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0521019176 - Engineering Fluid Dynamics: An Interdisciplinary Systems Approach

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This text provides a thorough treatment of the fundamental principles of fluid mechanics and convection heat transfer and shows how to apply the principles to a wide variety of fluid flow problems.

This book is intended for use primarily by senior and first-year graduate engineering students. The focus is on incompressible viscous flows with special applications to non-Newtonian fluid flows, turbulent flows, and free or forced convection flows. A special feature of the text is its coverage of generalized mass, momentum, and heat transfer equations; cartesian tensor manipulations; scale analyses; mathematical modeling techniques; and practical solution methods. The final chapter is unique in its case-study approach, applying general modeling principles to analyze nonisothermal flow systems found in a wide range of engineering disciplines.

Numerous end-of-chapter sample problem solutions, homework assignments, and mathematical aids are provided to enhance the reader's understanding and problem-solving skills.

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Engineering
Fluid Dynamics
An Interdisciplinary Systems Approach

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*Dedicated to the memory of Mother Barbara
and to my family, Christin, Nicole, and Joshua*

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Preface

Engineering fluid dynamics is considered (here) to be synonymous with fluid mechanics and convection heat transfer with engineering applications. The textbook is written for intermediate to advanced readers: for professionals as well as selected seniors and first-year graduate students in mechanical, biomedical, nuclear, and chemical engineering.

The main objective of the textbook is to provide the reader with sufficient background to enable him/her

- (i) to bridge fluid mechanics and convection heat transfer material on an introductory graduate level with specialized advancements in hydrodynamic instability, turbulence, multiphase flows, or computational fluid dynamics; and
- (ii) to tackle basic research projects in fluid mechanics and convection-heat-transfer related fields.

Although the text contains the basic engineering concepts, physical explanations, exercises, and mathematical aids necessary to succeed, it is the experience students will gain with the homework assignments, in-class discussions, journal article reviews, and course project reports that will move them to a deeper understanding and a higher level of proficiency. Specifically, the in-depth understanding of (the) *basics* in fluid mechanics/convection heat transfer and the skills to apply fundamental knowledge to the solution of interdisciplinary fluid dynamics problems are more valuable than presentation of large amounts of material within, typically, a very restricted time frame. Thus, in addition to the potentially unique learning experience provided with the text, the advanced student is given powerful tools, including computational fluid dynamics (CFD) software (cf. App. F), which may lead him/her to a level of maturity to solve challenging (industrial) fluid flow and heat transfer problems. In order to complete this task, a few interesting topics that usually overload a first-year graduate course had to be omitted; for example, three-dimensional boundary-layer flows, stability theory and statistical applications in turbulence, inviscid fluid flow theory, and advanced concepts of vorticity dynamics. Instead, topics such as scale analysis, non-Newtonian fluid flows; mathematical aids, including computer programs for CFD assignments; and selected (real-world) case studies were included. In summary, the text stresses pedagogical aspects of achieving a good understanding of basic viscous flows *balanced* by mathematical skills necessary to set up and solve or simulate interdisciplinary flow problems of increasing complexity.

The book consists of five major chapters plus several appendixes. Every chapter closes with illustrative sample problem solutions, practical problem assignments, and useful references. Chapter 1 reviews fluid kinematics and fluid dynamics aspects as well as fluid flow properties. The preliminary concepts of Chapter 1 in conjunction with the tensor applications of Appendix A provide the background for the main material presented.

Chapter 2 introduces the conservation laws in integral and differential forms. In discussing a number of illustrative examples, the mathematical/physical understanding of the basic transport equations is advanced. Instead of following the general trend toward

completeness in listing solutions to variations of (the) basic thermal flow problems, some space is devoted to the repeated analyses and discussions of *fundamental* problems employing *different* solution approaches.

After very brief reviews of compressible flow and ideal flow, Chapter 3 treats incompressible viscous flows following the common thread of low-, moderate-, and high-Reynolds number flows, including turbulent flows. Again, sample problem solutions and specific problem assignments may enlighten the reader and strengthen his/her problem-solving skills. *A written review of an appropriate journal article and subsequently oral (i.e., viewgraph) presentation should be a course requirement. End-of-the-semester reports on challenging computer projects using the listed ordinary differential equation (ODE) and partial differential equation (PDE) solvers are highly recommended.* The information and mathematical aids given in Appendixes A–F make the book rather self-contained.

Many engineering fluid mechanics problems are nonisothermal and hence Chapter 4 provides the basics in forced and free convection heat transfer. The emphasis is on free or forced external (i.e., thin shear-layer) flows and forced internal flows, considering both the laminar and the fully turbulent regimes. The material presented in Chapter 4 and several sections of Chapter 5 should be sufficient to cover a first-year graduate course in convection heat transfer if, again, reviewing and presenting selected journal articles as well as computer course projects are part of the course requirements.

Chapter 5 is quite unique when compared to existing texts, in applying mathematical modeling approaches to set up and simulate real-world thermal flow systems. Selected applications and case studies, drawn primarily from the author's publications, illustrate the use of general principles to solve complex, multidisciplinary viscous flow as well as convection heat transfer problems. Hence, the engineering equations and solution methods introduced, the problem solutions presented, and the modeling steps discussed in Chapters 2–4 and Section 5.1 as well as the material provided in Appendixes A–F, form the background for Chapter 5. Furthermore, the project solutions in Chapter 5 stress the similarities between seemingly very different fluid flow systems ranging from biomedical to nuclear engineering applications. Specifically, the topics include laminar, moving boundary flows, turbulent boundary-layer flows, biofluid mechanics applications, and thermal flow of power-law fluids. Background information is provided in introductory sections for every case study. Whatever is selected from Chapter 5, it should help in the review and presentation of refereed journal articles as well as in the development of computer projects that are important requirements for each course.

The text can be used for a one-semester course in fluid mechanics (i.e., Ch. 1–3 plus Appendixes and Ch. 5 selectively), or, more economically, for a two-course sequence in fluid mechanics *and* convection heat transfer, where the latter course relies on Chapters 2, 4, and 5. Prerequisites include an introductory knowledge of fluid mechanics (e.g., texts by Sabersky et al. (1989), White (1986), or Gerhart et al. (1992)), thermodynamics (cf. Cengel and Boles (1993)), heat/mass transfer (e.g., texts by White (1988) or Bird et al. (1960)), and applied mathematics (e.g., texts by Boyce & DiPrima (1977), Habib (1975), Rieder and Busby (1986), or Wylie and Barrett (1982)). Corequisite courses, typically in numerical analysis and partial differential equations, are highly recommended.

The present book is based on lecture notes that were periodically extended and improved while I taught first-year graduate courses in fluid mechanics and convection heat transfer at Rensselaer Polytechnic Institute (RPI), Troy, New York, and at North Carolina State

University (NCSU), Raleigh. Naturally, I have borrowed (and referenced) some material from introductory texts, advanced reference works, and complementary books. In addition, homework set solutions, prepared by my former students in chemical, mechanical, environmental, nuclear, and biomedical engineering, were modified and used in Sections 1.4, 2.4, 3.3, and 4.3.

The author has received many helpful comments and suggestions during the preparation of the manuscript from former students and from colleagues in academic circles, research labs, and industry. I would like to mention Mr. Al Powell, who helped to put together Appendixes F.1 through F.4, and Mr. Kenneth Comer who greatly assisted in proofreading the manuscript and who finalized the nomenclature and index. I am especially grateful to Ms. Mary Molly Taylor, who did an excellent job in typing the manuscript, and to Mr. David Farmer, who expertly generated the figures, graphs, and sketches on his lap-top computer. Of the many people who were instrumental in publishing this book, I would like to mention Ms. Florence Padgett, Physical Sciences Editor, Ms. Katharita Lamoza, Production Editor, and Ms. Ellen Tirpak, Project Manager.

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Symbols and Abbreviations

Abbreviations

F	Force (N)
J	Joules (J)
L	Length (m)
M	Mass (kg)
T	Time (sec, s)
θ	Temperature ($^{\circ}\text{C}$, K)
W	Watts (J/s)
1	dimensionless
—	not applicable

Roman Symbols

a	acceleration	LT^{-2}
A	area	L^2
A	Van Driest dampening length	L
A	aspect ratio	1
\vec{a}_p	particle acceleration vector	LT^{-2}
A_D^+	Van Driest constant (inner variable)	1
B	bulk modulus	1
B	empirical constant	various
BC	boundary condition	various
Bi	Biot number	1
BL	boundary layer	—
BVP	boundary value problem	—
C, c	heat capacity	$J\theta^{-1}$
C, c	constant	various
C_d, C_D	drag coefficient	1
C_f	local skin friction coefficient	1
C_p	specific heat (constant pressure)	$JM^{-1}\theta^{-1}$
C_v	specific heat (constant volume)	$JM^{-1}\theta^{-1}$
CFD	computational fluid dynamics	—
CVM	control volume method	—
d	diameter	L
D_{AB}	binary mass diffusion coefficient	L^2T^{-1}
D_{ij}	diffusive transport term	L^2T^{-1}
D_h	hydraulic diameter	L
D_m	mass diffusivity	L^2T^{-1}
DNS	direct numerical simulation	—

\hat{e}_i	unit vector in i -direction	1
erf	error function	1
$erfc$	complementary error function	1
exp	exponential (math operator)	—
E or \dot{E}	energy or thermal energy rate	J or W
Ec	Eckert number	1
EVM	eddy viscosity modeling	—
F	force	F
f	friction factor	1
fct, f	function	1
\vec{f}_b	body force	FL^{-3}
\vec{f}_{ext}	external body force	FL^{-3}
$f(\eta)$	similarity function	1
F	Helmholtz free energy	J
FDM	finite difference method	—
FEM	finite element method	—
g	gravitational constant	LT^{-2}
\vec{g}	gravitational vector	LT^{-2}
G	extensive system property	various
G	Gibbs free energy	J
Gr	Grashof number	1
GTE	General Transport Equation	—
h_f	friction head loss	L
h	specific enthalpy	JM^{-1}
h	small gap (position)	L
h	heat transfer coefficient	$JL^{-2}\theta^{-1}$
\hat{h}	specific enthalpy	JM^{-1}
H	total enthalpy	J
H	shape factor	1
H	height	L
HFD	hydrodynamically fully developed	—
\hat{i}	unit vector for the x -direction	1
I.C.	initial condition	various
IVP	initial value problem	—
J	momentum flux	MLT^{-1}
\hat{j}	unit vector for y -direction	1
\hat{k}	unit vector for z -direction	1
k	thermal conductivity	$JL^{-1}\theta^{-1}$
k	turbulence kinetic energy	J
k	permeability	LT^{-1}
l	Prandtl's mixing length	L
l_e	entry length	L
l_k	Kolmogorov length scale	L
l_m	mixing length	L
ln	natural log	—
l_τ	viscous wall length	L

l^+	dimensionless mixing length	L
L	length	L
L	biharmonic differential operator	—
Le	Lewis number	1
LES	large eddy simulation	1
LHS	left-hand side	—
m	mass	M
\dot{m}	mass flow rate	MT^{-1}
\dot{m}_0	initial mass flow rate	MT^{-1}
M	total momentum	MLT^{-1}
Ma	Mach number	1
MLH	Prandtl's mixing length hypothesis	—
n	power-law exponent	1
N	mole number	1
Nu	Nusselt number	1
ODE	ordinary differential equation	—
p	pressure	FL^{-2}
P	perimeter	L
p	pressure parameter	1
P_{ij}	stress production term	L^2T^{-3}
PDE	partial differential equation	—
Pe	Peclet number	1
Pr	Prandtl number	1
Pr_t	turbulent Prandtl number	1
p_∞	pressure in a free approach stream	FL^{-2}
q	heat flux	JL^{-2}
\dot{Q}	(heat) flow rate	J, W/s
\dot{Q}	fluid flow rate	L^3T^{-1}
\dot{Q}_z	flow rate in z -direction	L^3T^{-1}
r	radial position	L
r_0^+	dimensionless radius	1
\vec{r}_p	position vector	L
\hat{r}	dimensionless radial position	1
R	specific gas constant	$JM^{-1}\theta^{-1}$
Ra	Rayleigh number	1
Re	Reynolds number	1
REV, REV	representative elementary volume	—
RHS	right-hand side	—
Ri	Richardson number	1
ROMA	relative order of magnitude analysis	—
RSM	Reynolds stress modeling	—
RSTE	Reynolds stress transport equation	—
S	surface	L^2
S, s	entropy	$JM^{-1}\theta^{-1}$
S	source term	various
Sc	Schmidt number	1

SGSM	subgrid scale modeling	—
t	time	T
t_c	convection time scale	T
T_{ij}	turbulent diffusion transport term	L^2T^{-3}
t_h	heat diffusion time scale	T
t_k	Kolmogorov time scale	T
t_e	representative fluid element travel time	T
t_v	viscous diffusion time scale	T
T	temperature	θ
TFD	thermally fully developed	—
TSL	thin shear layer	—
T_b	bulk temperature	θ
T_f	reference temperature of fluid	θ
T_∞	free stream or ambient temperature	θ
T_m	mean or mixing cup temperature	θ
T_w	wall temperature	θ
T^+	inner variable (turbulent temperature)	1
tr	transpose (math operator)	—
u	velocity component in x -direction	LT^{-1}
u_{av}	average axial velocity	LT^{-1}
u^+	turbulent boundary layer velocity	LT^{-1}
u_e	boundary-layer edge velocity	LT^{-1}
u_m	mean velocity	LT^{-1}
u_0	average inlet velocity	LT^{-1}
u_τ	friction velocity	LT^{-1}
u^+	inner variable (turbulent velocity)	1
u_∞	free stream or approach velocity	LT^{-1}
U	outer (potential flow) velocity	LT^{-1}
U	internal energy	$J, J/M$
\hat{u}	specific internal energy	—
v	velocity component in y -direction	LT^{-1}
\vec{v}	velocity vector	LT^{-1}
v_r	radial velocity	LT^{-1}
v_z	axial velocity	LT^{-1}
v_θ	circumferential velocity	LT^{-1}
V, \forall	volume	L^3
\hat{v}	specific volume	L^3M^{-1}
w	velocity component in z -direction	LT^{-1}
W, w	width	L
w.r.t.	with respect to	—
W	work	W, FL
W	wake function	various
x	axial position	L
x_m	unheated starting length	L
\hat{x}	dimensionless position	L
y	position	L
y^+	inner variable (wall Reynolds number)	1
z	axial or vertical position	L

Greek Symbols

α	coefficient	various
α	angle	radians or degrees
α	thermal diffusivity	$L^2 T^{-1}$
β	angle	radians or degrees
β	pressure gradient parameter	1
β	volumetric expansion coefficient	θ^{-1}
Γ	circulation	$L^2 T^{-1}$
γ	ratio of specific heats	1
γ	intermittency factor	1
$\dot{\gamma}$	rate of deformation tensor	T^{-1}
δ	small displacement	L
δ	small increment	—
δ	boundary layer or thin shear layer thickness	L
δ_1, δ_2	displacement, momentum thicknesses	L
δ_{ij}	Kronecker delta	1
δ_{th}	thermal boundary layer	L
$\vec{\delta}$	unit vector	1
$\hat{\delta}$	unit tensor	1
$\Delta()$	change of ()	—
ε	turbulent eddy diffusivity	1
ε_{ij}	viscous dissipation function	$L^2 T^{-3}$
ε_{ijk}	permutation symbol	1
$\vec{\varepsilon}$	internal source function	—
$\dot{\varepsilon}$	strain rate tensor	T^{-1}
$\zeta, \vec{\zeta}$	vorticity, vorticity vector	T^{-1}
η_0	zero shear rate viscosity	$ML^{-1} T^{-1}$
η	non-Newtonian fluid viscosity	$ML^{-1} T^{-1}$
η	similarity or combined variable	1
η	dimensionless coordinate	1
θ	angle	rad
θ	dimensionless temperature field	1
θ_m	mean nondimensionless temperature	1
θ_{th}	enthalpy thickness	L
κ	Von Karman universal constant	1
λ	dimensionless system parameter	1
$-\lambda^2$	arbitrary separation constant	—
λ_n	eigenvalues	various
μ	chemical potential	$J \text{ mol}^{-2}$
μ	dynamic viscosity	$ML^{-1} T^{-1}$
ν	kinematic viscosity	$L^2 T^{-1}$
ν_t	turbulent eddy viscosity	$L^2 T^{-1}$
$\vec{\pi}$	total stress tensor	FL^{-2}
π	pi (3.14159.)	1
Π	wake parameter	—
Π_{ij}	pressure–strain correlation term	$L^2 T^{-3}$

ρ	density	ML^{-3}
$\vec{\sigma}$	normal stress vector	FL^{-2}
\sum	math operator (to sum)	—
$\vec{\Sigma}$	source term (vector)	various
τ	dissipation time scale	T
$\vec{\tau}$	shear stress vector	FL^{-2}
$\underline{\underline{\tau}}$	shear stress tensor	FL^{-2}
ϕ	velocity potential	L^2T^{-1}
ϕ, Φ	viscous dissipation function	various
ξ	dimensionless streamwise distance	1
ξ	integration dummy variable	various
ψ	generalized transport V function	various
ψ	stream function, arbitrary quantity	L^2T^{-1} , various
$\vec{\Omega}$	diffusional flux vector	1
$\omega, \vec{\omega}$	angular velocity (vector)	rad/time

Special Symbols

$:=$	equivalent to, here equal to
$\nabla()$	gradient of ()
$\nabla \cdot ()$	divergence of ()
$\nabla \times ()$	curl, rotation of ()
\perp	perpendicular to
1- D	one-dimensional flow
2- D	two-dimensional flow
3- D	three-dimensional flow
∞	infinite
$O()$	order of ()
\propto, \sim	proportional to
#	arbitrary variable (various units)
\approx	approximately
$()^{\text{tr}}$	transpose of
\angle	angle between
$\hat{=}$	equivalent to

Subscripts

o	initial (time or location); wall
abs	absolute
b	body
c	concentration
cL., \mathcal{d}	centerline
d	with respect to diameter
ext	external
f	fluid
g, grav	gravity

Hf	Helmholtz free energy
kin	kinetic
max	maximum value
n	normal
pr	pressure
r	radial
rel	relative
s	solid or surface
t	turbulent
T	temperature
visc	viscous
w, o	wall
x	with respect to the x-coordinate
y	with respect to the y-coordinate
z	with respect to the z-coordinate
,t	derivative with respect to time
,x	derivative with respect to x
,y	derivative with respect to y
,z	derivative with respect to z
θ	with respect to the θ -coordinate
ϕ	with respect to the ϕ -coordinate
∞	free stream, far field or infity

Superscripts

–	mean value
^	distinguishes variables; nondimensional
+	turbulent, law of the wall variable
/	differentiation (with respect to η)