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052101736X - Quantum Geometry: A Statistical Field Theory Approach

Jan Ambjorn, Bergfinnur Durhuus and Thordur Jonsson

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