Parametric Sensitivity in Chemical Systems

The behavior of a chemical system is affected by many physicochemical parameters. The sensitivity of the system’s behavior to changes in parameters is known as parametric sensitivity. When a system operates in a parametrically sensitive region, its performance becomes unreliable and changes sharply with small variations in parameters. Thus, it is of great value to those who design and operate chemical reactors and systems to be able to predict sensitivity behavior.

This book is the first to provide a thorough treatment of the concept of parametric sensitivity and the mathematical tool it generated, sensitivity analysis. The emphasis is on applications to real situations. The book begins with definitions of various sensitivity indices and describes the numerical techniques used for their evaluation. Extensively illustrated chapters discuss sensitivity analysis in a variety of chemical reactors – batch, tubular, continuous-flow, fixed-bed – and in combustion systems, air pollution processes, and metabolic processes. In addition, various plots and simple formulas are provided to readily evaluate the operational behavior of reactors. Chemical engineers, graduate students, researchers, chemists and other practitioners will welcome this valuable resource.

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To our parents
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Preface

The behavior of physical and chemical systems depends on values of the parameters that characterize the system. The analysis of how a system responds to changes in the parameters is called parametric sensitivity. For the purposes of reliable design and control, this analysis is important in virtually all areas of science and engineering. While similar concepts and techniques can be applied in different types of systems, we focus on chemical systems where chemical reactions occur.

In many cases, when one or more parameters are varied slightly, while holding the remaining parameters fixed, the response of a chemical system also changes slightly. However, under other sets of parameter combinations, the chemical system may respond with an enormous change, even if one or more parameters are varied only slightly. In this case, we say that the system behaves in a parametrically sensitive manner. Clearly, it becomes difficult to control the chemical system when it operates in a parametrically sensitive region, and sometimes this leads to so-called runaway behavior that ends up with catastrophic results. This book is concerned with parametric sensitivity and parametrically sensitive behavior of chemical systems, analyzed with a unified conceptual and theoretical framework.

In Chapter 2, we define various sensitivity indices and illustrate numerical techniques that are commonly used for their evaluation. Then, in Chapters 3 to 4, sensitivity analysis is used to identify the parametrically sensitive regions in various types of reactors, such as batch, tubular, continuous-flow stirred tank, and fixed-bed, where either a single or complex reactions occur. In Chapter 7, we use explosions in hydrogen–oxygen mixtures as an example to show that the same analysis can be used to quantify critical ignition conditions in combustion systems. Chapters 8–10 comprise the second part of the book, where sensitivity analysis is employed as an effective mathematical tool to analyze various chemical systems. These include mechanistic studies and model reduction in chemical kinetics, air pollution, and metabolic processes.

This book should appeal to all who are interested in the behavior of chemical systems, including chemists and chemical, mechanical, aerospace, and environmental
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engineers. Also, the applied mathematicians should find here a rich source of interesting mathematical problems. Finally, we hope that industrial practitioners will find the concepts and results described in this book to be useful for their work.

This book can be used either as a text for a senior graduate-level specialized course, or as a supplementary text for existing courses in reaction engineering, applied mathematics, design, and control. In this context, although we do not provide unsolved problems at the end of chapters, there are a relatively large number of examples illustrating the concepts and results. The book can also be used as a reference for industrial applications in reactor design, operation and control.

It is a pleasure to acknowledge here our debt of gratitude to Professor John H. Seinfeld of the California Institute of Technology. He encouraged our writing from the beginning, and looked over drafts of Chapters 2 and 9, providing valuable suggestions for improvements. In addition, Dr. Vassily Hatziimanikatis of du Pont Central Research Department kindly provided a keen evaluation of our draft of Chapter 10.

The last thought goes to our families. Our wives (Karen, Luisella, and Guixian) and children (Anita and Sophia; Melissa and Orestes; Xi and Dino) deeply support us and our work, even as they suffer some neglect during the course of writing projects such as this. We cherish their love and affection.

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