

Scattering of Waves from Large Spheres

This book describes the scattering of waves, both scalar and electromagnetic, from impenetrable and penetrable spheres. It provides an extensive introduction for those first studying the field, as well as a guide to further analysis and application for researchers in this area.

Although the scattering of plane waves from spheres is an old subject, there is little doubt that it is still maturing as a broad range of new applications demands an understanding of finer details. In this book attention is focused primarily on spherical radii much larger than incident wavelengths, along with the asymptotic techniques required for physical analysis of the scattering mechanisms involved. Applications to atmospheric phenomena such as the rainbow and glory are included, as well as a detailed analysis of optical resonances. Extensions of the theory to inhomogeneous and nonspherical particles, collections of spheres, and bubbles are also discussed.

This book will be of primary interest to graduate students and researchers in physics (particularly in the fields of optics, the atmospheric sciences, and astrophysics), electrical engineering, physical chemistry, and some areas of biology.

WALTER T. GRANDY, JR is Professor of Physics, Emeritus, at the University of Wyoming, where he has been a member of the Department of Physics and Astronomy for over 36 years, serving as Chairman 1971–1979. He has also been a visiting professor at the University of Arizona (1974), the Universidade de São Paulo (1966–1967, 1982), the Universität Tübingen (1979), and the University of Sydney (1988). His major professional interests have been in the fields of statistical mechanics, electrodynamics, classical and quantum scattering theory, and relativistic quantum mechanics. He has numerous publications in these fields and is author of several earlier books, including *Introduction to Electrodynamics and Radiation* (1970), a two-volume work titled *Foundations of Statistical Mechanics* (1987, 1988), and *Relativistic Quantum Mechanics of Leptons and Fields* (1991).

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Walter T. Grandy, Jr
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Preface

Although the scattering of plane waves from spheres is an old subject, there is little doubt that it is still maturing as a broad range of new applications demands an understanding of finer details. The classical theory of electromagnetic scattering from dielectric spheres is due to Lorenz, Mie, and Debye, and has proved to be enormously rich; it is still being developed and continues to yield new insights. Much of this development has been motivated by the availability of small silicon spheres that can be probed precisely with laser light, as well as by new techniques in acoustics, in atmospheric physics, and in the study of biological molecules.

The classic treatise in the subject has long been van de Hulst's *Light Scattering by Small Particles* (1957), supplemented in later years by the application-oriented works of Kerker, *The Scattering of Light and Other Electromagnetic Radiation* (1969), and Bohren and Huffman, *Absorption and Scattering of Light by Small Particles* (1983). These volumes, and others, have contributed greatly to the subject, while concerning themselves primarily (though not exclusively) with scattering from particles whose dimensions are on the order of an incident wavelength or less. Among my reasons for writing the present book, however, is a long-time interest in understanding the detailed physics of the rainbow and glory in terms of modern scattering theory, and these phenomena arise from water droplets whose dimensions are a great deal larger than optical wavelengths. Thus, the time seems ripe for a theoretical exposition extending the earlier works to encompass a broader range of phenomena.

The complete mathematical solution to the problem of light scattering from a sphere was obtained over a century ago as the well-known infinite series of partial waves. This series, which is generically known as the Mie solution, contains in principle all of the physics of the problem for any size of particle and all wavelengths. For particle radii less than a wavelength only

a few terms of the series need be retained and a satisfying physical picture is readily constructed. As the particle size increases relative to the wavelength, however, the series becomes very slowly convergent and the job of extracting the important physics from it becomes quite difficult.

For these reasons much of the theoretical work in this field over the past 30 years has been computational in nature, out of necessity. While this is often a useful approach, providing insightful graphical representations of the solutions, it is a difficult (and sometimes impossible) way to uncover the physical origins of many of the interesting phenomena. The computational results represent only a part of the complete picture, and in practice should complement an effective analytic treatment when possible. In the present work the emphasis is therefore on exploring analytically that vast region of relative particle sizes between those for which a pure wave picture of the interaction is effective, and the particle-like regime of geometric optics, where the Mie series tends to obscure its own physical content. In analogy with the quantum-mechanical theory of scattering, this can be referred to as the *semiclassical* domain. Such a task has been aided immensely by the work of Moysés Nussenzveig who, beginning some 30 years ago, developed the relevant asymptotic methods through analytic continuation of the Mie sum into the complex angular momentum plane.

My goal in this monograph has been to provide a self-contained discussion of the scattering of scalar and electromagnetic waves from spherical targets, yet one that calls upon the prospective reader to be somewhat familiar with classical electromagnetic theory and optics, as well as with some elementary quantum mechanics. The latter reflects a desire for mathematical maturity and physical background, rather than for a mastery of content. It is hoped that the level of sophistication asked of the reader is only what is found at the advanced undergraduate and early graduate levels in the physical sciences. The aim is to emphasize strongly the physical mechanisms at work and, in contrast with more application-oriented works, to view the subject more from the approach and language of the theoretical physicist. In this sense Chapter 1 should effectively be a review for the reader, in that the general ideas will be rather familiar. We establish notation and conventions here, as well as provide a perspective for the overall intent of the book, and also set the stage for what follows.

Chapter 2 continues in this vein, but at a higher and more important level. The special case of scalar waves scattering from an impenetrable sphere is treated here by way of an introduction to the mathematical techniques and physical mechanisms that lead to a complete analysis for the transparent sphere in the ensuing chapters. This analysis is carried out in Chapters 3, 4,

5, and 7, with a significant digression in Chapter 6 providing applications to meteorological phenomena. Further applications and extensions of the theory, such as illumination by Gaussian beams rather than plane waves, are reviewed in Chapter 8.

There are six mathematical appendices dedicated to self-containment, which calls for some further comment. In a number of places throughout the text the mathematical development is extensive enough that it is simply not possible to include all the detailed steps in an argument. Much of this detail must be supplied by the reader and involves the asymptotic and other properties of Bessel and Legendre functions. To ease the pain somewhat I have collected together in Appendices A–D what I hope is all the required information on these functions. Appendix E provides a brief introduction to the asymptotic analysis of functions defined by definite integrals, including the saddle-point method of steepest descents. A number of computational issues are discussed briefly in Appendix F, and we have employed the *Mathematica*[®] system for doing mathematics on a computer for almost all the computations in the book, although not necessarily for all the plotting.

I would like to absolve myself of not forewarning the reader by calling attention here to a number of mathematical notational conventions, some standard, some not, even though they are re-stated later in context. Primes on a function always denote differentiation with respect to the argument, as in $f'(x)$, whereas primes on a variable distinguish x' from x . An asterisk on a variable or function always denotes complex conjugation, and vectors are always denoted by boldface type. In addition, a caret over a vector identifies it as a unit vector, such as \hat{r} . Gaussian units are employed throughout, so that E , D , H , and B all have the same dimensions. Some authors include a factor $(-1)^m$ in the definition of the associated Legendre function $P_l^m(\cos \theta)$; we do not, preferring to include that factor in the definition of spherical harmonics. Finally, a *caveat* regarding our notation for Bessel, spherical Bessel, and Ricatti–Bessel functions: other choices are sometimes used by other authors.

Portions of the book were written with the help of some resources provided by the University of Hawaii, which I gratefully acknowledge. I am also indebted to Jennifer Cash for assistance with a number of plots. Professor Lee Schick carefully read the entire manuscript and provided many editorial as well as technical suggestions. We both know what a chore that is, but it is difficult to express my appreciation adequately here, other than to acknowledge that debt.

W. T. Grandy, Jr