

Index

SYMBOLS

- $A \otimes B$ product sigma-field, 82
 $A \setminus B$ difference of sets, 7
 $A \Delta B$ symmetric difference of sets, 7
 $BL(X)$ bounded Lipschitz functions, 170
 $\mathcal{B}(X)$ Borel sigma-field on X , 22
 $\mathcal{C}^k(\mathbb{R})$ smoothness class of functions, 177
 \mathbb{C} complex plane
 $\mathcal{C}(T)$ continuous, real functions on T , 215
 $D_+(x)$ right-hand derivative, 309
 $\Delta(x) := 2(e^x - 1 - x)/x^2$, 265
 $dv/d\mu$ density, 53
 $D(P\|Q)$ relative entropy, 61
 \mathcal{E}_δ (\mathcal{E}_σ) countable intersections (unions) of sets from a class \mathcal{E} , 20
 $:=$ equality, by definition
 $\overset{\circ}{f}$, \bar{f} semicontinuous envelope, 173
 \mathcal{F}_τ pre- τ sigma-field, 142
 $H(P, Q)$ Hellinger distance, 61
 \Rightarrow NOT weak convergence, 171
 \mathbb{I}_A indicator function of a set
 $\langle \cdot, \cdot \rangle$ inner product, *see* Appendix B
 $\mathcal{L}^1(\mu)$ functions integrable wrt μ , 28
 L^1 equivalence classes of \mathcal{L}^1 , 35
 L^∞ , 49
 \mathcal{L}^p , L^p , 36
 $L(x) := (2x \log \log x)^{1/2}$, 261
 \vee maximum (pointwise, of functions)
 \wedge minimum (pointwise, of functions)
 m Lebesgue measure (usually), 29
 $\mathcal{M}^+(\mathcal{X}, \mathcal{A})$ cone of nonnegative, \mathcal{A} -measurable functions on a set \mathcal{X} , 24
 $\mathcal{M}_{\text{bdd}}^+$ bounded members of \mathcal{M}^+ , 59
 $\mu^x \lambda_x^y$ iterated integral, 84
 μf same as $\int f(x)\mu(dx)$, 27
 $(\mu_1 - \mu_2)^+$, 73
 $\mu_1 \wedge \mu_2$ minimum of measures, 61
 $\mu * \nu$ convolution, 91
 $\mu \otimes \lambda$ product measure, 88
 $\mu \otimes \Lambda$ product of measure and kernel, 86
 \mathbb{N} natural numbers (1, 2, ...)

 $\bar{\mathbb{N}}_0 := \{0\} \cup \mathbb{N}$

 $\bar{\mathbb{N}} := \mathbb{N} \cup \{\infty\}$

 $\bar{\bar{\mathbb{N}}}_0 := \{0\} \cup \mathbb{N} \cup \{\infty\}$

 $\nu \perp \mu$ mutual singularity, 57
 $O_p(\cdot)$, $o_p(\cdot)$ stochastic order symbols, 183
 $\phi(x)$ standard normal density, 317
 $\bar{\Phi}(x) := \mathbb{P}\{N(0, 1) > x\}$, 317
 $P_n \rightsquigarrow P$ weak convergence, 171
 $\mathbb{P}(X | T = t)$ conditional expectation, 125
 $\mathbb{P}(X | \mathcal{G})$ conditional expectation, 126
 $\psi(x) := 2 \left((1+x) \log(1+x) - x \right) / x^2$, 264
 $\bar{\mathbb{R}}$ extended real line, $\mathbb{R} \cup \{-\infty, \infty\}$
 \mathbb{R} real line
 $\rho(x) := \phi(x)/\bar{\Phi}(x)$, 317
 $\sigma(\mathcal{E})$ sigma-field generated by a class of sets \mathcal{E} , 19
 $\sigma(\mathcal{H})$ sigma-field generated by a class of functions \mathcal{H} , 23
 $T\mu$ image measure, 40
 2^{\aleph_0} a particular cardinality, 103
 \mathbb{W} Wiener measure, 215
 $X_n \rightsquigarrow P$ convergence in distribution, 171
 $\mathcal{X} \times \mathcal{Y}$ product space, 82

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