Statistical energy analysis: a critical overview

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For the benefit of the 'enquirer within', who may not be familiar with the background and concepts of SEA, this overview opens with a discussion of the rationale for the use of probabilistic energetic models for high-frequency vibration prediction, and introduces the postulate upon which conventional SEA is based. It compares and relates the modal and travelling wave approaches, discusses the strengths and weaknesses of SEA as currently practised and points out needs and directions for future research. Critical discussions of individual contributions to the development of the subject are presented only in as much as they treat specific matters of concept, principle or reliability. The roles of SEA in providing a framework for experimental investigations of the high-frequency dynamic behaviour of systems and in interpreting observations on operating systems, although equally important, are not substantially addressed. Nor are specific experimental techniques which involve considerations of transducers, spatial sampling, signal processing, error analysis and data interpretation, which require a critical review in their own right.

1. Origins

The development of statistical energy analysis (SEA) arose from a need by aerospace engineers in the early 1960s to predict the vibrational response to rocket noise of satellite launch vehicles and their payloads. Although computational methods for predicting vibrational modes of structures were available, the size of the models which could be handled (i.e. the number of degrees of freedom), and the speed of computation, were such as to allow engineers to predict only a few of the lowest order modes of rather idealized models. This posed a serious difficulty because the frequency range of significant response encompassed the natural frequencies of a multitude of higher order modes, the structure could support a number of different wave types, the payload structures were indirectly excited via structural wave transmission, and the transmission paths were circuitous and involved many different forms of structure and contained fluids. It is estimated that the Saturn launch vehicle possessed approximately 500000 natural frequencies in the range 0 to 2000 Hz.

Many previous years of experience with the analysis of sound fields in rooms suggested that the application of deterministic methods of analysis to the prediction of broadband vibrational response of such systems would neither be appropriate nor effective: a concomitant was that it would not be appropriate to estimate the response in terms of local values of kinematic or dynamic variables. Consequently, a form of analysis was introduced in which a system was divided up into subsystems, the subsystem parameters were expressed probabilistically, and the vibrational state of the system was expressed in terms of the time-average total vibrational energy of

each of the subsystems, i.e. a global, rather than a local, measure. Vibratory inputs were expressed in terms of time-average input powers, rather than in terms of external forces or displacements. This approach became known as statistical energy analysis.

2. Limitations of deterministic models and analysis

Modern theoretical vibration analysis of complex mechanical structures depends heavily on numerical procedures demanding large, fast computational facilities that can deal with mathematical models representing very detailed idealizations of the physical structures. The computational demands increase with geometric and material complexity, and with increase in analysis frequency. Even today, when computational methods are highly developed and optimized, it is not generally practicable to predict the detailed vibrational behaviour of such structures at frequencies beyond a few hundred Hertz (see, for example, Roozen (1992) who uses 550000 degrees of freedom in a finite element model to study the vibrational behaviour of a 2 m length of aircraft fuselage at frequencies up to 225 Hz).

It is, in principle, possible to extend deterministic computational forms of analysis to higher frequencies, at the expense of rapidly increasing demands in terms of the size of the model and consequent analysis time and cost. However, there is a fundamental physical reason why such extension is ultimately doomed to failure: it is associated with an unavoidable uncertainty about the precise dynamic properties of a complex assemblage of structural components. As frequencies increase, the results of deterministic prediction of frequency response become more and more unreliable. The fundamental reason is that the sensitivity of modal resonance frequencies and relative modal phase response to small changes in structural detail, especially boundary conditions and damping distribution, increases with mode order; and as frequency increases, the responses of almost all systems at any one frequency comprise contributions from an increasing number of modes. The probability distributions of the lower natural frequencies of a simply-supported beam of which the mass per unit length is randomly perturbed are presented by Manohar & Keane (this book). The distributions of the higher order modes begin to overlap, indicating that there is increasing uncertainty in even the order of occurrence of the modal natural frequencies.

There is unavoidable uncertainty about structural detail and material properties associated with manufacturing tolerances and fabrication imperfections, together with environmental and operational influences such as temperature and static load; the high-frequency dynamic properties of joints between components are especially uncertain. As a result, high frequency vibrational responses of individual examples of nominally identical structures are observed to differ, sometimes greatly, as illustrated by figure 1, after Kompella & Bernhard (1993). It is clear that the apparent precision of prediction of response offered by large computational models of complex engineering structures is illusory at frequencies where more than a few modes contribute significantly to the response, and that the high costs of computational procedures based upon deterministic models cannot be justified, particularly since the wealth of detail provided by such procedures is always indigestible and usually unnecessary.

What then is the alternative? It is essentially to treat response prediction from the outset as a probabilistic problem. The high frequency response characteristics of a population of grossly similar systems of which the individual members differ in

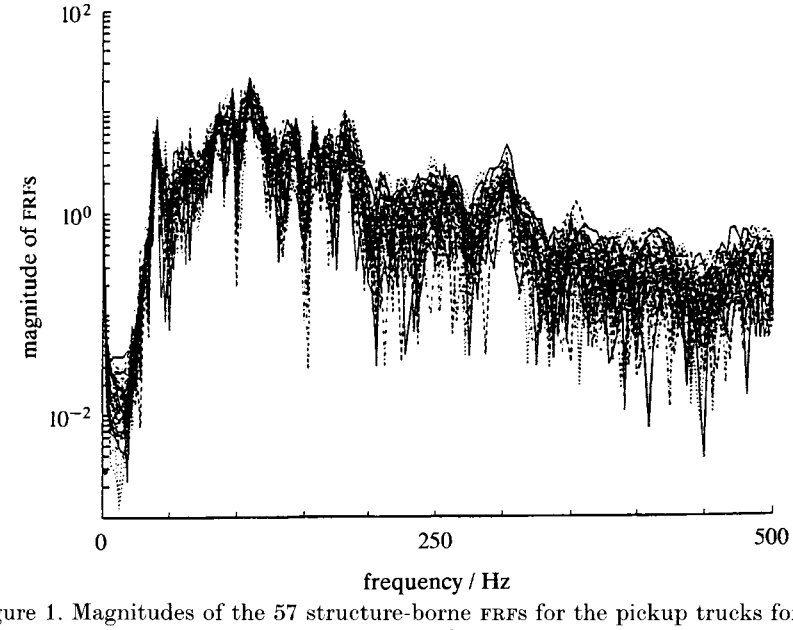

Figure 1. Magnitudes of the 57 structure-borne FRFs for the pickup trucks for the driver microphone.

unpredictable detail may be characterized by their ensemble-average behaviour, together with statistical measures of the distribution of responses about this average. One possible approach to generating an estimate of such a distribution would be to apply a Monte Carlo procedure to a system by randomizing its parameters and properties according to some assumed distributions and repeatedly applying deterministic computational analysis to each member of the set so-generated. A little thought will soon reveal that such an approach is impractical, not only for reasons of cost and time, but because there is no way to model the multi-dimensional joint probability distributions of the large number of parameters and variables.

3. An alternative energy response model

Even though it is impossible to predict precisely the detailed response behaviour of any one physical system to any specific form of high-frequency excitation, it is clear that the gross physical properties such as geometric form and dimensions, together with the material properties, determine the ensemble-average behaviour of a population that shares those properties. It is therefore sensible to seek an approach which utilizes the minimum system description necessary for a prediction of ensemble-average behaviour consistent with the aims of the analyst and with the degree of uncertainty inherent in the prediction on account of imperfect knowledge of the detailed properties of any individual system. Because it is unrealistic, and generally unnecessary, to attempt to predict the response at every point of a system, it is sensible to seek a global description of the response of individual components of the system within which the ensemble-average response is expected to be reasonably uniform: this is a necessary condition for the usefulness of a global response prediction, since it would not be sensible to attribute a single response level to contiguous systems which are likely to exhibit greatly different levels.

These considerations lead to the adoption of a gross parameter model. This is

conceptually divided up into subsystems that either have significantly different gross properties, or are separated from contiguous subsystems by structural elements, which form significant barriers to transmission of vibration from the source(s) of excitation. Because the relative amplitude and phase of the frequency response at individual points within any subsystem are essentially unpredictable, it is reasonable to treat these response quantities as being random, thereby introducing the concept of treating the frequency response functions at individual points as a population of which only statistical descriptors such as the ensemble mean value and measures of the spatial variation about this mean are meaningful.

The total time-averaged energy of vibration is an attractive global descriptor of subsystem response since it is phase-independent and is subject to the fundamental constraint imposed by the principle of conservation of energy. The multi-degree-of-freedom model or finite element model of a subsystem is replaced by a single degree of freedom, and the conventional frequency response function is replaced by a subsystem energy response function. Naturally there is a penalty; no information is available about spatial distributions of response variables such as strain or acceleration: however, these can also be represented by probabilistic models (see, for example, Stearn 1970). Once this energy response descriptor is adopted, the description of subsystem interaction in terms of rate of energy exchange (or 'power flow') and the description of subsystem excitation in terms of power input from external sources naturally follow.

In summary, the energy response model comprises a set of subsystems described by their gross geometric forms and dynamic material properties, which are subject to external power inputs, and which receive, dissipate and transmit vibrational energy. The output of an energy response analysis is an estimate of the equilibrium time-average energies of vibration of the various subsystems.

4. Statistical energy analysis

(a) *Frequency-average estimates*

Statistical energy analysis (SEA) in its current state of development provides a means for estimating the equilibrium energies of a network of subsystems which are subject to an assumed distribution of external sources of time-stationary vibrational power input. For historical and practical reasons, it does not deal directly with ensemble-mean energy response functions for a population of systems that share the same gross parameters, but which differ in detail, as discussed above. Instead, it is used to estimate the frequency-average value of the energy response functions of individual archetypal subsystems over intervals of frequency that are adjudged large enough to justify simplifying assumptions concerning the coefficients which relate time-averaged vibrational power flow between subsystems to their equilibrium energies.

The relation between frequency-average and ensemble-average energy response is the subject of current research and is not yet fully resolved. However, the use of frequency-average energy response is more practical for the purposes of experimental SEA, which plays a vital complementary role to predictive SEA. In many cases of engineering interest, the power transmission and dissipation coefficients associated with selected subsystems are not computable, and consequently they must be derived from experiments on physical systems. It is not generally practicable to measure ensemble-average coefficients on populations of physical systems, but it is

simple to implement frequency averaging of experimental response data obtained from one system. It is worth noting here that SEA may also be used to interpret response data collected from operating systems for the purposes of selecting appropriate vibration and noise control measures. It has also been used in an inverse mode to infer the locations and magnitudes of sources of external power input from experimental estimates of subsystem energies and a previously established SEA model.

The evaluation of the coefficients that relate power transfer between subsystems to their equilibrium energies lies at the heart of the engineering application of SEA. There are three main approaches to this problem. In the *modal* approach, the equations governing the dynamic behaviour of the subsystems are expressed explicitly in terms of expansions of the vibrational fields in series representing the uncoupled natural modes of the subsystems: the choice of boundary conditions which express the decoupling is a matter of analytical simplicity and physical characteristics (Maidanik 1976). It is then possible, in principle, to express the multi-mode power transfer coefficient as an estimate of the average of the individual mode-pair coupling coefficients, subject to certain assumptions regarding modal frequency distributions and coupling strengths using the results of coupled oscillator analysis. This approach is rather well suited to vibroacoustic problems involving acoustic interaction between enclosed volumes, gases and solid structures, because coupling is normally weak, the *in vacuo* modes of the structure can be used, and the statistical properties of rigidly bounded fluid volumes have been exhaustively researched (see, for example, Fahy 1970).

The explicit modal approach is not ideally suited to coupling between solid structures, especially where they are not uniform and isotropic. Consequently a travelling *wave* approach is used, in which the vibrational fields are modelled as superpositions of travelling waves (usually plane), and transfer between subsystems is evaluated from considerations of wave transmission and reflection at subsystem interfaces. The wave intensities distribution is related to subsystem energy density, to the dispersion relation of the waves and to the subsystem geometry. Wave power transmission coefficients can be evaluated using a combination of statistical models of incident, reflected and transmitted wave fields with relatively small-scale deterministic models of interface dynamics analysed by computational procedures such as finite element analysis. A modern development of the wave approach is presented by Langley & Bercin (this book).

The *mobility* approach utilizes the concept of dynamic mobility, or impedance (point, line or wave) to express the result of interaction between coupled subsystems. To deal with probabilistic models of the subsystems, averages of various forms (spatial, frequency, ensemble) are applied to these dynamic characterizations of subsystems. This approach is explained by Manning (this book) and evaluated by comparison with experiment by Cacciolati & Guyader (this book).

(*b*) *The modal approach to* SEA

Where two subsystems interact to exchange energy, it is possible to express the interaction in terms of the modes of the uncoupled subsystems, of which selection of the appropriate boundary conditions depends upon the physical natures of the subsystems. SEA has traditionally been developed in terms of this modal interaction model, and the SEA equations have been developed as an extension of the relationship between the energies of vibration of two conservatively coupled, viscously damped

oscillators subjected to white noise excitation, and the rate of exchange of energy, or power flow. This exact relation is (Scharton & Lyon 1967)

$$P_{12} = g[E_1 - E_2], \tag{1}$$

where P_{12} is the time-average power flow, E_1 and E_2 are the time-average equilibrium energies of the oscillators when subject to the given excitation but with the motion of the other oscillator 'blocked', and g is a power transfer coefficient. (In this simple case, the relation also holds for the actual coupled energies, but, of course, g is different.) The coefficient g is a function of the properties of the coupling elements, the oscillator half power bandwidths (damping), and the natural frequencies of the uncoupled oscillators: it is particularly sensitive to the difference of natural frequencies of the blocked oscillators.

Formal extension of this analysis to power exchange between two sets of oscillators is analytically straightforward, but the resultant expressions for power exchange involve terms that express correlations between the motions of all the oscillators and is not only algebraically unwieldy, but useless for practical purposes. If it is assumed that the motion of each oscillator within one set is uncorrelated with the motion of every other within the same set, these terms disappear and the total set-to-set power exchange can be expressed as the sum of the power exchanges between individual pairs of oscillators, one from each set.

Since the motion of an individual oscillator is determined by the combined effects of any external excitation and the reaction forces produced by interaction with other oscillators, neglect of intra-set correlation requires two assumptions: (i) that the external forces on each oscillator are uncorrelated; (ii) that coupling is sufficiently 'weak' to ensure that indirect intraset oscillator interaction via any one oscillator of the other set is negligible compared with direct inter-set oscillator interaction. Extension of this weak coupling model to energy exchange between the modes of distributed elastic systems is based upon an assumption of uncorrelated modal generalized forces and weak coupling between the two subsystems. Uncorrelated modal forces can only be generated by delta-correlated or 'rain-on-the-roof' force fields: the basic SEA relation does not hold in the case of locally concentrated excitation, except in the sense of an ensemble-average over all locations. Further, to use the concept of a 'modal-average' coupling coefficient, it is assumed that energy is shared equally by modes resonant within the band of analysis (equipartition of modal energy). Under these (rather restrictive) conditions, the two-oscillator result may be extended to an analogous relation between the power flow between the two sets of oscillators which represent the uncoupled modes of the subsystems, and the average stored energy per oscillator (modal energy) of each subsystem, thus (Scharton & Lyon 1967);

$$P_{12} = M_{12}[E_1/n_1 - E_2/n_2], \tag{2}$$

in which P_{12} is the net power flow between the subsystems, E represents total subsystem energy, n represents modal density, which is the inverse of the local average frequency spacing between successive modal natural frequencies and M_{12} is a modal-average 'power transfer coefficient'. Newland (1969) derives an equivalent relation for weakly coupled oscillator sets on the basis of a small parameter, perturbation approach and Zeman & Bogdanoff (1969) do the same for extended elastic systems. On the basis of a criterion for 'weak' coupling which is closely related to that adopted by Newland, namely that the Green functions of the subsystems are little affected by coupling, Langley (1989) also confirms the general relation

expressed by (2). Gersch (1968) generalizes the results of Scharton and Lyon and of Newland to cases involving non-conservative coupling, giving exact expressions for a three-oscillator case. The term 'power transfer coefficient' used herein is not standard; it is conventional in SEA to express (2) in terms of a quantity known as the 'coupling loss factor' which is defined by analogy with the dissipation loss factor thus:

$$P_{12} = \eta_{12}\,\omega E_1 - \eta_{21}\,\omega E_2. \tag{3}$$

It transpires that the coupling loss factor is a function of the spatial extent of a subsystem, and that the product of coupling loss factor and modal density is a physically more significant quantity which is both independent of subsystem extent and independent of the direction of power flow considered.

Comparison of (2) and (3) indicates that the power transfer coefficient M_{12} is equivalent to $\eta_{12}\,\omega n_1 = \eta_{21}\,\omega n_2$. It represents a form of mode-pair average of the coupling coefficient g, and is formally a function of the physical coupling between the subsystems, the degree of spatial matching of the mode pair shapes integrated over the common interface, the modal dampings and the relative distributions of mode pair natural frequencies. M_{12} is independent of modal damping only in the case of weak coupling, and only then if a certain form of probability distribution of mode pair frequency difference obtains. A condition of 'strong' coupling, in SEA terms, produces very similar values of average modal energy in all strongly coupled subsystems, irrespective of the distribution of input power. It will also necessarily produce correlated modal responses and non-uniform modal energies in subsystems which are not subject to external forces, but are driven only through couplings to other subsystems. It may be argued that the condition of 'weak coupling', necessarily requires that $M_{12} \ll M_1$ and M_2 for all pairs of connected subsystems (Smith 1979). Keane & Price (1987) present an analysis of power flow between multi-mode systems strongly coupled at a single point in which subsystem receptances are used to derive an exact analytic expression for the power transfer coefficient. Simplification of the exact expression is achieved by ensemble averaging and using a Monte Carlo approach to evaluating the behaviour of various terms in the expression. They demonstrate that the basic power flow-energy difference relation holds for all strengths of coupling, but that the expressions for power transfer coefficient which obtain under conditions of weak coupling must be modified to account for strong coupling, in agreement with Lyon (1975) and Mace (1992). The transition from weak to strong coupling in a system of spring-coupled rods is clearly illustrated by the work of Keane & Price (1991), in which the power flow increases monotonically with increase of spring stiffness up to the transition point, after which it becomes independent of coupling strength.

Except in the cases of highly idealized mathematical models, explicit averaging of mode pair coupling coefficients is not possible in practical systems for the reasons discussed above; modal parameters are normally only describable in a probalistic sense. The dependence of the power transfer coefficient on the probability distribution of mode pair natural frequency differences suggests that it will be rather sensitive to perturbations of the subsystem physical parameters under conditions where the frequencies are well separated in terms of typical modal bandwidths, i.e. low modal overlap. In this respect, modal natural frequency distribution statistics for practical systems have been inadequately studied.

The relationship between power flow and modal energy difference expressed by (2) forms the central 'postulate' upon which the edifice of SEA has been constructed.

It is analogous to the equation of conductive heat flow in which the average modal energy is analogous to temperature.

One implication of (2) for practical applications is that SEA cannot be expected to give accurate estimates in cases where the contiguous subsystem modal energies are very similar (close to equipartition of modal energy) since given fractional errors in the estimate of either or both will give rise to much larger large fractional errors in the difference, and hence in the power flow estimate. As shown by Heckl & Lewit (this book), this is especially relevant where SEA power balance equations are used, via loss factor matrix inversion, to estimate coupling loss factors from measured subsystem responses. This requirement has implications for the selection of subsystem definitions and boundaries, and rules out the use of SEA for systems through which vibrational waves travel with so little attenuation or reflection that modal energies are all very similar. On the other hand, excessive damping renders inappropriate the concept of modal energy as a global descriptor of vibrational state.

The modal (oscillator) interaction model is reasonably satisfactory as a conceptual basis for SEA but it is generally not well suited to computation in structural engineering applications and it raises difficulties of physical understanding of the power exchange process since uncoupled modes are generally chosen to satisfy boundary conditions which allow no energy transmission across the boundary. Consequently, in the practice of SEA, the modal model is rarely used directly to predict structural vibration, although numerous SEA validation exercises have been performed using the known modal characteristics of uniform subsystems such as rods, beams, rectangular plates and circular cylindrical shells in various combinations (see, for example, Keane & Price 1987; Davies 1981; Dimitriadis & Pierce 1988). The modal approach is, however, more practical in cases of vibroacoustic interaction between extended structures and volumes of contiguous fluid, in which power transfer takes place over the surface of the common boundary, and the coupling loss factor is influenced by the structural boundary conditions. In practice, the energy distributions and the processes of subsystem interaction are more commonly expressed in terms of propagating wave models.

Woodhouse (1981) presents a treatment of vibration transmission between coupled multimode systems which is based upon the classical multi-degree-of-freedom generalized coordinate representation and which facilitates the introduction of extra degrees of freedom as arbitrary coupling elements. This paper, together with the thought provoking overview of high-frequency structural vibration of Hodges & Woodhouse (1986), is recommended to the reader who wishes to acquire greater physical insight into the foundations of SEA.

(c) *The wave approach to SEA*

In the wave approach to SEA, the vibrational fields of subsystems are represented in terms of superpositions of travelling waves, and the power transfer coefficients between subsystems are evaluated from consideration of wave reflection and transmission at their junctions. In the case of spatially uniform junctions and subsystems, the harmonic plane wave (single wavenumber) transmission coefficients are simple functions of the input wave impedances (or mobilities) of the uncoupled subsystems evaluated at the joint. It should be note carefully that, as explained by Manning (this book), the mobility of a spatially extended joint is not equal to the sum of the mobilities of the connected subsystems.

Junction mobilities are conventicnally evaluated on the assumption that

transmitted waves returning to the junction after reflection from other boundaries are uncorrelated with the directly transmitted waves. This assumption is clearly unjustified where systems exhibit distinct resonances, because resonances and modes are the result of interference between coherent waves travelling in different directions: hence, power transfer coefficients for systems of low modal overlap are likely to be rather sensitive to perturbations of subsystem parameters.

Just as mode shapes, dampings and frequencies are described in a statistical sense in the modal approach, so the distribution of amplitudes, phases, attenuation and directions of wave propagation must be represented by statistical distributions in the wave approach. For example, the wave equivalent of the uncorrelated modal response assumption is an assumption of zero correlation between the set of travelling plane waves by which a subsystem vibration field is modelled: the equivalent of equipartition of subsystem modal energy is a smooth, sometimes uniform (diffuse), angular distribution of wave intensity. It may be shown that wave intensity may be directly related to modal energy, which relation provides the link between the two approaches. A profound implication of the assumption of uncorrelated travelling wave components is that reactive wave intensity (non-propagating energy density) does not exist. This is in stark contrast to the modal model, because modes involve predominantly reactive intensity. A related distinction is that, unlike the modal coupling model, in which coupling coefficients are, in general, functions of subsystem damping, the power transfer coefficients derived from wave transmission models of coupled semi-infinite subsystems are independent of damping. Given these gross differences in the assumed energetic characteristics of the modal and wave models, it is somewhat surprising that they produce compatible predictions.

There is a fundamental distinction between the modal and wave representation. Uncoupled subsystem modes, although unpredictable in a deterministic sense, are entities whose characteristics depend inherently upon the assumed geometric and dynamic boundary conditions, and which are associated with particular natural frequencies. Propagating waves are controlled only by the dynamic properties of the wave-bearing medium and are essentially independent of boundary conditions: the latter only influence wave scattering at the boundary. Consequently, the travelling wave representation involves no consideration of natural frequencies or assumptions about their distribution. In its favour, it raises fewer problems with regard to the representation of the action of distributed dissipative mechanisms acting within the body of a subsystem, and at boundaries, than the modal approach, where the question of the representation in terms of real or complex modes is a vexed one. These distinctions make the wave approach more attractive in terms of qualitative appreciation of the physical behaviour of a complex system. In fact, the most commonly applied method of evaluating the coupling loss factor in (3) is based upon wavefield modelling, and not modal modelling.

However, although the travelling wave approach is more appropriate at frequencies where numerous modes make significant contributions to the response at any one frequency (i.e. the modal overlap factor greatly exceeds unity) it may prove to be less helpful in developing means of making confidence estimates for SEA predictions in frequency ranges where distinct individual resonance peaks are observable in the energy-response functions, since a probabilistic wave model excludes the phenomenon of resonance. When the modal overlap factor is less than unity, a probabilistic model of the occurrence of modal resonance frequencies will be

a necessary ingredient of the analysis of variance of response estimates. (It should be noted that the subsystem impedances which control the wave power transmission coefficient will exhibit strong frequency dependence under conditions of low modal overlap (because of coherent (modal) interference), and statistical models of the distribution of impedance components may also be used to study uncertainty in SEA response estimates.)

(d) *The mobility approach to SEA*

The dynamic properties of system may be characterized by means of a transfer function which relates a harmonic input force or displacement field to the resulting response. In vibration analysis, these transfer functions take two principal forms: impedance relates force input to velocity response, and mobility is the corresponding inverse; where both quantities are specified at the same point or line, they are known as 'direct' or 'input' impedance and mobility. The most common forms correspond to 'point', 'uniform line' and 'plane wave' inputs. When a vibrational wave in a subsystem impinges upon an interface between it and a subsystem having different dynamic properties, wave reflection and transmission occur, which may or may not involve non-specular scattering, diffraction and refraction. The resulting forces and displacements at the interface depend upon the form of the incident field and the dynamic properties of both systems. The power transmitted through the interface is given by the time-average product of the associated forces and velocities. These are related by the interface mobility (or impedance) appropriate to the form of incident wave. Hence, wave power transmission coefficients can be expressed in terms of mobilities. Because the incident wave power can be related to the modal energy of the associated system as explained in the previous section, the power transfer coefficient of (2) can be evaluated in terms of the junction mobilities. These, in turn may be determined in a frequency or ensemble average sense for specific physical forms of subsystem, together with assumptions concerning the statistical distributions of the subsystem parameters. As explained by Manning (this book), an average over an exact power transfer expression is approximated by inserting average mobilities into the expression. This constitutes the essential approximation at the heart of the mobility approach to SEA.

(e) *Modal energy and wave intensity*

As seen from (2), the fundamental SEA relation between power flow and modal energy difference is analogous to a heat flow equation for bodies at different temperatures. Modal energy may be thought of as a measure of subsystem temperature. It is not immediately obvious how this analogy may be explained in terms of the physics of vibrational energy transfer between subsystems. An examination of analytical expressions for the modal energies of simple, uniform, isotropic subsystems provides some elucidation of this issue. (Non-isotropic or curved shell structures require special analysis because they do not possess direction-independent wave group speeds and therefore do not support diffuse fields. The wave intensity distribution analysis of Langley & Bercin (this book) is relevant to such cases.) The average density of modal natural frequencies associated with any vibrational wave-type is directly proportional to the spatial extent of the bounded medium (length, area or volume) and inversely proportional to the wave group speed (energy propagation speed). For example, the modal density $n(\omega)$ of a uniform one-dimensional system is given by $2L/c_g$, of a uniform, isotropic two dimensional plane structure of surface area S is given by $kS/\pi c_g$ and of a uniform, isotropic volume V