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Introduction

1.1 Background and motivation

Small particles in the size range from approximately one micron (10^{-6} m) up to one millimeter (10^{-3} m) are very important in today's technological world. Though often hidden from our view, they serve as tireless workhorses in many mechanisms and devices, from electrostatic copiers and printers to powder couplings to fluidized beds. Particles are used in new colloidal suspensions called *electrorheological fluids,* which respond to an applied electric field by rapidly changing their apparent viscosity. Particles are also employed in manufacturing operations including packed and fluidized bed reactors, powder coating machines, powder injection molding, etc. Many of the raw materials used in the agricultural, food, mining, and metallurgical industries are received in particulate form to be separated, beneficiated, or processed. Likewise, modern chemical technology is heavily based upon the processing of feedstocks into powdered, granular, or pelletized dry products.

Particulates, so useful and necessary in modern materials and manufacturing, can also be a nuisance or outright hazard in other situations. For example, particulate pollution is a recognized environmental and industrial health hazard. Characterization of pollutants in particulate form is an important aspect of modern environmental health science. The collection and removal of particulate matter from combustion gases is the goal of electrostatic precipitators, packed bed filters, and other pollution control apparatus. Similarly, preservation of water quality in lakes and rivers depends on removal of certain particulate matter from industrial waste water. Another example, vital in today's electronics industry, is control of submicron contaminants during fabrication and processing of solid-state devices. This hard-to-control contamination is a significant contributing factor to the high rejection rates often experienced in the fabrication of very large scale integrated (VLSI) electronic chips. Finally, airborne dust is a well-known fire and explosion hazard in certain polymer and metallurgical manufacturing operations.

One rapidly emerging branch of particulate science and technology concerns particles of biological origin, such as cells and DNA. Cells sized from less than a micron on up to several hundreds of microns make up all living organisms. The characterization, handling, and manipulation of individual cells and DNA molecules have become major thrusts of modern biomedical science and engineering. At the same time, flow cytometry has revolutionized biological assay methods by making it possible to sort and separate literally millions of cells in minutes.

Some materials technologists have labeled the decade of the 1990s the "particle age" – a fitting recognition of the tremendous advances in the manufacture of new particulate materials and the applications being discovered for them in new products and processes.

1.2 Objectives of this book

Because all particles have electrical and magnetic properties associated with their shape and with the materials of which they are constituted, they experience forces and torques when subjected to electric and/or magnetic fields. Furthermore, particles will exhibit mutual interactions – often quite strong – through the agency of their own electrical charge, polarization, or magnetization. *Particle electromechanics*, the subject of this book, may be defined as follows:

Particle electromechanics: Forces and/or torques exerted on small particles (and collections of such particles) less than approximately 10^{-3} meters in diameter through the action of an electric or magnetic field, and also the mechanics and dynamics induced by these forces and torques. The electric or magnetic field may be imposed by external means (via electrodes, magnetic pole pieces, etc.) or by other nearby charged, polarized, or magnetized particles or particle ensembles. This definition extends to the mechanics of static particle beds and to the dynamics of moving particle beds when subject to electric or magnetic fields.

The above definition encompasses the subjects of electrophoresis and dielectrophoresis; electrorheological fluids; the mechanics of electrofluidized, electrospouted, and electropacked beds; electrostatic precipitation; electrostatic particle adhesion; high-gradient magnetic separation; the magnetostabilized bed; magnetic powder couplings; magnetic field–coupled particle flow control devices; and the magnetic brush electrophotographic copier/printer. It is not the author's objective to stake out or otherwise mark such a broad ground, but rather to offer some common terminology for the subject of field–particle interactions

and to provide a framework for the contributions of many engineers and researchers – past, present, and future – who work in these diverse fields.

What impresses the student of particle electromechanics is an immediately recognizable set of common phenomena manifested in diverse physical situations. For example, the same dielectrophoretic force experienced by biological cells in aqueous suspensions can also be significant in electrostatic precipitation and certain electrophotographic development processes. An analogous magnetophoretic force is exploited in high-gradient magnetic separators to filter out magnetizable particles. The exemplar of commonality is the ubiquitous phenomenon of particle chaining, which can be anticipated whenever uncharged dielectric particles, loose or in fluid suspension, are subjected to a strong electric field, or when magnetizable particles are placed in a magnetic field. Another kind of unity is found in the fundamental connection of both electromechanical forces and torques to the effective dipole moment. The point of view taken in this book is that these commonalities and interrelationships are not mere academic curiosities, but the mortar binding together a large collection of seemingly unrelated phenomena into a viable scientific and technical discipline, namely, particle electromechanics.

1.3 Limitations and caveats

As defined here, the subject of particle electromechanics can hardly be done full justice by any single volume. Therefore, certain limitations have been imposed in writing this monograph, the focus of which is the electromechanics of dielectric, conducting, and magnetizable particles in the diameter size range from about 1 μm (~10^{-6} m) to about 1 mm(~10^{-3} m). This book does not supplant the late Prof. Herbert Pohl's classic text on dielectrophoresis (Pohl, 1978), but rather places that subject into a larger context. In fact, Chapter 3 covers the fundamentals of dielectrophoresis and will serve as a graduate-level introduction to the subject; however, the serious investigator of biological dielectrophoresis will be drawn inevitably to Pohl's definitive volume.

Excellent works on the physics of *electrically charged* particles are widely available in technical libraries; therefore, except in Chapter 7 where electrostatic adhesion is briefly reviewed, particle charge is not considered. The lower size limit (~10^{-6} m) is imposed because the mechanics of submicron particles are strongly influenced by random thermal (Brownian) motions and van der Waals forces, while the upper limit (~10^{-3} m) is based on a reasonable working definition of what constitutes a classical particle. We may confidently predict the rapid emergence of ultrafine particle technology and, thus, the need for a volume

on the mechanics of particles smaller than 1 μm. The author of such a book will be faced with the challenging task of folding the subjects of particle electromechanics, aerosol science, and adhesion science into the unique and somewhat perplexing set of physical properties exhibited by ultrafine particles.

Another subject not covered in this book is colloidal electro-optics (and magneto-optics), which concerns the influence of electric (or magnetic) fields on the optical properties of colloidal suspensions. While particle orientation (the subject of Chapter 5) and field-induced particle chaining (discussed in Chapters 6 and 7) are electromechanical mechanisms responsible for some of the important electro-optic effects, no attention to the optics side of the problem is given here. The reader interested in electro-optics is referred to the excellent treatise on this subject by S. P. Stoylov (1991).

The subjects of electrohydrodynamics (EHD) and electroconvection, as they relate to particles, droplets, and bubbles, are not covered in the present volume. Therefore, bubble and droplet deformations induced by an electric (or magnetic) field are not considered. Likewise, no treatment of the important subject of electrophoresis of particles in aqueous suspension is provided. Only simple dielectric models for biological cells and particles in aqueous media are examined; the neglect of surface charging guarantees that the dielectric models for particles in aqueous suspension, especially biological cells, are deficient at very low frequencies. One more limitation of this book is that chain interactions among nonspherical particles have not been considered.

2

Fundamentals

2.1 Introduction

The definition of particle electromechanics offered in Chapter 1, Section 1.2, is very broad, precluding any possibility of definitive treatment in a single volume. Accordingly the scope of this book is restricted primarily to field–particle interactions involving (i) uncharged, lossy, dielectric and electrically conductive particles with AC and DC electric fields and (ii) magnetizable, electrically conductive particles with AC and DC magnetic fields. The particle electromechanics of interest here are a consequence of either the field-induced polarization of dielectric particles or the field-induced magnetization of magnetic particles. The forces and torques governing particle behavior result from the interaction of the dipole and higher-order moments with the field.

A. Electromechanics of particles

Two distinct types of electromechanical interactions may be identified: *imposed field* and *mutual particle* interactions. Imposed field interactions reign when a single particle, or an ensemble of noninteracting particles, is influenced by an externally imposed field. Examples include the dielectrophoretic force or the alignment torque exerted on an isolated particle. Here, it is customary to assume that the particle does not influence the field, though such an assumption is not always justified. Mutual particle interactions occur where particles are so closely spaced that the local field of a particle influences its neighbors. For particles in close mechanical contact, mutual interactions can be very strong, leading to significant changes in the equilibrium structure of particle ensembles (e.g., chain formation and cooperative electrorotation), as well as strong cohesive forces.

For the purpose of convenience in presentation, this monograph is organized into sections on imposed field interactions (Chapters 2 through 5) and mutual interactions (Chapters 6 and 7). However, the distinctions drawn between these

two branches of particle electromechanics are in fact artificial and have no real formal basis, because it is typical for both types of interactions to be present simultaneously. A common thread running throughout this book is the application of field–multipole interactions to calculate forces and torques on particles and particle ensembles in the cases of either imposed field or mutual interactions.

A very close analogy exists among electrostatic problems involving lossless dielectrics, DC conduction problems involving ohmic conductors, and magnetostatic problems with (linear) lossless magnetizable materials. This analogy makes it possible to start with expressions for electrical quantities such as potential and field variables, dipole moments, etc., and then to formulate the solution to the analogous conduction or magnetostatic problem by a simple change of variables. Appendix A briefly explains the origins of the analogy between electrostatic, conduction, and magnetostatic problems and contains two useful tables. Table A.1 identifies analogous quantities and their appropriate SI units, while Table A.2 summarizes the more important analogous formulas for calculation of the effective dipole moment, the gradient force, and the torque.

The approach consistently taken in this book is to cast problems in their electrostatic form first and then to consider equivalent magnetics problems second. The reader's ability to perform the appropriate changes of variables is relied upon to obtain solutions to equivalent conduction or magnetostatic problems. However, the student of particle electromechanics must also learn to recognize the limits of this analogy, which breaks down due to the properties of many common magnetizable particles. The analogy's failure should be immediately evident in the case of ferromagnetic particles exhibiting nonlinear magnetization, such as hysteresis or saturation, and permanent magnetization. But failure also occurs for the case of a conductive particle in a time-varying magnetic field, where eddy currents are induced that create a nonzero curl magnetic field within the particle. Furthermore, there is no behavior comparable to superconductivity in the realm of dielectrics.

B. Force on an infinitesimal dipole

An obvious starting point in formulating the electromechanics of particles is to estimate the net force upon a small physical dipole. This approach reveals the essential implications for particle electromechanics of the so-called ponderomotive force exerted upon dielectric materials by a nonuniform electrostatic field. The dipole consists of equal and opposite charges $+q$ and $-q$ located a vector distance $\bar{d}$ apart, and it is located in an electric field of force $\bar{E}$. Refer to Figure 2.1a. For the present, we choose to say nothing about how the electric field is created, except to stipulate that $\bar{E}$ includes no contributions due to the dipole itself.

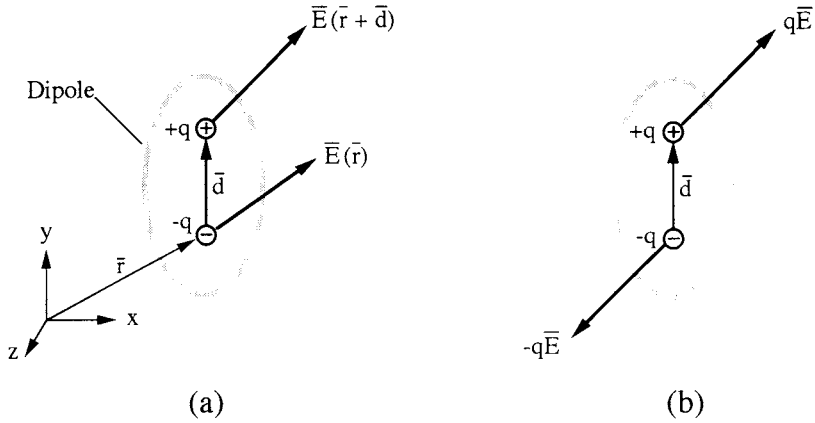

Fig. 2.1 Representation of the force and torque exerted upon a small dipole by an electric field of force.

a) Net force on a small dipole of strength $p = qd$ in a nonuniform electric field.

b) Coulombic force components creating net torque on a small dipole of strength $p = qd$ in a uniform electric field.

If the electric field is nonuniform, then in general the two charges ($+q$ and $-q$) will experience different values of the vector field $\bar{E}$ and the dipole will experience a net force. Performing a sum of the forces on the particle, we have

$$\bar{F} = q\bar{E}(\bar{r} + \bar{d}) - q\bar{E}(\bar{r}) \tag{2.1}$$

where $\bar{r}$ is the position vector of $-q$. Equation (2.1) can be simplified when $|\bar{d}|$ is small compared to the characteristic dimension of the electric field nonuniformity. In this case, the electric field can be expanded about position $\bar{r}$ using a vector Taylor series expansion; that is

$$\bar{E}(\bar{r} + \bar{d}) = \bar{E}(\bar{r}) + \bar{d} \cdot \nabla\bar{E}(\bar{r}) + \cdots \tag{2.2}$$

where all additional terms, of order d^2, d^3, and so forth, have been neglected. Using the expansion of Equation (2.2) in Equation (2.1), the following result is obtained.

$$\bar{F} = q\bar{d} \cdot \nabla\bar{E} + \cdots \tag{2.3}$$

If the limit $|\bar{d}| \to 0$ is taken in such a way that $\bar{p} \equiv q\bar{d}$ (the dipole moment) remains finite, then the following well-known expression for the force on an infinitesimal dipole results:

$$\bar{F}_{\text{dipole}} = \bar{p} \cdot \nabla\bar{E} \tag{2.4}$$

Equation (2.4) teaches that no net force is exerted on a dipole unless the externally imposed electric field is nonuniform.

The above derivation reveals that Equation (2.4) is an approximation for the force exerted upon any physical dipole, such as a polarized particle of finite size. This approximation, referred to here as the *dielectrophoretic approximation*, is usually quite adequate for imposed field interactions because the dimensions of practical electrodes are ordinarily much larger than the particles, meaning that the scale of the electric field nonuniformity is large compared to the particle dimensions. On the other hand, the DEP approximation often leads to significant error in the case of mutual interactions between closely spaced particles because the nonuniformities of the induced field due to the particles themselves are comparable to the particles' size. When this is true, Equation (2.4) becomes very inaccurate and higher-order multipolar terms must be retained. A special case where the higher-order terms can be taken into account easily is considered and analyzed in Section 2.4B below. Furthermore, in Chapters 6 and 7, which deal with mutual particle interactions, the treatment of higher-order terms is given special attention.

C. Torque on a dipole

The torque exerted by an electric field on an infinitesimal dipole may be derived by considering the net force couple acting about the center of the small dipole, as shown in Figure 2.1b. There are two contributions to this torque, one due to each charge.

$$\bar{T}^{e} = \frac{\bar{d}}{2} \times q\bar{E} + \frac{-\bar{d}}{2} \times (-q\bar{E}) = q\bar{d} \times \bar{E} \tag{2.5}$$

or, in terms of the dipole moment $\bar{p}$ as previously defined,

$$\bar{T}^{e} = \bar{p} \times \bar{E} \tag{2.6}$$

Note that the torque on the infinitesimal dipole is dependent only on the electric field vector, not its gradient, and so an electrical torque can be exerted on a dipole by a uniform field. The only requirement for the existence of this torque is that $\bar{p}$ and $\bar{E}$ are not parallel. It should be evident that, like Equation (2.4) for the dipole force, the torque expression, Equation (2.6), is an approximation for any finite dipole if the electric field is nonuniform. Any error incurred in its use becomes significant only when the scale of the electric field nonuniformities is comparable to the dimension of the dipole.

2.2 Lossless dielectric particle in an electric field

A. Induced multipolar moments

Equations (2.4) and (2.6) were derived in Section 2.1 without reference to the nature of the dipole moment $\bar{p}$. This vector quantity might be the permanent moment of a polar molecule or poled particle, or it might be induced by the imposed electric field itself. In this book, interest focuses primarily on the latter case, where the dipole moment and all higher-order moments are induced by the electric field and its derivatives. This limitation is relaxed in order to consider the mechanics of a permanently magnetized particle in a magnetic field. In general, the moment-inducing field is a combination of externally imposed and mutual field contributions (due to other nearby particles). Thus, it is necessary to relate these moments to the electric field and the particle parameters so that these values can then be used in Equations (2.4) and (2.6) to predict the forces and torques on particles.

A crucial objective of this chapter is to introduce the *effective moment* method of calculating electromechanical forces and torques exerted by electric fields upon particles. This method is based on identification of the "correct" expression for $\bar{p}$ to be used in Equations (2.4) and (2.6). Subsequent chapters on such topics as dielectrophoresis and magnetophoresis, rotation, and orientation will all employ this method whenever possible for the calculation of force and torque. The effective moment method, despite certain pitfalls to be discussed later, is valuable because it is easy to use and readily provides valid predictive results in many important cases where rigorous derivations based on the Maxwell stress tensor seem difficult or impossible.

Imagine a particle suspended in some dielectric fluid and subject to a uniform electric field. The field polarizes the particle, inducing a moment in it. The effective dipole moment $\bar{p}_{\text{eff}}$, here aligned parallel to the imposed field, is then defined as the moment of an equivalent, free-charge, point dipole that, when immersed in the same dielectric liquid and positioned at the same location as the center of the original particle, produces the same dipolar electrostatic potential. From Appendix B, the electrostatic potential Φ_{dipole} due to a point dipole of moment $\bar{p}_{\text{eff}}$ in a dielectric medium of permittivity ε_1 is

$$\Phi_{\text{dipole}} = \frac{p_{\text{eff}} \cos\theta}{4\pi\varepsilon_1 r^2} \tag{2.7}$$

where θ and r are, respectively, the polar angle and radial position in spherical coordinates. For a dielectric particle, the effective dipole moment is determined by solving the appropriate boundary value problem and then comparing the

induced (dipole) term of the electrostatic potential solution to Equation (2.7). We provide an example of this procedure in the next section.

B. Effective dipole moment of dielectric sphere in dielectric medium

Let an insulating dielectric sphere of radius R and permittivity ε_2 be suspended in a fluid of permittivity ε_1 and be subjected to a uniform z-directed electric field of magnitude E_0. Refer to Figure 2.2. It is assumed at the outset that the electric field is uniform, that there is no free charge anywhere in the sphere or the dielectric liquid, and that the presence of the particle does not significantly disturb the system of source charges that creates E_0. The more general case of a nonuniform electric field is considered in Section 2.2E.

The electrostatic potential satisfies Laplace's equation everywhere because of the divergence- and curl-free properties of the electrostatic field. The assumed solutions for the potential outside Φ_1 and inside Φ_2 the sphere take the form

$$\Phi_1(r, \theta) = -E_0 r \cos\theta + \frac{A \cos\theta}{r^2}, \; r > R \tag{2.8a}$$

$$\Phi_2(r, \theta) = -Br \cos\theta, \; r < R \tag{2.8b}$$

where A and B are unknown coefficients to be determined using the boundary conditions. Note that the first term in Equation (2.8a) is the imposed uniform electrostatic field, while the second term is the induced dipole term due to the

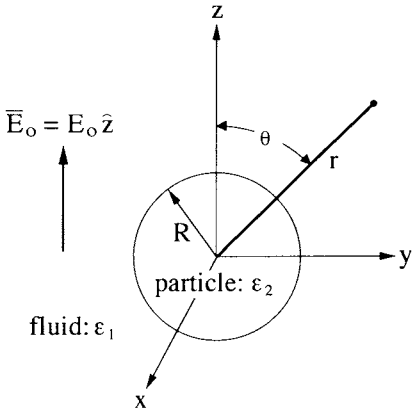

Fig. 2.2 Dielectric sphere of radius R and permittivity ε_2 immersed in a dielectric liquid of permittivity ε_1 and subjected to a uniform electric field of magnitude E_0.