

An Introduction to Formal Logic

Formal logic provides us with a powerful set of techniques for criticizing some arguments and showing others to be valid. These techniques are relevant to all of us with an interest in being skilful and accurate reasoners. In this highly accessible book, Peter Smith presents a guide to the fundamental aims and basic elements of formal logic. He introduces the reader to the languages of propositional and predicate logic, and then develops formal systems for evaluating arguments translated into these languages, concentrating on the easily comprehensible ‘tree’ method. His discussion is richly illustrated with worked examples and exercises. A distinctive feature is that, alongside the formal work, there is illuminating philosophical commentary. This book will make an ideal text for a first logic course, and will provide a firm basis for further work in formal and philosophical logic.

PETER SMITH was formerly Senior Lecturer in Philosophy at the University of Cambridge. His other books include *Explaining Chaos* (1998) and *An Introduction to Gödel’s Theorems* (2007), and he is a former editor of the journal *Analysis*.

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Preface

The world is not short of good introductions to logic. They differ widely in pace, style, the coverage of topics, and the ratio of formal work to philosophical commentary. My only excuse for writing another text is that I didn't find one that offered quite the mix that I wanted for my own students (first-year philosophy undergraduates doing a compulsory logic course). I hope that some other logic teachers and their students will find my particular combination of topics and approach useful.

This book starts from scratch, and initially goes quite slowly. There is little point in teaching students to be proficient at playing with formal systems if they still go badly astray when faced with ground-level questions about the whole aim of the exercise. So I make no apology for working hard at the outset to nail down some basic ideas.

The pace picks up as the book proceeds and readers get used to the idea of a formal logic. But even the more symbol-phobic students should be able to cope with most of the book, at least with a bit of judicious skipping. For enthusiasts, I give soundness and completeness proofs (for propositional trees in Chapter 19, and for quantifier trees in Chapter 30). The proofs can certainly be skipped: but I like to think that, if explained in a reasonably relaxed and accessible way, even these more 'advanced' results can in fact be grasped by bright beginners.

I have kept the text uncluttered by avoiding footnotes. You can follow up some of the occasional allusions to the work of various logicians and philosophers (such as Frege or Russell) by looking at the concluding notes on further reading.

The book has a web-site at www.logicbook.net. You will find there some supplementary teaching materials, and answers to the modest crop of exercises at the end of chapters. (And I'd like to hear about errors in the book, again via the web-site, where corrections will be posted.)

I am very grateful to colleagues for feed-back, and to the generations of students who have more or less willingly road-tested versions of most of the following chapters. Special thanks are due to Hilary Gaskin of Cambridge University Press, who first encouraged my plan to write this book, and then insisted

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that I didn't keep revising it for ever; to Dominic Gregory and Alexander Paseau, who read late drafts of parts of the book, and provided many corrections; and to Laurence Goldstein, who did much more than it was reasonable to expect of a publisher's reader.

Not least, I must thank Patsy and Zoë Wilson-Smith, without whose love and support this book would never have been finished.

Additional warm thanks are due to all those who kindly told me about mistakes in the first printed version of the book. I took the opportunity of an initial reprint to make the needed corrections and to make many other minor changes to improve clarity. Joseph Jedwab then gave me a long list of further errors, which have now also been corrected.