

## Defects and Geometry in Condensed Matter Physics

This book describes the key role played by thermally excited defects such as vortices, disclinations, dislocations, vacancies and interstitials in the physics of crystals, superfluids, superconductors, liquid crystals and polymer arrays.

Geometrical aspects of statistical mechanics become particularly important when thermal fluctuations entangle or crumple extended line-like or surface-like objects in three dimensions. In the case of entangled vortices above the first-order flux lattice melting transition in high-temperature superconductors, the lines themselves are defects. A variety of theories combined with renormalization-group ideas are used to describe the delicate interplay among defects, statistical mechanics and geometry characteristic of these problems in condensed matter physics.

This indispensable guide has its origins in Professor Nelson's contributions to summer schools, conference proceedings and workshops over the past twenty years. It provides a coherent and pedagogic graduate-level introduction to the field of defects and geometry.

DAVID NELSON is Mallinckrodt Professor of Physics and Professor of Applied Physics at Harvard University. He received his Ph.D. in 1975 from Cornell University. His research focuses on collective effects in the physics of condensed matter, particularly on the connections between thermal fluctuations, geometry and statistical physics. In collaboration with his Harvard colleague, Bertrand I. Halperin, he has proposed a theory of dislocation- and disclination-mediated melting in two dimensions. Professor Nelson's other interests include the statistical mechanics of metallic glasses, the physics of polymerized membranes, vortex phases in high-temperature superconductors and biophysics.

Professor Nelson is a member of the National Academy of Sciences, a member of the American Academy of Arts and Sciences and a Fellow of the American Physical Society; he has been an A. P. Sloan Fellow, a Guggenheim Fellow and a Junior and Senior Fellow in the Harvard Society of Fellows. He is the recipient of a five-year MacArthur Prize Fellowship, the National Academy of Sciences Prize for Initiatives in Research and the Harvard Ledlie Prize.

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**David R. Nelson**

Lyman Laboratory of Physics,  
Harvard University



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**To Patricia, Meredith, Christopher and Peter**

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## Preface to the book

Considerable relief accompanies completion of a project like this, especially one pursued (intermittently) for a period of eighteen years! There are many people to thank, not the least of which are the numerous graduate students, postdocs and other colleagues mentioned in Chapter 1 and in the acknowledgements of the remaining chapters. However, I owe a special debt to Michael Fisher, Leo Kadanoff and Bert Halperin, who provided inspiring examples of how to do theoretical physics early in my career. I am also grateful to Harvard University and the National Science Foundation of the USA, for creating the environment and freedom to do curiosity-driven research for the past twenty-five years.

I was fortunate to find a sympathetic publisher in Cambridge University Press. Simon Capelin and Rufus Neal provided expert editorial guidance and extraordinary patience in face of distractions caused by, among other things, my three years as Chair of the Harvard Physics Department. The original documents on which the last eight chapters are based illustrate the recent history of scientific publishing. At least one chapter evolved from an old-fashioned typewritten manuscript. Others originally appeared via a photographic offset printing process. The later chapters were created using LaTeX and at least one is available (in an early form) on the World Wide Web (<http://arXiv.org/abs/cond-mat/9502114>). I appreciate the willingness of the original publishers to allow me to adapt my contributions to various proceedings, summer schools and workshops. I owe a special debt to Sally Thomas, Steven Holt, Jo Clegg and Jayne Aldhouse for the expert way in which they created a seamless high-quality book from these disparate media. Saul Teukolsky, Paul Horowitz and Renate D'Archangelo kindly provided advice and assistance with the index.

Farid Abraham generously assisted with the preparation of the illustration on the front cover. For more beautiful images created by Farid and his collaborators, see his gallery (<http://www.almaden.ibm.com/vis/membrane/gallery.html>). The pictures on the back cover are double-sided decorations of flux lines in high-temperature superconductors, due to Zhen Yao, Charles Lieber and their associates. I am grateful to Zhen and Charlie for permission to use their remarkable images, which

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led to the first experimental measurement of a bosonic “phonon–roton” spectrum for entangled vortex lines. For a more detailed discussion, see the article with George Crabtree in the April 1997 issue of *Physics Today* and references therein. While going over the final page proofs, I was struck again by the many striking and experimentally observable manifestations of geometry, defects and statistical mechanics in condensed-matter physics. I hope others will be able to capture some of this excitement while reading this book.

David R. Nelson  
Rhineland, Wisconsin  
September, 2001

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