

SEISMIC RAY THEORY

Seismic Ray Theory presents the most comprehensive treatment of the seismic ray method available. This method plays an important role in seismology, seismic exploration, and the interpretation of seismic measurements.

The book presents a consistent treatment of the seismic ray method, based on the asymptotic high-frequency solution of the elastodynamic equation. At present, this is the most general and powerful approach to developing the seismic ray method. High-frequency seismic body waves, propagating in complex three-dimensional, laterally varying, isotropic or anisotropic, layered and block structures are considered. Equations controlling the rays, travel times, amplitudes, Green functions, synthetic seismograms, and particle ground motions are derived, and the relevant numerical algorithms are proposed and discussed. Many new concepts, which extend the possibilities and increase the efficiency of the seismic ray method, are included. The book has a tutorial character: derivations begin with a relatively simple problem in which the main ideas are easier to explain and then advance to more complex problems. Most of the derived equations in the book are expressed in algorithmic form and may be used directly for computer programming. The equations and proposed numerical procedures find broad applications in numerical modeling of seismic wavefields in complex 3-D structures and in many important inversion methods (tomography and migration among others).

Seismic Ray Theory will prove to be an invaluable advanced textbook and reference volume in all academic institutions in which seismology is taught or researched. It will also be an invaluable resource in the research and exploration departments of the petroleum industry and in geological surveys.

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Preface

The book presents a consistent treatment of the seismic ray method, applicable to high-frequency seismic body waves propagating in complex 3-D laterally varying isotropic or anisotropic layered and block structures. The seismic ray method is based on the asymptotic high-frequency solution of the elastodynamic equation. For finite frequencies, the ray method is not exact, but it is only approximate. Its accuracy, however, is sufficient to solve many 3-D wave propagation problems of practical interest in seismology and seismic exploration, which can hardly be treated by any other means. Moreover, the computed rays may be used as a framework for the application of various more sophisticated methods.

In the seismic ray method, the high-frequency wavefield in a complex structure can be expanded into contributions, which propagate along rays and are called elementary waves. Individual elementary waves correspond, for example, to direct P and S waves, reflected waves, various multiply reflected/transmitted waves, and converted waves. A big advantage of the ray method is that the elementary waves may be handled independently. In the book, equations controlling the rays, travel times, amplitudes, Green functions, seismograms, and particle ground motions of the elementary waves are derived, and the relevant numerical algorithms are developed and discussed.

In general, the theoretical treatment in the book starts with a relatively simple problem in which the main ideas of the solution are easier to explain. Only then are the more complex problems dealt with. That is one of the reasons why pressure waves in fluid models are also discussed. All the derivations for pressure waves in fluid media are simple, clear, and comprehensible. These derivations help the reader to understand analogous derivations for elastic waves in isotropic and anisotropic solid structures, which are often more advanced. There is, however, yet another reason for discussing the pressure waves in fluid media: they have often been used in seismic exploration as a useful approximation for P waves in solid media.

Throughout the book, considerable attention is devoted to the seismic ray theory in inhomogeneous anisotropic media. Most equations derived for isotropic media are also derived for anisotropic media. In addition, weakly anisotropic media are discussed in some detail. Special attention is devoted to the qS wave coupling.

A detailed derivation and discussion of paraxial ray methods, dynamic ray tracing, and ray propagator matrices are presented. These concepts extend the possibilities of the ray method and the efficiency of calculations. They can be used to compute the travel times and slowness vectors not only along the ray but also in its vicinity. They also offer numerous other important applications in seismology and seismic exploration such as solution of boundary-value ray tracing problems, computation of geometrical spreading, Fresnel volumes, Gaussian beams, and Maslov-Chapman integrals.

Most of the final equations are expressed in algorithmic form and may be directly used for programming and applied in various interpretation programs, in numerical modeling of seismic wave fields, tomography, and migration.

I am greatly indebted to my friends, colleagues, and students from Charles University and from many other universities and institutions for helpful suggestions and valuable discussions. Many of them have read critically and commented on certain parts of the manuscript. I owe a special debt of thanks to Peter Hubral, Bob Nowack, and Colin Thomson for advice and constructive criticism. I am further particularly grateful to Ivan Pšenčík and Luděk Klimeš for everyday discussions and to Eva Drahotová for careful typing of the whole manuscript. I also wish to express my sincere thanks to the sponsors of the Consortium Project “Seismic Waves in Complex 3-D Structures” for support and to Schlumberger Cambridge Research for a Stichting Award, which was partially used in the preparation of the manuscript. Finally, I offer sincere thanks to my family for patience, support, and encouragement.