EPIDEMIC MODELLING: AN INTRODUCTION

This is a general introduction to the ideas and techniques required to understand the mathematical modelling of diseases. It begins with an historical outline of some disease statistics dating from Daniel Bernoulli's smallpox data of 1760. The authors then describe simple deterministic and stochastic models in continuous and discrete time for epidemics taking place in either homogeneous or stratified (non-homogeneous) populations. A range of techniques for constructing and analysing models is provided, mostly in the context of viral and bacterial diseases of human populations. These models are contrasted with models for rumours and vector-borne diseases like malaria. Questions of fitting data to models, and the use of models to understand methods for controlling the spread of infection are discussed. Exercises and complementary results at the end of each chapter extend the scope of the text, which will be useful for students taking courses in mathematical biology who have some basic knowledge of probability and statistics.

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Epidemic Modelling: An Introduction

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521640794

First published 1999
First paperback edition 2001
Reprinted 2005

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Daley, Daryl J.
Epidemic modelling : an introduction / D.J. Daley and J.M. Gani.
p. cm. – (Cambridge studies in mathematical biology ; 14)
Includes bibliographical references and index.
ISBN 0 521 64079 2
1. Epidemiology–Mathematical models. 2. Epidemiology–
Statistical methods. I. Gani, J. M. (Joseph Mark) II. Title.
III. Series.
RA652.2.M3D34 1999
614.4′01′5–dc21 98-44051 CIP

ISBN 978-0-521-64079-4 hardback
ISBN 978-0-521-01467-0 paperback

Transferred to digital printing 2007

To Nola, for constant support and understanding [DJD]

To my late wife Ruth, who first directed my interest
to biological problems [JMG]

... The history of malaria contains a great lesson for humanity—that we should be more scientific in our habits of thought, and more practical in our habits of government. The neglect of this lesson has already cost many countries an immense loss in life and in prosperity.

Ronald Ross,
The Prevention of Malaria (1911)

... It follows that epidemic theory should certainly continue to search for new insights into the mechanisms of the population dynamics of infectious diseases, especially those of high priority in the world today, but that increased attention should be paid to formulating applied models that are sufficiently realistic to contribute directly to broad programs of intervention and control.

Norman T. J. Bailey,
The Mathematical Theory of Infectious Diseases (1975, p. 27)

... The level of economic development of communities generally determines the level of health services. The higher the level of economic development, the more effectively did surveillance and containment principles apply and the earlier was *variole major* [smallpox], in particular, eliminated from the country.

F. J. Fenner,
Smallpox and its Eradication (1988)

... Statistical science has made important contributions to our understanding of AIDS. Statistical methods were used in the earliest studies of the etiology of AIDS, and evidence for sexual transmission came from case-control studies among gay men, in which AIDS cases were compared to matched controls. It was found that high numbers of sexual contacts were a risk factor for AIDS.

R. Brookmeyer and M. H. Gail,
in *Chance* **3**(4), 9–14, 1990

Contents

Preface

This monograph is designed to introduce probabilists and statisticians to the diverse models describing the spread of epidemics and rumours in a population. Not all epidemic type processes have been included. With minor exceptions, we have restricted ourselves to the spread of viral and bacterial infections, or to the propagation of rumours, by direct contact between infective and susceptible individuals. Host–vector and parasitic infections have been mentioned only very briefly.

Throughout the book, the emphasis is on the mathematical modelling of epidemics and rumours, and the evolution of this modelling over the past three centuries.

Chapter 1 is a historical introduction to the subject, with illustrations of the most common approaches to modelling. This is followed in Chapter 2 by an account of deterministic models, in both discrete and continuous time. Chapter 3 analyses stochastic models in continuous time, and includes detailed studies of the simple, general and carrier-borne epidemics. In Chapter 4, the main stochastic models in discrete time, namely the chain binomial models are studied, and a pairs-at-parties and related models outlined. Chapter 5 considers models for the propagation and cessation of rumours, and exploits some of the techniques introduced earlier to analyse them; the results highlight differences between these and the classical epidemic models. Chapter 6, which is essentially statistical, is concerned with the fit of various models to observed epidemic data. The book ends with Chapter 7, which describes three main methods of controlling epidemics. A list of references that also incorporates an author index, and a subject index are provided at the end.

While the monograph cannot claim to be comprehensive, our hope is that readers who master its contents should have little difficulty in reading the current literature on epidemic modelling. The two main treatises on the subject are Bailey (1975) and Anderson and May (1991); both are often referred to in the text. The former considers most of the classical epidemic models, both deterministic and stochastic, while the latter concentrates essentially on deterministic results. We believe that both types of models have a role to play in describing the spread of infections in large and small populations, and have attempted to give due weight to each.

Current epidemic modelling relies on a great variety of mathematical methods; we have endeavoured to emphasize the intrinsic interest of such methods, as well as demonstrate their practical usefulness. Exercises and complements have been provided at the end of each Chapter, with the dual intentions of extending the text and of providing opportunities for readers to practise their skills in modelling and the analysis of models. The exercises are not of uniform difficulty.

For those who wish to reach the forefront of current research, the volumes of papers edited by Mollison (1995) and Isham and Medley (1996), both arising from a six-month research programme on Epidemic Models at the Newton Institute in Cambridge in 1993, provide further illustrations of epidemic modelling and many challenging problems in the field.

Finally we thank all of our colleagues who have collaborated with us over many years in both this area of applied probability and others; their contributions are too many to mention individually, save that DJD pays tribute to David Kendall who first introduced him to the topic of this book, and JMG expresses his gratitude to Norman Bailey and Maurice Bartlett, pioneers of stochastic epidemic modelling.

We also thank David Tranah, and the Copy Editor and others at Cambridge University Press for their cooperation in producing this book without invoking LaTEX.

Daryl Daley, Joe Gani
Canberra, October 1998