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This book describes the manifestations of chaos in atoms and molecules.

About ten years ago, atomic physics received a rejuvenating jolt from chaos theory with far-reaching implications. The study of chaos is today one of the most active and prolific areas in atomic physics. This is the first attempt to provide a coherent introduction to this fascinating area. In line with its scope, the book is divided into two parts. The first part (chapters 1–5) deals with the theory and principles of classical chaos. The ideas developed here are then applied to actual atomic and molecular physics systems in the second part of the book (chapters 6–10) covering microwave driven surface state electrons, the hydrogen atom in a strong microwave field, the kicked hydrogen atom, chaotic scattering with CsI molecules and the helium atom. The book contains many diagrams and a detailed reference list.

The book will be of interest to graduate students and researchers in atomic, molecular and optical physics.

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General Editors: A. Dalgarno, P. L. Knight, F. H. Read, R. N. Zare

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# Chaos in Atomic Physics

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Frontmatter  
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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press  
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521455022](http://www.cambridge.org/9780521455022)

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First published 1997  
This digitally printed first paperback version 2005

*A catalogue record for this publication is available from the British Library*

*Library of Congress Cataloguing in Publication data*

Blümel, R. (Reinhold)  
Chaos in atomic physics / R. Blumel, W. P. Reinhardt.  
p. cm. (Cambridge monographs on atomic, molecular, and chemical physics: 10)  
Includes bibliographical references.  
ISBN 0 521 45502 2  
1. Chaotic behavior in systems. 2. Atoms. 3. Molecules.  
I. Reinhardt, William P. II. Title. III. Series.  
QC174 17.C45B47 1997  
539.7d-c21 96-39292 CIP

ISBN-13 978-0-521-45502-2 hardback  
ISBN-10 0-521-45502-2 hardback

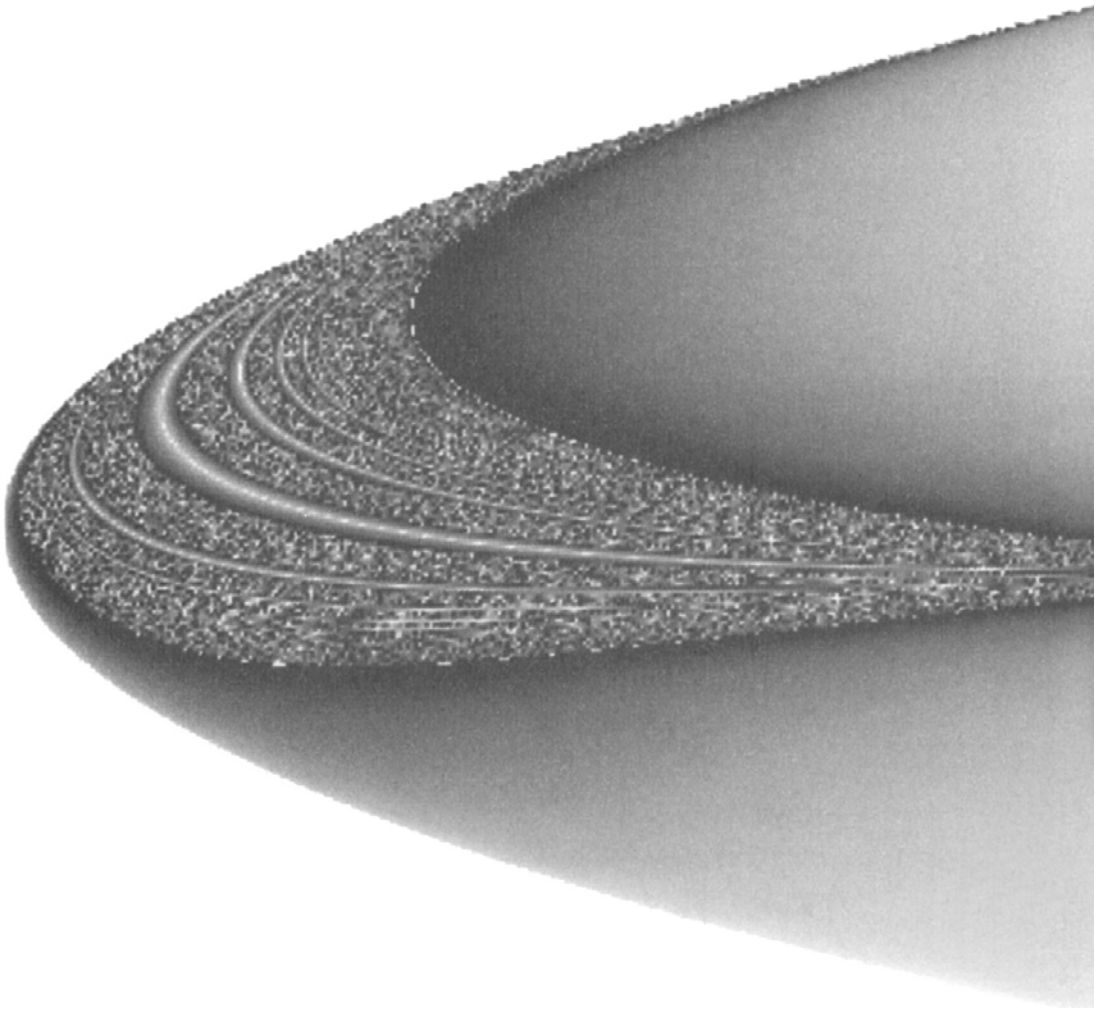
ISBN-13 978-0-521-01790-9 paperback  
ISBN-10 0-521-01790-4 paperback

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## Preface

Atomic physics is one of the oldest fields of physics. A barren and “academic” discipline? Not at all! About ten years ago, atomic physics received a rejuvenating jolt from chaos theory with far reaching implications. Chaos in atomic physics is today one of the most active and prolific areas in atomic physics. This book, addressed at interested students and practitioners alike, is a first attempt to provide a coherent introduction into this fascinating area of contemporary research. In line with its scope, the book is essentially divided into two parts. The first part of the book (Chapters 1 – 5) deals with the theory and philosophy of classical chaos. The ideas and concepts developed here are then applied to actual atomic and molecular physics systems in the second part of the book (Chapters 6 – 10).

When compiling the material for the first part of the book we profited immensely from a number of excellent tutorials on classical and quantum chaos. We mention the books by Lichtenberg and Lieberman (1983), Zaslavsky (1985), Schuster (1988), Sagdeev *et al.* (1988), Tabor (1989), Gutzwiller (1990), Haake (1991), Devaney (1992) and Reichl (1992).

The illustrative examples for the second part of the book were mostly taken from our own research work on the manifestations of chaos in atomic and molecular physics. We apologize at this point to all the numerous researchers whose work is not represented in this book. This has nothing to do with the quality of their work and is due only to the fact that we had to make a selection. We were not even able to devote a separate chapter to every one of the most important classically chaotic systems currently under active investigation. The most important omission is probably the hydrogen atom in a strong magnetic field, although we took care to mention it several times in this book, and we discuss its potential for future research in Chapter 11.

For the experienced practitioner of atomic physics there appears to be an enigma right at this point. What does *nonlinear* chaos theory have to do with *linear* quantum mechanics, so successful in the classification of atomic states and the description of atomic dynamics? The answer, interestingly, is the enormous advances in atomic physics itself. Modern day experiments are able to control essentially isolated atoms and molecules to unprecedented precision at very high quantum numbers. Key elements here are the development of atomic beam techniques and the revolutionary effect of lasers. Given the high quantum numbers, Bohr's correspondence principle tells us that atoms are best understood on the basis of classical mechanics. The classical counterpart of most atoms and molecules, however, is chaotic. Hence the importance of understanding chaos in atomic physics.

During the past ten years the area of chaos in atomic physics has matured to an established field with a rapidly expanding literature of hundreds of published papers. Therefore, we feel that the time has come where it makes sense to include this field in the set of courses routinely offered to students at the graduate level. In fact, this book is based on a course on chaos in atomic physics delivered at the University of Freiburg in the Winter Semester 1995/1996.

Excellent reviews exist addressing the topic of chaos in atomic physics. The most comprehensive attempt at a review of the entire subject of quantum chaology and applications is a recent collection of reprints and original articles edited by Casati and Chirikov (1995). Its section on atoms in strong fields especially is highly relevant for the topic of this book. Additional material on chaos and irregularity in atomic physics can be found, e.g., in a collection of articles edited by Gay (1992).

Many of the subjects discussed in this book were investigated and developed in collaboration with Prof. Uzy Smilansky. We owe many thanks to Uzy for sharing his continuous stream of ideas with us, and for hosting uncounted extended visits at the Weizmann Institute. This fruitful collaboration started in 1983 and was always generously supported by the MINERVA foundation.

The topic of Chapter 5 was originally investigated in collaboration with Prof. Shmuel Fishman. His hospitality on the occasion of many visits to the Technion is very much appreciated.

Prof. Peter Koch made it possible for one of us (R.B.) to visit his lab on several occasions for extended periods of time. This was a unique opportunity to see the inner workings of one of the most productive modern atomic physics labs. Many thanks!

Many thanks are due to our colleagues at the University of Maryland, Professors Tom Antonsen, Celso Grebogi, Ed Ott and Richard Prange for a very productive collaboration in 1995.

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Last but not least we would like to thank our colleagues at Freiburg, especially Dr G. Alber, Prof. J. S. Briggs, Dr A. Bürgers, Dr F. Großmann and Dr J.-M. Rost for valuable discussions and suggestions. We owe special thanks to T. R. Neal, our science editor at Cambridge University Press for continued encouragement.