A variety of models can be used to study nuclear structure. This book gives a comprehensive overview of these various models, concentrating in particular on a description of deformed and rotating nuclei. Emphasis is given throughout to the important physical features, rather than to esoteric theoretical topics.

Beginning with a treatment of the semi-empirical mass formula and nuclear stability, the liquid-drop model is then described and its use in the study of nuclear deformation and fission is discussed. The spherical nuclear one-particle potential is introduced and developed to cover the case of deformed nuclei. The main features of the shell correction method are described, with applications to nuclear deformation, fission, superheavy elements and rotation. A detailed discussion of terminating rotational bands and superdeformation is included. Finally, the nucleon–nucleon interaction is briefly described and the main features of the nuclear pairing interaction are discussed.

As well as treating important experimental and theoretical aspects of this fundamental subject, many problems and solutions are included, which help to illustrate key concepts. The book will be invaluable to graduate students of nuclear physics and to anyone engaged in research in this field.
SHAPES AND SHELLS IN NUCLEAR STRUCTURE
SHAPES AND SHELLS IN NUCLEAR STRUCTURE

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Preface

It is our intention in this volume to describe in an elementary way some of the achievements of the nuclear shell model, especially in its form used for deformed nuclei and rotating nuclei. In recent applications on nuclear deformation, fission and rotation, the microscopic shell model is merged with macroscopic models, which are thus also briefly discussed here. We try to concentrate on physical features rather than theoretical methods, not introducing more sophisticated models than are needed to understand the basic principles.

We have tried to put the presentation on such a level that the book should be suitable as a first course in theoretical nuclear physics. The reader is supposed to be familiar with the very basic concepts of experimental nuclear physics and to have some knowledge of quantum mechanics including central forces and angular momentum, the spin formalism and some perturbation theory. We also believe that this volume should be useful for people doing research in nuclear physics, not least for experimentalists.

The introductory chapter defines in a very elementary way the building blocks of nuclei, how these building blocks are held together and how nuclei might decay. It then also becomes natural to discuss the boundaries of nuclear stability and the abundance of different nuclei.

The subjects of the three following chapters relate to the macroscopic properties of nuclei. The size and average matter distribution of nuclei are discussed in chapter 2. The semi-empirical mass formula is introduced in chapter 3 and the liquid-drop model of nuclear deformation and fission is treated in chapter 4.

The single-particle concept is then introduced in chapter 5, making comparisons between the electron system and the nucleon system. The mean field concept leading to the introduction of a single-particle potential is discussed. The single-particle potential of a spherical nucleus is treated in some
detail in chapter 6. As the reader is not assumed to be familiar with the quantum-mechanical formalism for coupling of angular momentum vectors (the Clebsch–Gordan formalism), this formalism is presented as an appendix. Some elementary applications of the spherical single-particle potential are taken up in chapter 7 where the simplest static moments of nuclei with few particles outside closed shells are calculated.

The measured quadrupole moments indicate that it is not enough to study a spherical potential. Thus, in chapter 8, the orbitals of a deformed single-particle potential are discussed, i.e. this chapter deals with the so-called Nilsson model and related subjects, which probably more than anything else have made Sven Gösta Nilsson’s name well-known among nuclear physicists.

In chapter 9, it is shown how realistic calculations on nuclear ground state properties can be carried out if the single-particle model and the macroscopic model are combined. The methods introduced here make it possible to calculate the nuclear energy as a function of the most important shape degrees of freedom. The energy surfaces can be used as input for barrier penetration calculations of nuclear fission. This is demonstrated in chapter 10 where alpha-decay, which can be treated by similar methods, is also discussed. The success of these methods is demonstrated on heavy and superheavy nuclei. The hunt for an ‘island’ of superheavy nuclei has stimulated the imagination of nuclear structure physicists for many years and the possibility of such an island is also discussed in other parts of the present volume.

The most recent success of the macroscopic–microscopic method has been the application to fast nuclear rotation and this subject is treated in some detail in chapters 11 and 12. It is described how a unifying picture of single-particle excitations and collective rotation emerges from straightforward generalisations of the methods introduced in earlier chapters. In particular, recent applications on band terminations and superdeformation are discussed within a cranking formalism, which is first illustrated on the conceptually much simpler sd-shell nuclei. This chapter leads up to the present research front with a discussion on identical rotational bands and how they might be described.

We felt that even though it was not our primary goal, we should still present some basic concepts about the nucleon–nucleon interaction, which is thus the subject of chapter 13. This chapter is only intended for the inexperienced reader to make the present volume reasonably self-contained.

The final chapter deals with the pairing interaction in a way that we hope should be understandable even for readers who have no previous knowledge of the so-called second quantisation formalism. This chapter should, we
hope, clarify some of the discussion in earlier parts of the book, where the
importance of pairing has been mentioned and some consequences of pairing
have been anticipated.

The text is accompanied by a number of problems, some trivial and some
that might be quite tough for the ordinary reader. In some cases we have
felt that the text becomes more transparent without too many derivations.
Thus, some simple derivations are put as problems, which we also hope
will encourage the reader to try to carry them through by him-/herself. In other
problems the more general formalism of the text might be applied to more
concrete cases. If not very trivial, solutions are given to the problems.

In the applications of the shell model, we have used those models that we
think demonstrate best the surprising success of the concept of individual
particle motion in an average field. We have thus concentrated on the so-
called modified oscillator potential in order not to hide the simple physical
arguments with too much mathematical complexity. We are convinced that,
having understood the physical arguments, the reader will be well prepared
to find his way through more ‘realistic’ potentials of e.g. Woods–Saxon or
Hartree–Fock type.

The reference list is far from complete. We have only put in the papers
whose results and ideas we specifically refer to and some more general papers
so that the reader should easily find his way through the current literature.
Except for some older and generally recognised papers, we have thus not
tried to trace the origin of many of the arguments put forth here. Naturally,
the reference list is weighted toward our own papers and those of our closest
collaborators.

This volume has grown from a course given at the Institute of Technology
in Lund. Many people in our Department have contributed at different
stages of this course and thus also studied and given constructive criticism to
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Sven Gösta Nilsson died in 1979. As a student, colleague and friend of
his, it is with a deep sense of loss that I remember his warm and enthusiastic
personality and all the inspiration I have had from numerous discussions
with him. To bring the present volume to an end has been a much harder job than I realised when I started. The long delay means that several developments from the 1980s are covered. However, these subjects are all natural continuations of the original manuscript. I hope that the present book is in line with Sven Gösta's vision and that it will be useful and inspiring for its readers.

_Inge

mar Ragnarsson_