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To my wife, Mary, and to my daughters, Laura and Audrey

Contents

Preface

As a consequence of their electrical and/or magnetic properties, all particles experience forces and torques when subjected to electric and/or magnetic fields. Furthermore, when they are electrically charged, polarized, or magnetized, closely spaced particles often exhibit strong mutual interactions. In this book, I focus on these particle–field interactions, referred to collectively as *particle electromechanics*, by delineating common phenomenology and by developing simple yet general models useful in predicting electrically and magnetically coupled mechanics. The objective is to bring together diverse examples of field–particle interactions from many technologies and to provide a common framework for understanding the relevant electromechanical phenomena. It may disappoint some readers to learn that, despite the rather general definition offered for particle electromechanics, I restrict attention to particles in the size range from approximately 1 micron (10^{-6} m) to 1 millimeter (10^{-3} m). Though many of the ideas developed here indeed carry over into the domain of ultrafine particles, the lower limit recognizes that other phenomena, such as van der Waals forces and thermal (Brownian) motion, become important below one micron. The upper limit is consistent with a reasonable definition for a classical particle.

Chapter 1 introduces the subject, provides a definition for particle electromechanics, and adds some caveats to inform the reader of the book's limitations. Chapter 2 unveils the fundamental *effective moment* concept employed throughout the book in the calculation of electromechanical forces and torques. It also uses multipolar expansion methods to solve for the induced moments for a particle experiencing a strongly nonuniform field and exploits the analogy between electrostatic and magnetostatic problems to reveal how the results for a dielectric particle can be applied to a magnetizable particle. Chapter 3 harnesses the effective moment method to derive dielectrophoretic (DEP) and magnetophoretic (MAP) force expressions for small particles in nonuniform AC electric and magnetic fields, respectively. The frequency-dependent DEP force on dielectric particles with ohmic or dielectric loss is examined with reference to

dielectrophoretic levitation. Particle rotation is the subject of Chapter 4, where the effective moment method aids us in calculating the frequency-dependent torque on a particle due to a rotating electric field. Spontaneous Quincke rotation (in a static, DC electric field) is considered as a special case. The behavior of magnetizable and electrically conductive particles in a rotating magnetic field is examined by drawing on the close analogy to rotating machines. Chapter 5 concerns nonspherical (ellipsoidal) particles in uniform fields, principally the frequency-dependent orientation of lossy dielectric ellipsoids in AC electric fields and the alignment of magnetic crystals in a DC magnetic field.

The remainder of the book focuses on the mutual interactions between closely spaced particles. Chapter 6 describes methods for determining the effective multipolar moments of linear particle chains. These effective moments provide the means to calculate net DEP forces and electrical torques exerted upon chains by electric or magnetic fields. Several different approaches useful in modeling the particle interactions are covered, including the *method of images* and *method of inversion*, both applicable only to conductive particles, and a straightforward *multipolar expansion* technique, applicable to dielectric and magnetizable particles. This chapter also examines the effect of nonlinear magnetization upon the interactions of ferromagnetic particle chains and introduces a simple model for heterogeneous mixtures of particles agglomerated into short chains.

Chapter 7 exploits the particle interaction models of the previous chapter to determine the mutual forces of attraction between closely spaced particles. Experimental force measurements obtained with magnetizable spheres are compared to predictions of a linear multipolar expansion. One section of this chapter, devoted to the electrostatic contributions to surface adhesion, highlights practical methods for estimating these forces in realistic situations. An extensive review of some particulate technologies where chaining is important concludes this chapter.

A nomenclature section lists all important algebraic symbols used in the mathematical expressions. All literature references, cited by author and date of publication in the text, are listed alphabetically at the end of the book. A large set of appendixes at the end of the book covers the details of certain mathematical derivations and results. The choice of SI units for all variables and mathematical expressions throughout the text reflects the significant interdisciplinary character of particle electromechanics as well as the personal choice of the author.

This book evolved from lecture notes used by the author in a graduate course entitled "Particle Electromechanics" first offered at the University of Rochester in 1985. Among many who helped bring it into being, I am privileged to mention Dr. William Y. Fowlkes, Dr. Kelly S. Robinson, and Dr. Bijay Saha of Eastman Kodak Company; Dr. Ruth D. Miller of Kansas State University; John Kraybill

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Having benefited from those who have encouraged, taught, or studied with me, I can lay full claim only to the errors found in this book. Concerning these errors, readers are kindly requested to bring them to my attention.

Nomenclature

A	area of charge patch, Equation (7.14)
$\bar{A}$	magnetic vector potential, Equation (3.61)
A, B	coefficients of dipole solution for potential used in Equations (2.8a,b)
A_n, B_n	coefficients of multipolar potential terms in Equations (2.17a,b)
A, B, C, D	coefficients of dipole solution for potential of layered sphere in uniform field (Appendix C)
A_n, B_n, C_n, D_n	coefficients of multipolar terms for potential of layered sphere in field of point charge (Appendix C)
$\bar{B}$	magnetic flux density
D	spacing of source charges $\pm Q$ shown in Figure 6.2
$\bar{D}$	electric displacement vector
$\underline{\underline{D}}(\omega)$	complex quantity for conducting magnetizable sphere in time-varying field defined by Equation (3.64)
$\bar{E}$	electric field vector
E_{gap}	electric field in gap between closely spaced particles
E_{max}	breakdown-limited electric field between closely spaced particles
E_0	externally applied electric field
$E_{\parallel}, E_{\perp}$	parallel and perpendicular components of electric field
$\bar{F}$	force vector
$F_{m,n}$	attractive force between two aligned linear multipoles of orders m and n in Equation (7.1)
G	feedback gain coefficient in Equation (3.43)
$\bar{H}$	magnetic intensity vector
H_c	coercive magnetic field value
H_{gap}	magnetic field in gap between closely spaced particles
I	moment of inertia of spherical particle used in Equation (4.23)
$I_{\pm n/2}$	modified half-integer order Bessel functions of first kind

$\bar{J}$	electric current density vector
K	Clausius–Mossotti functions as defined by Equation (2.13)
$K^{(n)}$	polarization coefficient of nth order linear multipole defined by Equation (2.23)
K_0, K_∞	low- and high-frequency limits of complex Clausius-Mossotti function
K_x, K_y, K_z	polarization factors along three principal axes of ellipsoidal particle, Equation (5.27)
$\bar{K}$	surface electric current density (Appendix C)
$\underline{L}(\omega) = R^3\underline{D}/2$	magnetization coefficient for conductive magnetizable sphere, Equation (3.63)
L_e	characteristic length of levitation electrodes
L_x, L_y, L_z	depolarization factors along x, y, and z axes for ellipsoid defined by Equation (5.4)
$L_{\parallel}$, $L_{\perp}$	depolarization factors parallel and perpendicular to long axis of prolate spheroidal particle, Equation (5.13)
$\bar{M}$	magnetization vector
M_n	normal component of magnetization vector
M_{rem}	remanent magnetization per unit volume
M_{sat}	saturation magnetization per unit volume
N	integer, number of spheres in linear chain
N_p	number of particles distributed in sphere of radius R_0
P	levitation equilibrium point: (x_0, y_0, z_0)
$\bar{P}$	polarization vector
$P_n(\cos\theta)$	Legendre polynomial of order n
Q	source charge for method of images
R	radius of spherical particle
R_α	radii of two identical intersecting spheres shown in Figure 6.10
R_a, R_b	radii of two interacting spherical particles shown in Figure 6.8b
$R_m = R\sqrt{\omega\mu_2\sigma_2/2}$	magnetic Reynolds number of conductive spherical particle used in Equation (3.68)
R_0	radius of equivalent sphere of homogeneous permittivity ε_{eff} defined in Figure 6.22
S_1, S_2	functions defined by Equations (7.19a,b)
T	temperature
$\bar{T}^e$	vector torque of electrical origin
T^η	viscous drag torque on rotating particle defined by Equation (4.7)
$\bar{T}^m$	vector torque of magnetic origin
V	voltage, or particle volume

V_0	equilibrium voltage for levitated particle at point P defined by Equation (3.34c)
X, Y	coefficients of magnetostatic potential solution in Equations (3.46a,b)
Z_1, Z_2	coefficients defined by Equations (G.1) and (G.2)
$a = R_1/R_2$	ratio of radii of spherical shell in Figures 2.3 and 3.4a
a, b, c	semimajor axes of ellipsoidal particle shown in Figure 5.1
b	damping coefficient in Equations (3.33a,b)
c	constant used in Equation (6.59)
c_m	cell membrane capacitance per unit area in Figures 3.3a and 3.4a
c_x, c_y, c_z	direction cosines
$\bar{d}$	vector distance between charges in finite dipole
d_n	distance of nth image charge from midplane of particle chain
e	eccentricity of spheroidal particle used in Equation (5.15)
$f = q_p/q$	patch fraction defined in Section 7.4B
$f(N), f(\theta)$	distribution functions for chain mixtures used in Section 6.5B
$\bar{g}$	acceleration due to gravity (9.81 m/s^2)
g_m	transmembrane conductance
i, j	integer indices
$\mathrm{j} = \sqrt{-1}$	square root of minus one
k	Boltzmann's constant ($1.38 \cdot 10^{-23}$ J/deg K)
$\bar{m}$	magnetic dipole moment vector
m, n	integer indices
m_b	buoyant mass of particle in fluid
m_{eff}	effective magnetic dipole moment
m_{perm}	permanent moment of magnetized particle in air, Section 5.6D
m_{rem}	remanent moment of magnetized particle in Equation 4.37
$\hat{n}$	unit normal to surface
$\bar{p}$	electric dipole moment vector
$p^{(n)}$	moment of nth order linear multipole
p_{eff}	effective electric dipole moment
$\bar{p}_{eff} = p_{eff}/p_0$	normalized effective electric dipole moment
p_N	dipole moment of linear chain of N perfectly conducting spheres
$p_0 = 4\pi\varepsilon_1 R^3 E_0$	dipole moment of perfectly conducting sphere

$p_{\parallel}, p_{\perp}$	components of dipole moment parallel and perpendicular to long axis of prolate spheroid or chain
$\underline{p}_{\parallel}, \underline{p}_{\perp}$	parallel and perpendicular effective electric dipole moments normalized to p_0
q	electric charge
q_c	charge at center of dielectric particle, Figure 7.15
q_n	magnitude of nth image charge within conducting particle chain
q_p	charge on dielectric particle, Figure 7.15
$\bar{r}$	position vector
s	complex frequency variable used in Equation (E.1); variable of integration in Section 5.2
$s_\alpha, s_\beta, \cdots$	poles of $K(s)$ defined in Appendix E
t	time
$u(t)$	unit step function
$v'(t)$	feedback voltage defined by Equation (3.34c)
x	variable of integration in Kramers–Krönig relations
$\hat{x}, \hat{y}, \hat{z}$	unit vectors in Cartesian coordinate system
x_1, x_2, x_3	Cartesian coordinates (x, y, z) in indicial notation (Section 3.4A)
Δ	uniform thickness of layer on spherical particle, Appendix C
$\Delta_x, \Delta_y, \Delta_z$	thicknesses of confocal layers in ellipsoid shown in Figure 5.4a
Ξ	center-to-center spacing of interacting spheres in Figure 7.2
$\Sigma_1, \Sigma_2, \Sigma_3$	summations defined by Equations (6.28a,b,c)
Σ	closed surface of integration
Φ	electrostatic potential function
Ψ	electromechanical potential function in Equation (3.24)
Ψ_1, Ψ_2	magnetostatic potential functions defined by Equations (3.46a,b)
Ω	angular velocity of particle
Ω_{eq}	equilibrium rotational speed of particle defined by Equation (4.6)
Ω_f	terminal angular velocity of Quincke rotation defined by Equation (4.27)
α	lagging angle between rotating field and dipole moment shown in Figure 4.2; angle of intersection for spheres in Figure 6.10
α, β, γ	indices representing right-hand, ordered sequence of Cartesian coordinates: $x, y, z, x, \cdots$

$\alpha_1, \alpha_2, \cdots$	coefficients of field expansion along axis of axisymmetric field from Equation (3.32)
$\beta = R_b/R_a$	ratio of radii of chained spheres shown in Figure 6.8a
$\gamma = a/b$	ratio of axes of spheroid shown in Figure 5.2a
$\delta = \Xi - 2R$	gap spacing between two interacting particles
ε	dielectric permittivity
$\bar{\bar{\varepsilon}}$	tensor permittivity of anisotropic particle
$\underline{\varepsilon} = \varepsilon + \sigma/j\omega$	complex permittivity
ε'	real part of complex permittivity
ε''	negative imaginary part of permittivity
ε_2'	effective permittivity of spherical shell, Equation (C.4)
$\underline{\varepsilon}_\Sigma$	complex surface permittivity used in Equation (C.14)
ε_s	permittivity of dielectric substrate in Figure 7.12
ε_0	permittivity of free space ($8.854 \bullet 10^{-12}$ F/m)
ε_c, σ_c	permittivity and conductivity of cell cytoplasm, Section 3.3E
ε_w, σ_w	permittivity and conductivity of cell wall, Section 3.3E
ζ	distance of charge q from center of sphere in Figure 2.4
$\zeta(n)$	Riemann-zeta function of integer argument n
η_1	dynamic viscosity of fluid medium
θ	polar angle in spherical coordinates
$\vartheta_\parallel, \vartheta_\perp$	polarization coefficients for chains of unequal conductive particles in Equations (6.27a,b)
$\kappa = \varepsilon/\varepsilon_0$	dielectric constant
κ_s	dielectric constant of substrate in Figure 7.15
$\underline{\kappa} = \kappa' - j\kappa''$	complex dielectric constant
λ	exponent of electric field defined in Section 7.2C
λ_n	coefficients of neutralizing charges, Equation (6.12)
μ	magnetic permeability
μ_0	permeability of free space ($4\pi \bullet 10^{-7}$ H/m)
$\mu_R = \mu_2/\mu_1$	relative permeability
$\xi = (\kappa_s - \kappa_1)/(\kappa_s + \kappa_1)$	coefficient of images in dielectric plane used in Section 7.4B
ξ_n	distance of neutralizing charges from midplane in Figure 6.4c
ρ	radial coordinate in cylindrical coordinates
ρ_1, ρ_2	mass densities of fluid and particle
ρ_{ij}	planar spacing parameter in Figure 7.16b
σ	electrical conductivity
σ_f	free electrical surface charge
σ_Σ	surface electrical conductivity
$\tau = \varepsilon/\sigma$	charge relaxation time
$\tau_\alpha, \tau_\beta, \ldots, \tau_N$	relaxation times of layered particle
$\tau_c = \varepsilon_c/\sigma_c$	time constant used in Equation (3.12)

$\tau_m = c_m R/\sigma_c$	time constant used in Equation (3.12)
τ_{MW}	Maxwell–Wagner relaxation time defined by Equation (2.32)
τ_0	relaxation time constant defined by Equation (2.32)
τ_x, τ_y, τ_z	Maxwell–Wagner relaxation time constants along the three principal axes of ellipsoid, Equation (5.45)
ϕ	azimuthal angle in spherical coordinates
$\bar{\bar{\chi}}$	magnetic susceptibility tensor
$\chi_{\parallel}$, $\chi_{\perp}$	parallel and perpendicular susceptibilities of prolate spheroids, Equation (5.53)
χ^*	critical value of magnetic susceptibility in Equation (5.54a,b)
ω	radian frequency of electric or magnetic field
ω_c	critical transition frequency between + and – DEP defined by Equation (3.41)