

CHAPTER 1

Preliminary Concepts**1.1 Introduction and Overview**

As indicated in the Preface, the main objectives of this book are to provide the reader, in a unifying way, with classical material in fluid mechanics and convection heat transfer and to introduce him or her to basic techniques for modeling engineering fluid dynamics systems. Thus, studying the book in a formal graduate course setting may enhance the student's physical understanding, increase problem-solving skills, and build up confidence to solve other thermal flow problems not discussed in this text. The approach and objectives of problem-solving steps, or in more complex cases "model development" in the engineering sciences, are summarized in Fig. 1.1. This sequence will be highlighted throughout.

The material in Chapters 1 and 2 together with Appendices A and B may equalize readers' different entry levels in fluid dynamics, systems analysis, and engineering mathematics. Specifically, in Section 1.2.1, the two fundamental flow field descriptions (i.e., Lagrange vs. Euler) are reviewed; Sections 1.2.2–1.2.4 discuss the *kinematics* of shear flow (i.e., fluid element translation, rotation, and deformation), *thermodynamic* properties (e.g., pressure, temperature, density, and entropy), and *transport* properties (e.g., viscosity, conductivity, and diffusivity). Some basics of particle dynamics are extended to *fluid particle* dynamics in Section 1.3. Differential operators and cartesian tensor applications, useful for Sections 1.2.2 and 1.3 as well as for Chapter 2 are summarized in Appendix A. Fluid flow systems under consideration in Chapters 1–5 are *restricted* to single-phase flow, continuum mechanics, deterministic processes, and Eulerian flow descriptions (cf. Sect. 1.4). Selected problem solutions illustrating the material presented in Chapter 1, are given in Section 1.4.

The material in Appendix A should be frequently consulted and the different notations (vector, tensor, etc.) should be swiftly absorbed.

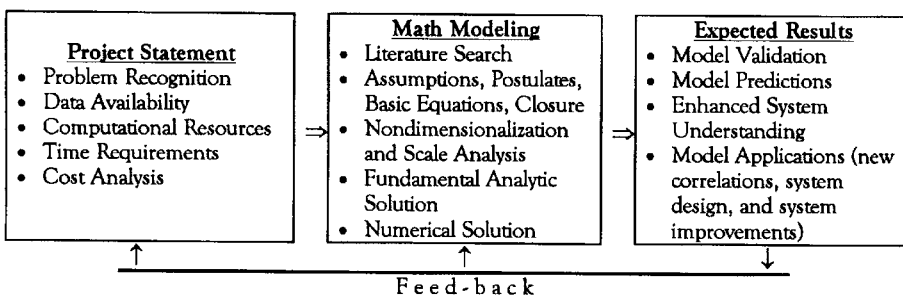


Fig. 1.1. Sequence of model development.

1.2 Fluid Particle Kinematics and Fluid Flow Properties

The *kinematics* of fluid motion is described in terms of velocity fields and their derivatives such as acceleration and vorticity. In contrast, the *dynamics* of fluid motion describes the forces acting on a fluid element. *Fluid properties*, such as density and viscosity, “correlate” forces with fluid motion. Typically for flow field descriptions, the *Eulerian* framework is preferred by engineers, whereas the *Lagrangian* point of view is often taken by (atmospheric) scientists.

1.2.1 Flow Field Descriptions

The method for describing fluid flow systems associated with Lagrange considers a *closed* system (cf. Fig. 1.2), which consists of an identifiable quantity of mass. The system boundaries may be fixed or movable but no mass crosses them. An example is the nonleaking piston–cylinder device, where the system mass is the enclosed gas, that is, $m_{\text{gas}} = \text{constant}$. The Lagrangian viewpoint of fluid mechanics is an extension of particle mechanics. The independent variables are the position vector and the time. Position vectors, $\vec{r}_p(t)$, give

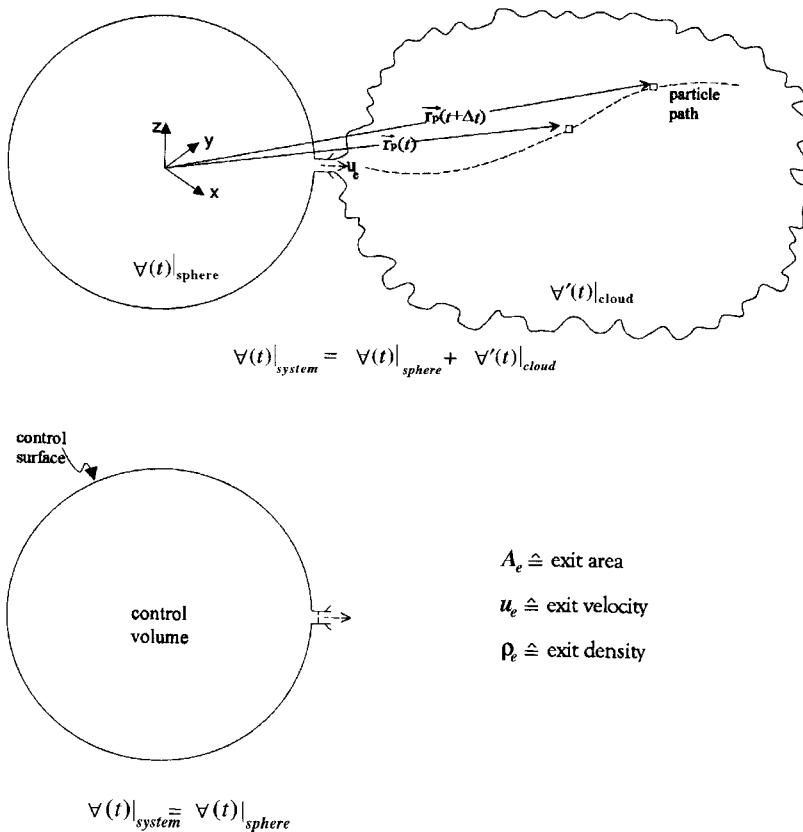


Fig. 1.2. Closed versus open system: (a) Closed system (Lagrange) (system mass is constant); (b) Open system (Euler) (control volume approach).

the paths of all particles with time as a parameter. Hence even “steady” processes are time-dependent in this framework. Once $\vec{r}_p(t)$ measured from a fixed origin is known, the particle velocity or particle ensemble velocity, $\vec{v}_p = (d\vec{r}_p/dt)$, and acceleration, $\vec{a}_p = (d\vec{v}_p/dt) = (d^2\vec{r}_p/dt^2)$, can be readily computed.

In contrast, the *open* system or *control volume* approach, associated with Euler, considers an arbitrary, typically fixed region in space. Its boundary is called a control surface through which flow quantities (mass, momentum, energy) enter and leave. Examples include all mass, force, or energy balances taken for a finite (control) volume, which usually is a slice (or shell) of the given system. From the Eulerian viewpoint, all flow properties are functions of a fixed point in space and time. Hence, the particle position vector in Eulerian variables is simply the fixed space coordinates. In order to express the time rate of change of a particle property in Eulerian variables, the substantial, Stokes, or material derivative $(D\#/Dt) \equiv (\partial\#/\partial t) + (\vec{v} \cdot \nabla)\#$ is employed (cf. App. A). This is a shorthand expression that states that the (total) time derivative of variable “#” (i.e., a vector or scalar flow property), evaluated as we follow a material particle, fluid element, or closed system, is equal to the *local* time rate of change of “#” and a convective or spatial change in “#” (cf. Problem Solution 1.3.3 in Sect. 1.4). Clearly, the operator (D/Dt) in fluid mechanics should be distinguished from (d/dt) employed in solid particle dynamics (cf. Chapter 2).

In summary, the Lagrangian viewpoint is most beneficial when tracking the history of a particle or a particle cloud as required, for example, in air pollution analyses. The Eulerian viewpoint is, in general, more useful because physical laws written in this framework do not contain the position vector, and the velocity appears as the major (dependent) variable.

1.2.2 Fluid Kinematics

Forces (i.e., body and surface forces) set fluids into motion as in turn fluid motion may exert forces. For example, stresses that are surface forces per unit area continuously deform a fluid, that is, generate fluid motion. On the other hand, wind and waves produce periodic forces on structures. The types of fluid motion and the relationships between imposed shear stress and the rate of fluid element deformation are of interest in kinematics. Certain materials such as Bingham plastics are actually not fluids since they can sustain, like solids, a *limited* shear stress while remaining in static equilibrium. Fluid motion created by changes (i.e., gradients) in pressure and stresses can be decomposed into

- translation, which is fluid element displacement or convection expressed by the velocity vector $\vec{v} = (u, v, w)$ or (v_r, v_θ, v_z) , and so on;
- rotation, where a fluid element spins about its own axis causing a changing *orientation*, measured by the vorticity vector $\vec{\zeta} \sim \vec{\omega} \sim \nabla \times \vec{v}$; and
- deformation, which is the dilatation plus distortion of a fluid element, measured with the strain rate tensor $\vec{\varepsilon} \sim \nabla \vec{v}$ or $\vec{\gamma}$, which is the rate-of-deformation tensor.

Clearly, once the velocity field, \vec{v} , is known, all forms of fluid motion can be readily computed. Observing a plane fluid element in rectangular coordinates at time t and then after forces acted upon it, at time $t + \Delta t$, we can describe the resulting motion as follows (cf. Fig. 1.3a).

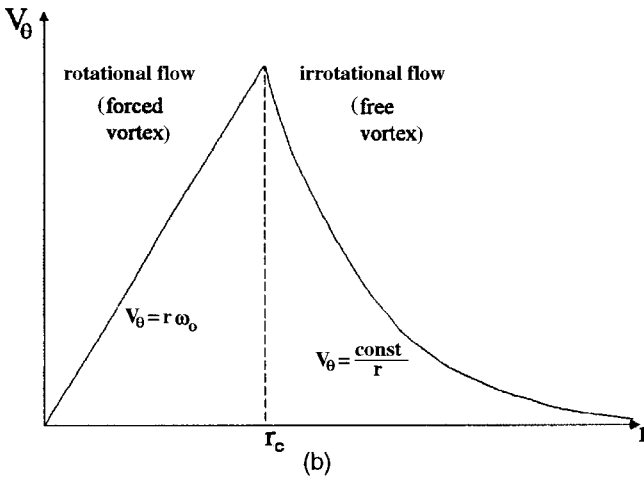
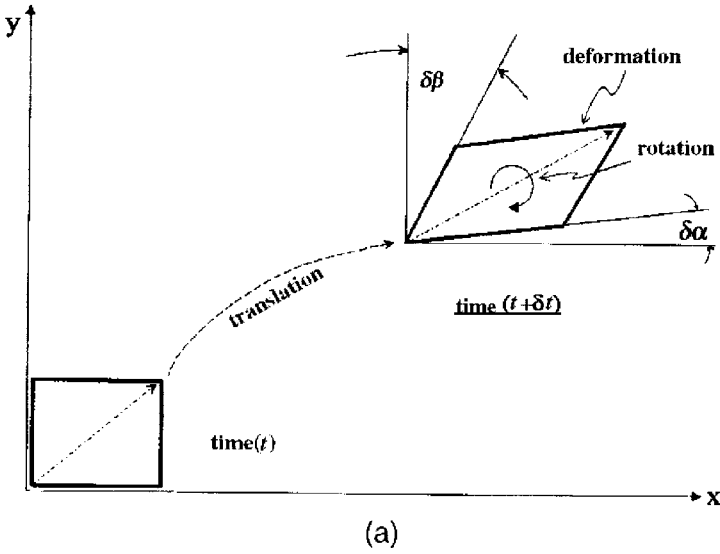


Fig. 1.3. Elements of fluid motion: (a) translating, rotating, and deforming fluid element during time δt ; (b) rotational versus irrotational flow.

A Translation

Rate of fluid element displacement in rectangular coordinates.

$$u = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}; \quad v = \lim_{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta t} \quad \text{and} \quad w = \lim_{\Delta t \rightarrow 0} \frac{\Delta z}{\Delta t}$$

◆ where $u\hat{i} + v\hat{j} + w\hat{k} = \vec{v}$ (1.1)

B Rotation

An angular velocity about the z -axis, ω_z , can be defined as the *net* average rate of counterclockwise spin of the fluid element.

$$\omega_z = \frac{1}{2} \left(\frac{d\alpha}{dt} - \frac{d\beta}{dt} \right)$$

where $d\alpha$ and $d\beta$ are related to the planar velocity derivatives as (cf. Fig. 1.3a and Sect. 1.4)

$$d\alpha = \frac{\partial v}{\partial x} dt \quad \text{and} \quad d\beta = \frac{\partial u}{\partial y} dt$$

so that

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

or, in general,

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} = \frac{1}{2} \nabla \times \vec{v} = \frac{1}{2} \vec{\zeta} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \quad (1.2)$$

where $\vec{\zeta}$ is the vorticity vector. An example of rotational versus irrotational flow is given in Fig. 1.3b.

Example: “Idealized Hurricane”

- (i) Eye of the hurricane is rotational, i.e., with solid body-like rotation, $v_\theta \sim r$
- (ii) Outside a critical radius r_c , the flow field is basically irrotational, i.e., $v_\theta \sim r^{-1}$

C Deformation

Fluid elements may experience a volume change (i.e., elongation or compression) due to pressure and normal stresses as well as shape distortion due to shear stresses, that is, tangential forces. The possible fluid element *elongation* or *compression* is expressed in terms of the tensional/compressional strain rates

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} \quad \text{and} \quad \varepsilon_{zz} = \frac{\partial w}{\partial z}, \quad \text{i.e.,} \quad \varepsilon_{ii} = \frac{\partial u_i}{\partial x_i}$$

For example, in unidirectional flow a fluid volume rate change in x -direction can be written as

$$\frac{1}{A_x} \frac{\Delta \dot{V}}{\Delta x} \approx \frac{\Delta u}{\Delta x} \approx \varepsilon_{xx}$$

The average *distortion* rate is the arithmetic mean of the rate change in angles α and β , that is, $1/2(d\alpha/dt + d\beta/dt)$, looking onto the x - y plane as given in Fig. 1.3a. Hence, with

$(\partial\alpha/\partial t) \approx (\partial v/\partial x)$ and $(\partial\beta/\partial t) \approx (\partial u/\partial y)$,

$$\varepsilon_{yx} = \varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

Combining fluid element dilatation and distortion and generalizing the analysis for three-dimensional elements yield the rate-of-strain tensor

$$\diamond \quad \varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) := \frac{1}{2} [\nabla \vec{v} + (\nabla \vec{v})^{\text{tr}}] \quad (1.3)$$

where $\nabla \vec{v} \equiv \text{grad } \vec{v}$ is a dyadic product, usually called the velocity field gradient, and $(\nabla \vec{v})^{\text{tr}}$ is its transpose. Note that $\nabla \times \vec{v} \sim \vec{\zeta}$ is a vector, whereas $\nabla \vec{v} \sim \vec{\bar{\varepsilon}}$ is a second-rank tensor with nine components (cf. App. A). It should be already transparent that $\nabla \vec{v} \sim \vec{\bar{\varepsilon}}$ can be directly related to the stress tensor $\vec{\bar{\tau}}$ (cf. Sect. 1.3.1).

1.2.3 Thermodynamic Properties

Thermodynamic properties of a system such as internal energy, entropy, pressure, and temperature are related to each other and define the *state* of the system. A *process*, being considered either quasi-reversible or irreversible, is any mechanism by which the system state is changed. *Extensive properties* such as volume, energy, and mole number are system-size-dependent, whereas *intensive properties* (e.g., pressure and temperature) are size-independent. In order to determine the state of simple compressible substances, three independent properties have to be fixed:

- internal energy U ,
- volume V ,

and, of special importance in chemical engineering, the

- mole number of a substance, N .

For example, knowing the functional form for the entropy S (i.e., an equation of state for a given system)

$$S = S(U, V, N) \quad (1.4)$$

we can form the total differential of S as

$$dS = \left. \frac{\partial S}{\partial U} \right|_{V,N} dU + \left. \frac{\partial S}{\partial V} \right|_{U,N} dV + \left. \frac{\partial S}{\partial N} \right|_{U,V} dN$$

where $(\partial S/\partial U)|_{V,N} \equiv (1/T)$ which implies that the temperature $T = T(U, V, N)$ is also an equation of state of the substance. With two other equations of state for the pressure $p = p(U, V, N)$ and chemical potential $\mu = \mu(U, V, N)$ we can define

$$p \equiv T \left. \frac{\partial S}{\partial V} \right|_{U,N} \quad \text{and} \quad \mu \equiv -T \left. \frac{\partial S}{\partial N} \right|_{U,V}$$

so that the total change in entropy reads

$$dS = \frac{1}{T} dU + \frac{p}{T} dV - \frac{\mu}{T} dN$$

or

$$\blacklozenge \quad TdS = dU + pdV - \mu dN \quad (1.5)$$

which is the *second law of thermodynamics* in a different form.

Alternative forms of the basic differential equation of thermodynamics can be obtained by introducing the

$$\text{enthalpy} \quad H \equiv U + pV$$

$$\text{Gibbs free energy} \quad G \equiv H - TS$$

$$\text{Helmholtz free energy} \quad F \equiv U - TS$$

Differentiating, for example, the enthalpy definition, we have

$$dH = dU + p dV + V dp$$

or, substituting dU from Eq. (1.5) results in

$$dH = T dS + V dp + \mu dN \quad (1.6)$$

that is, $H = H(S, p, N)$, which contains all thermodynamic information.

In classical thermodynamics, the theory starts directly from experimental results, using heat and work as fundamental concepts. The *first law of thermodynamics* for a *closed stationary system* undergoing a process of infinitesimally small changes received is

$$\blacklozenge \quad dU = \delta Q + \delta W \quad (1.7)$$

where the use of δ as a differential sign indicates that an infinitesimal amount of, say, work done, $\delta W = -p dV$, is an inexact differential. An alternative form of the first law would be

$$dH = \delta Q + \delta W + d(pV)$$

Defining the heat capacity C as

$$C = \lim_{T_2 \rightarrow T_1} \frac{Q}{T_2 - T_1}$$

we can write

$$C = \frac{\delta Q}{dT} \quad (1.8a)$$

or for the case of constant V and N

$$TdS = \delta Q$$

so that

$$\left. \frac{\delta Q}{dT} \right|_{V,N} \equiv C_V = T \left. \frac{\partial S}{\partial T} \right|_{V,N} \quad \text{and} \quad C_P \equiv \left. \frac{\delta Q}{dT} \right|_{P,N} \quad (1.8b,c)$$

is the specific heat at constant volume and constant pressure.

It is apparent that different choices of independent and dependent variables can be made in thermodynamics depending upon which is most advantageous for a given problem. The most common way to specify thermodynamics information is via the equations of state of a so-called pVT system. Any *isotropic*, that is, direction-invariant, system of constant mass and constant composition that exerts a uniform hydrostatic pressure on the surroundings, in the absence of surface, gravitational, electrical, and magnetic effects, is called a pVT system. Examples include a pure substance, that is, a single chemical species in any solid, liquid, or gaseous form, or a homogeneous mixture of different chemical species such as a mixture of gases (e.g., air) or liquids (e.g., water). Using simple theorems in partial differential calculus, one can express any thermodynamic property in terms of these three coordinates (pressure, volume, and temperature) of such a system, provided that it is in a state of equilibrium. The most common example is the *ideal gas law*,

$$\blacklozenge \quad \frac{p}{\rho} = RT \quad \text{or} \quad pV = mRT \quad (1.9a,b)$$

where R is (the) specific gas constant.

As all this holds for a *fluid at rest*, we extend these ideas to a *moving continuum* by assuming that the bulk motion of the fluid does not affect the thermodynamics state. Thus, the thermodynamic properties are determined by an observer moving with the local velocity (cf. Sects. 1.2.1 and 2.1.1).

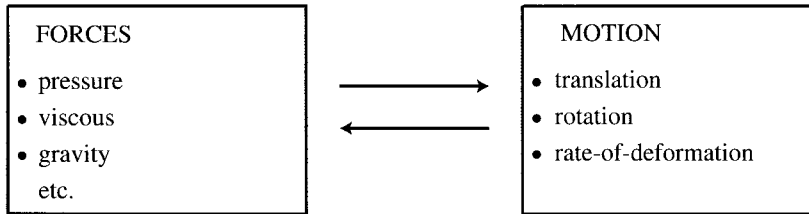
1.2.4 Transport Properties

The transport properties of interest include the fluid viscosity, μ , the thermal conductivity, k , and the binary mass diffusivity, D_{AB} . For laminar flow of Newtonian isotropic fluid, these properties are typically a function of temperature only. However, in practical applications, transport properties are also functions of pressure and concentration and they may exhibit (nonlinear) directional dependencies. Additional complexities arise when the *fluid* is non-Newtonian, such as particle suspensions and polymeric liquids, and when the *flow* is turbulent. In such cases, transport properties are not only dependent on the type of fluid but predominantly a function of local flow characteristics, such as the gradient of the velocity field, $\text{grad } \vec{v} \equiv \nabla \vec{v}$, and the presence of solid surfaces or other interfaces.

A solution of the transient or steady one-dimensional form of the transport equations is often employed to measure the transport properties for various fluids at different temperatures (cf. App. E). Theoretical expressions for μ , k , and D_{AB} have been established for fluids in shear flow; however, the transport properties are taken to be independent of pressure and density variations (cf. Kay and Nedderman 1985; Bird, Stewart, and Lightfoot 1960; Bird, Armstrong, and Hassager 1987; White 1991).

1.3 Fluid Particle Dynamics

As indicated in Section 1.2.2, surface and body forces acting on fluid elements generate fluid motion, and vice versa. (cf. illustration):



One could define *fluid flow* as a substance in a process of continuous deformation due to the interaction of driving forces (e.g., pressure, gravity, stress) and resisting forces (e.g., friction or drag). Some substances appear to be borderline solids–fluids, such as toothpaste, tar, sand, foodstuff, depending upon the magnitude of the forces applied. Clearly, *stresses* on fluid elements, either a normal force per surface area, like normal stress and pressure, or a tangential force per surface area, are flow properties of utmost importance. In mathematical terms, the interactions of *net* surface forces and change in total stress can be described as follows. Starting with a one-dimensional force balance for a rectangular fluid element depicted in Figure 1.4 yields

$$\Delta F_x = \left[\frac{\partial}{\partial x} \pi_{xx} + \frac{\partial}{\partial y} \pi_{yx} + \frac{\partial}{\partial z} \pi_{zx} \right] \Delta x \Delta y \Delta z$$

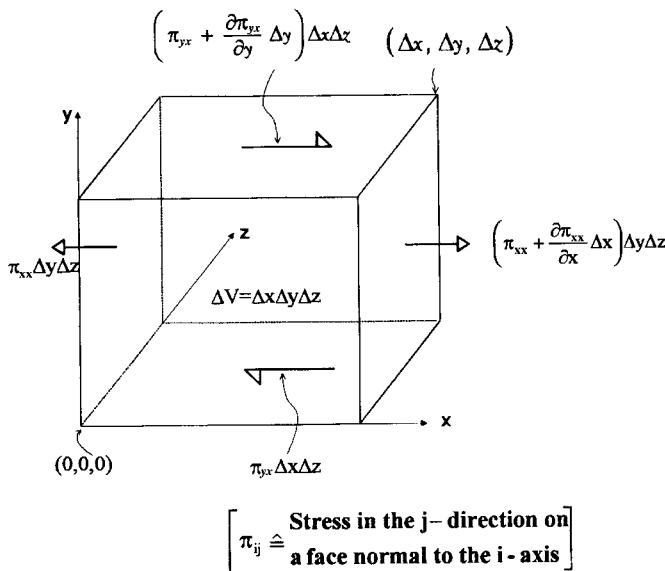


Fig. 1.4. Selected surface forces acting on a fluid element.

or, extended to three dimensions, we can write in the limit:

$$\left. \frac{d\vec{F}}{dV} \right|_{\text{surface}} = \left. \frac{d\vec{F}}{dV} \right|_{\text{pressure}} + \left. \frac{d\vec{F}}{dV} \right|_{\text{viscous}} = \nabla \cdot \vec{\pi} = \underbrace{-\nabla p}_{\vec{f}_p} + \underbrace{\nabla \cdot \vec{\tau}}_{\vec{f}_v} \quad (1.10)$$

Here, $\vec{\pi} = -p\vec{\delta} + \vec{\tau}$ is called the *total* stress tensor, encompassing the six shear stresses, τ_{ij} , as well as the thermodynamic pressure p (note, $\vec{\delta}$ is the unit tensor) and the three normal stresses, τ_{ii} ; $\nabla \cdot \vec{\pi} \equiv \text{div } \vec{\pi}$ is the divergence of the total stress tensor of rank 2, producing a vector (cf. App. A). Thus, together with gravity as an example of a body force per unit volume \vec{f}_b , we could apply Newton's second law of motion

$$\blacklozenge \quad m\vec{a} = \sum \vec{F}_{\text{ext}} \quad \text{or} \quad \rho \frac{d\vec{v}}{dt} = \sum \vec{f}_{\text{ext}} \quad (1.11a,b)$$

to an incompressible fluid flow field as

$$\rho \underbrace{\frac{D\vec{v}}{Dt}}_{\vec{a}_{\text{total}}} = \vec{f}_p + \vec{f}_v + \vec{f}_b \quad (1.12a)$$

or

$$\blacklozenge \quad \rho \left[\underbrace{\frac{\partial \vec{v}}{\partial t}}_{\vec{a}_{\text{local}}} + \underbrace{(\vec{v} \cdot \nabla) \vec{v}}_{\vec{a}_{\text{convective}}} \right] = -\nabla p + \nabla \cdot \vec{\tau} + \vec{g} \quad (1.12b)$$

The question now is, how can we relate the (unknown) stress tensor, $\vec{\pi}$ or $\vec{\tau}$, to the principal unknown, \vec{v} , or at least to $\nabla \vec{v} \equiv \text{grad } \vec{v}$, the velocity field gradient, naturally, also a tensor of rank 2. Such a constitutive equation, say, $\vec{\tau} \sim \nabla \vec{v}$, depends on the type of fluid and on the flow regime. For example, for laminar Newtonian fluid flow, Stokes (1845) postulated a *linear* correlation, which for *incompressible* flow is

$$\blacklozenge \quad \vec{\tau} = \mu[\nabla \vec{v} + (\nabla \vec{v})^T] \quad (1.13a)$$

where μ is the fluid's (constant) dynamic viscosity (cf. Section 1.2.4 and App. E). For non-Newtonian fluids such as polymeric liquids, exotic lubricants, foodstuff, paints, pastes, and slurries, the generalized fluid viscosity, η , is itself a function of $\nabla \vec{v}$ and other parameters, rendering $\vec{\tau}(\nabla \vec{v})$ *nonlinear*, that is,

$$\blacklozenge \quad \vec{\tau} \approx \eta(\nabla \vec{v})[\nabla \vec{v} + (\nabla \vec{v})^T] \quad (1.13b)$$

Expressions for the stress tensor are even more complicated in turbulent flows, where $\tau_{ij}^{\text{total}} = \tau_{ij}^{\text{laminar}} + \tau_{ij}^{\text{turbulent}}$ with $\tau_{ij}^{\text{turb}} \gg \tau_{ij}^{\text{lam}}$, except very near walls, namely,

$$\vec{\tau}_{\text{turb}} = \text{fct}(\vec{v}, \nabla \vec{v}, \mu, \text{geometry, etc.}) \quad (1.13c)$$

In the next section, the stress vector (three components) acting at a surface of a representative fluid element and the stress tensor (nine components) acting on a three-dimensional (3-D) fluid element are discussed. One should recall that fluid element *volume changes* and