response’
Thermal	Change in temperature, $\Delta T$	Change in entropy/vol., $\Delta S$
Electrical	Electric field, $\mathbf{E}$	Electric displacement, $\mathbf{D}$
Magnetic	Magnetic field, $\mathbf{H}$	Magnetic induction, $\mathbf{B}$
Mechanical	Stress, $\boldsymbol{\sigma}$	Strain, $\boldsymbol{\varepsilon}$

The thermal quantities  $\Delta T$  and  $\Delta S$  are both scalars, the electrical quantities  $\mathbf{E}$  and  $\mathbf{D}$  are vectors (or polar vectors), the magnetic quantities  $\mathbf{H}$  and  $\mathbf{B}$  are axial vectors, and, finally, the mechanical quantities  $\boldsymbol{\sigma}$  and  $\boldsymbol{\varepsilon}$  are symmetric second-rank tensors (with six independent components). Thus, tensor types of rank 0–2 are represented in the table above.

If we write the linear constitutive equations in component form, we have

$$\left. \begin{aligned} \Delta S &= (C/T)\Delta T + p_i E_i + q_i H_i + \alpha'_{ij} \sigma_{ij} \\ D_i &= p'_i \Delta T + \kappa_{ij} E_j + \lambda_{ij} H_j + d_{ijk} \sigma_{jk} \\ B_i &= q'_i \Delta T + \lambda'_{ij} E_j + \mu_{ij} H_j + Q_{ijk} \sigma_{jk} \\ \varepsilon_{ij} &= \alpha_{ij} \Delta T + d'_{ijk} E_k + Q'_{ijk} H_k + s_{ijkl} \sigma_{kl} \end{aligned} \right\} \quad (1-10)$$

This set of equations, which is nothing more than an explicit form of Eq. (3), covers all possible first-order interactions of the four types of physical

quantity presented in the table above. If we wish to take advantage of the tensor character of the quantities involved, we may group the ‘forces’ and ‘responses’ together as tensors and write the  $K_{ij}$  coefficients in the following block form:

$C/T$	$\mathbf{p}$	$\mathbf{q}$	$\alpha'$
$\mathbf{p}'$	$\kappa$	$\lambda$	$\mathbf{d}$
$\mathbf{q}'$	$\lambda'$	$\mu$	$\mathbf{Q}$
$\alpha$	$\mathbf{d}'$	$\mathbf{Q}'$	$s$

Here the matrices  $\kappa$  and  $\mu$  have dimensions  $3 \times 3$ ,  $s$  is  $6 \times 6$ ,  $\mathbf{q}$  is  $3 \times 1$ , and so on. The diagonal line drawn through the blocks is the symmetry line, about which the relation  $K_{ij} = K_{ji}$ , that is, Eq. (5), is applicable. The diagonal blocks:  $C/T$ ,  $\kappa$ ,  $\mu$  and  $s$ , are called *principal effects*, since they relate a given response tensor to its conjugate ‘force’ tensor. The off-diagonal blocks are called *cross effects* or *interaction effects*. They show that a given ‘force’ may produce non-conjugate responses. Not all cross effects need to occur for all crystals, in the sense of being significant in magnitude, but they must all be anticipated in order to allow for complete generality.

We next identify all the quantities in these equations, beginning with the principal (diagonal) matter tensors. Thus  $C$  is the specific heat per unit volume at constant fields  $\mathbf{E}$ ,  $\mathbf{H}$  and  $\sigma$  and, as such, is a generalization of the usual specific heat at constant pressure. It is a scalar crystal property since it relates two scalar quantities. The quantities  $\kappa_{ij}$  are components of the dielectric permittivity tensor already given by Eq. (9). This tensor relates two polar vectors and is therefore a tensor of second rank. Further, it is symmetric by virtue of Eq. (5). The quantities  $\mu_{ij}$  are components of the magnetic permeability tensor  $\mu$  which relates  $\mathbf{B}$  to  $\mathbf{H}$ , that is,

$$\mathbf{B} = \mu\mathbf{H} \tag{1-11}$$

The tensor  $\mu$  relates two axial vectors and thus transforms as a (polar) second-rank tensor. (The reason is that any transformation that changes the sign of  $\mathbf{H}$  must do the same for  $\mathbf{B}$ . See Appendix A.) It is also

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symmetric in view of Eq. (5). Note that these principal effects themselves include off-diagonal components, such as  $\kappa_{ij}$  and  $\mu_{ij}$  for  $i \neq j$ .

The last of the principal matter tensors in Eq. (10) are those which relate strain to stress, namely, the elastic compliance constants  $s_{ijkl}$ . In Appendix B, we have reviewed the definitions of the six components of strain  $\varepsilon_{ij}$  and of stress  $\sigma_{ij}$ , both of which are symmetric second-rank tensors, that is,  $\varepsilon_{ij} = \varepsilon_{ji}$  and  $\sigma_{ij} = \sigma_{ji}$ . It is shown there that  $\varepsilon_{11}$ ,  $\varepsilon_{22}$  and  $\varepsilon_{33}$  are tensile components while  $\varepsilon_{23}$ ,  $\varepsilon_{31}$  and  $\varepsilon_{12}$  are the shears, and similarly for the stress components  $\sigma_{ij}$ . Thus, if we write the generalized Hooke's law:

$$\boldsymbol{\varepsilon} = \mathbf{s}\boldsymbol{\sigma} \quad (1-12)$$

the matrix  $\mathbf{s}$  whose components are  $s_{ijkl}$  represents a fourth-rank tensor called the *elastic compliance* tensor. It is symmetric in the interchange of  $i$  and  $j$ , or  $k$  and  $l$ , and also that of  $i, j$  for  $k, l$ . (The first two symmetries are valid because  $\boldsymbol{\varepsilon}$  and  $\boldsymbol{\sigma}$  are symmetric, and the latter because of Eq. (5).) It follows from these symmetries that there are 21 components in this elastic compliance tensor. If Eq. (12) is inverted, we may write

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} \quad (1-13)$$

in which  $\mathbf{c}$  is a fourth-rank tensor called the *elastic stiffness* tensor, which has the same intrinsic symmetry and number of independent coefficients as the  $\mathbf{s}$  tensor.

We now turn to the off-diagonal or cross effects, keeping in mind the symmetry derived from thermodynamic considerations, Eq. (5). The coefficients  $p_i$ , which represent the entropy (or heat) produced by an electric field, are components of the *electrocaloric effect*, a matter tensor which is a vector quantity since it couples a scalar response to a vector field. The converse effect,  $p'_i$ , giving the electric displacement caused by a temperature change is called the *pyroelectric effect*. From the reciprocity relations, Eq. (5), it is clear that  $p'_i = p_i$ . Similarly, the quantities  $q_i$  constituting the *magnetocaloric effect* are components of an axial vector, as are the components  $q'_i$  of the converse effect, called the *pyromagnetic effect*, with  $q'_i = q_i$ .

The cross effect  $\alpha_{ij}$ , which relates strain to  $\Delta T$ , constitutes a set of *thermal expansion* coefficients. In general, there are six coefficients, meaning that, in an arbitrary crystal, a change in temperature can produce not only tensile but also shear deformations as well. Since the set of coefficients  $\alpha_{ij}$  comprise the components of a strain, they constitute a second-rank symmetric matter tensor. The converse coefficients  $\alpha'_{ij}$  constitute the *piezocaloric effect*, and by Eq. (5) and the symmetry of  $\alpha_{ij}$  we

must have

$$\alpha'_{ij} = \alpha_{ji} = \alpha_{ij} \quad (1-14)$$

so that the direct and converse coefficients are equal.

Still another cross-effect is represented by the coefficients  $\lambda_{ij}$  which relate a polar vector ( $\mathbf{D}$ ) to an axial vector ( $\mathbf{H}$ ). The quantity  $\lambda$ , called the *magnetolectric polarizability* tensor, is then a second-rank *axial* tensor, and it is not symmetric. (This can easily be seen from its position with respect to the diagonal in the block form on page 5.) Thus  $\lambda$  has, in general, nine components. The coefficients of the converse tensor  $\lambda'_{ij}$  must obey

$$\lambda'_{ij} = \lambda_{ji} \quad (1-15)$$

as a consequence of Eq. (5).

Next we turn to interactions between electrical and mechanical properties, the so-called 'piezoelectric effects'. The coefficients  $d_{ijk}$  relate to the electric displacement produced in a crystal subject to stress; as such they couple a second-rank tensor to a polar vector and must therefore form a tensor of third rank. Since stress is symmetric in its indices,  $d_{ijk}$  must be symmetric in the interchange of  $j$  and  $k$ . The existence of  $d_{ijk}$ , the *direct piezoelectric effect* implies (by the reciprocity relations) the existence of a converse effect, that is, that application of an electric field will give rise to an appropriate strain. From Eq. (5) the quantitative relationship between the *converse piezoelectric effect*  $d'_{pqr}$  and the direct effect is

$$d'_{pqr} = d_{rpq} \quad (1-16)$$

In addition, both the direct and converse quantities are symmetric in the interchange of  $p$  and  $q$  in this equation, leading to a total of 18 piezoelectric coefficients in the most general case.

The final cross effect which appears in Eqs. (10) is the *piezomagnetic effect*  $Q_{ijk}$  and its converse  $Q'_{pqr}$ . These matrices each relate a second-rank symmetric tensor to an axial vector and thus constitute a pair of third-rank axial tensors. As in the case of the piezoelectric effect,  $Q_{ijk}$  is symmetric in the interchange of  $j$  and  $k$  and is related to the converse effect by

$$Q'_{pqr} = Q_{rpq} \quad (1-17)$$

$Q'_{pqr}$  is often called the *magnetostrictive tensor*.

The various equalities of cross effects that follow from Eq. (5), such as those given by Eqs. (14)–(17), have the qualitative meaning that if a force  $X_j$  produces a response  $Y_i$ , a force  $X_i$  will of necessity produce a response  $Y_j$ . In addition, however, such equations give a *quantitative* relationship between what might have appeared to be unrelated effects. The

importance of these quantitative thermodynamic relationships for the experimenter is that it allows him/her to choose the measurement that is easier to carry out for evaluating a given cross-effect. For example, in piezoelectricity, it may be easier to measure the direct effect (electrical response produced by stress) than the converse effect (strain resulting from an electric field). It is reassuring that, by measuring such a property, the converse property is obtained as well.

Table 1–1 (located at the end of the chapter on page 18) lists these various matter tensors, as well as others which will be introduced later in this chapter. Included in this table (second column) is the tensor character of each property using notation  $T(n)$  for a tensor of rank  $n$ , a superscript ax for axial quantities and a subscript S to denote that it is symmetric (the fifth column showing exactly what intrinsic symmetry exists among the coefficients). Thus  $T(0)$ ,  $T(1)$  and  $T(1)^{\text{ax}}$  denote, respectively, a scalar, polar vector and axial vector property, respectively. The third column shows the tensor characters of the two measurable quantities that the given matter tensor relates. The final column gives the maximum number of independent components of the property in question, based on its intrinsic symmetry but in the absence of any crystal symmetry. The reader should recall that the intrinsic symmetry can come from either the definition of the property in question or the thermodynamic reciprocity relations of the type of Eq. (5).

#### 1–4 Condensation of indices: the ‘engineering’ stresses and strains

In much of the solid-state and engineering literature the awkwardness of the need for double indices to specify components of stress and strain is avoided by using a single-index notation running from 1 to 6 instead of from 1 to 3. The convention, which yields the so-called ‘engineering’ stresses and strains, converts  $\sigma_{ij}$  into  $\sigma_k^e$  (the superscript e standing for ‘engineering’) in the following way:

$$\begin{array}{rcccccc} ij: & 11 & 22 & 33 & 23 & 31 & 12 \\ k: & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

in which  $k = 1, 2, 3$  represents the tensile components and 4, 5, 6 the shear components. Thus,  $\sigma_1^e = \sigma_{11}$ , and  $\sigma_4^e = \sigma_{23}$ , etc. In the case of the strains, however, the conversion of indices is the same, but additional factors of 2 are introduced in the shear strains, as follows:

$$\begin{array}{lll} \varepsilon_1^e = \varepsilon_{11}, & \varepsilon_2^e = \varepsilon_{22}, & \varepsilon_3^e = \varepsilon_{33} \\ \varepsilon_4^e = 2\varepsilon_{23}, & \varepsilon_5^e = 2\varepsilon_{31}, & \varepsilon_6^e = 2\varepsilon_{12} \end{array}$$

In this way, two-index compliances are introduced in place of the four-index quantities of Eq. (10):

$$\epsilon_i^e = s_{ij}^e \sigma_j^e \tag{1-18}$$

and for the stiffness constants:

$$\sigma_i^e = c_{ij}^e \epsilon_j^e \tag{1-19}$$

the summations now being from 1 to 6. Thus  $s_{ij}^e$  and  $c_{ij}^e$  are  $6 \times 6$  matrices which are symmetric as a consequence of Eq. (5). This condensation of indices for stress and strain may also be extended to the piezoelectric and the piezomagnetic coefficients.

The entire set of Eqs. (10) may then be rewritten in terms of these condensed indices, with the introduction of appropriate engineering quantities, and allowing all indices for stresses and strains to run from 1 to 6:

$$\left. \begin{aligned} \Delta S &= (C/T)\Delta T + p_i E_i + q_i H_i + \alpha_i^e \sigma_i^e \\ D_i &= p_i \Delta T + \kappa_{ij} E_j + \lambda_{ij} H_j + d_{ij}^e \sigma_j^e \\ B_i &= q_i \Delta T + \lambda_{ji} E_j + \mu_{ij} H_j + Q_{ij}^e \sigma_j^e \\ \epsilon_i^e &= \alpha_i^e \Delta T + d_{ji}^e E_j + Q_{ji}^e H_j + s_{ij}^e \sigma_j^e \end{aligned} \right\} \tag{1-20}$$

In this equation, we have also taken advantage of reciprocity relations already discussed that relate the primed physical properties to the unprimed.

For later use, it is advantageous to give the conversions here between the original tensor properties of Eq. (10) and the corresponding ‘engineering’ properties in Eq. (20), taking cognizance of the factor of 2 which entered in the definition of the engineering shear strains. Thus,

$$\begin{aligned} s_{ijkl} &= s_{mn}^e && \text{when } m \text{ and } n \text{ are } 1, 2 \text{ or } 3 \\ 2s_{ijkl} &= s_{mn}^e && \text{when either } m \text{ or } n \text{ is } 4, 5 \text{ or } 6 \\ 4s_{ijkl} &= s_{mn}^e && \text{when } m \text{ and } n \text{ are } 4, 5 \text{ or } 6 \\ \alpha_{ij} &= \alpha_k^e && \text{when } k = 1, 2 \text{ or } 3 \\ 2\alpha_{ij} &= \alpha_k^e && \text{when } k = 4, 5 \text{ or } 6 \\ d_{ijk} &= d_{il}^e && \text{when } l = 1, 2 \text{ or } 3 \\ 2d_{ijk} &= d_{il}^e && \text{when } l = 4, 5 \text{ or } 6 \end{aligned}$$

while  $Q_{ijk} \rightarrow Q_{il}^e$  obeys the same relations as for  $d_{ijk} \rightarrow d_{il}^e$ . Note that the two-index matrices  $\mathbf{d}^e$  and  $\mathbf{Q}^e$  are  $3 \times 6$  (non-square) matrices. There are no factors of 2 and 4 in the conversion of the tensor stiffness constants  $c_{ijkl}$  to engineering quantities  $c_{mn}^e$ .

It is important to note that, while the set of Eqs. (20) looks simpler than that of Eqs. (10), the apparent simplicity is deceiving, because in making

the condensation of indices, we have given up our ability to easily handle the transformation of stresses and strains under a change of axes (as given, for example, in Appendix A). In other words, the proper tensor character of the crystal properties has been lost in the engineering notation. We will see later that group theory provides ways of obtaining the advantage of both condensed indices and knowledge of the transformation properties by using appropriate six-vectors for stress and strain. For the present, however, we need to introduce the engineering notation because, in the literature, it is customary to give numerical values of these properties based on this notation rather than on the tensor notation.

### 1–5 Effect of changing the conditions of measurement

In Eqs. (2) and (10), we have taken the ‘forces’ as independent variables and the ‘responses’ as dependent. Thus, the meaning of any quantity whose definition does not involve a given force is that it is to be measured with that force held constant. For example, the dielectric permittivity  $\kappa_{ij}$  in Eq. (10) is to be thought of as measured at constant values of  $T$ ,  $\mathbf{H}$  and  $\boldsymbol{\sigma}$  as well as those of  $E_k$  for  $k \neq j$ . Similarly, the quantity  $d_{ijk}$  is  $\partial D_i / \partial \sigma_{jk}$  for constant values of  $T$ ,  $\mathbf{E}$ ,  $\mathbf{H}$  and stress components other than  $jk$ .

Now, for each conjugate pair of quantities listed in the table on page 4, we may choose either the ‘force’ or the ‘response’ as the independent variable, the other being regarded as dependent. Sets of linear constitutive equations similar to Eqs. (10) may then be written in terms of the selected independent variables. In this way we can produce many variations of Eqs. (10), each with coefficients having different meanings. However, any one such set of equations contains, in principle, all the physical information from which the other sets can be derived.

The relationship between coefficients with different quantities held constant can be derived thermodynamically. First we illustrate, with a principal effect, the elastic compliances. Modern measurements of elastic constants often use high-frequency sound-wave propagation (Truell *et al.*, 1969), leaving no time for heat flow into or out of a strained region. Therefore, changes are adiabatic (constant  $S$ ) rather than isothermal. Writing the thermoelastic parts of Eq. (10) only, we have

$$\left. \begin{aligned} \Delta S &= (C^\sigma/T)\Delta T + \alpha_{ij}\sigma_{ij} \\ \varepsilon_{ij} &= \alpha_{ij}\Delta T + s_{ijkl}^T\sigma_{kl} \end{aligned} \right\} \quad (1-21)$$

where  $C^\sigma$  is specific heat at constant stress and superscript T indicates that the constants are isothermal. To obtain adiabatic constants, we set  $\Delta S = 0$