

Elements of Mathematical Ecology

Elements of Mathematical Ecology provides an introduction to classical and modern mathematical models, methods, and issues in population ecology. The first part of the book is devoted to simple, unstructured population models that, for the sake of tractability, ignore much of the variability found in natural populations. Topics covered include density dependence, bifurcations, demographic stochasticity, time delays, population interactions (predation, competition, and mutualism), and the application of optimal control theory to the management of renewable resources. The second part of this book is devoted to structured population models, covering spatially structured population models (with a focus on reaction-diffusion models), age-structured models, and two-sex models. Suitable for upper level students and beginning researchers in ecology, mathematical biology and applied mathematics, the volume includes numerous line diagrams that clarify the mathematics, relevant problems throughout the text that aid understanding, and supplementary mathematical and historical material that enrich the main text.

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Preface

Ecology is an old discipline. The discipline was christened in 1866 by Ernst Haeckel, a well-known German evolutionary biologist. Haeckel was a neologist – he loved to invent new scientific terms. His most famous gems are *phylogeny* and *oecologie*. *Oecologie* and *ecology* take their derivation from the Greek *oikos*, house or dwelling place. Ecology, as envisioned by Haeckel, is the study of the houses and the housekeeping functions of plants and animals. It is the scientific study of the interrelationships of organisms, with each other, and with their physical environment. The *idea* of ecology is even older (Worster, 1994). It is closely related to 18th century notions of the balance or economy of nature reflected, most clearly, in Linnaeus's 1749 essay *Oeconomia Naturae* (Stauffer, 1960).

Ecology is also a diverse discipline. After all, it has all of life to account for. In the old days, it was common to divide ecology into two subdisciplines: *autecology*, the ecology of individual organisms and of populations, and *synecology*, the study of plant and animal communities. Ecology is now divided into many subdisciplines (see Table 1).

Several subdisciplines use mathematics. For example, behavioral ecology makes extensive use of game theory and of other brands of optimization. It is impossible to cover all of these subdisciplines in one short book. Instead, I focus on population ecology and engage in occasional forays into community ecology and evolutionary ecology. This book could, and perhaps should, have been entitled *The Dynamics of Biological Populations*.

The material in this book has been used to teach a two-semester course. There is, therefore, a dichotomy in these notes. The first semester of the course is devoted to unstructured population models, models that, in effect, treat organisms as 'homogeneous green gunk'. Unstructured population models have the advantage, at first, of simplicity. As one adds extra bits

viii *Preface*Table 1. *Branches of ecology*

Synecology	Landscape ecology Systems ecology Community ecology
Autecology	Population ecology Evolutionary ecology Behavioral ecology Physiological ecology Chemical ecology

of biology, these models become more realistic and more challenging. The topics in the first half of the book include density dependence, bifurcations, demographic stochasticity, time delays, population interactions (predation, competition, and mutualism), and the application of optimal control theory to the management of renewable resources.

Variety, and variability, are the spice of life. We frequently ascribe differences in the success of individuals to differences in age, space (spatial location), or sex. The second half of this book is devoted to structured population models that take these variables into account. I begin with spatially-structured population models and focus on reaction-diffusion models. There is also tremendous interest in metapopulation models, coupled lattice maps, integrodifference equations, and interacting particle systems (Turchin, 1998; Hanski, 1999). However, my colleagues and I tend to leave this material for our advanced course. I follow with an overview of age-structured population models in which I compare integral equations, discrete renewal equations, matrix population models, and partial differential equations. I conclude with a brief introduction to two-sex models.

The emphasis in these notes is on strategic, not tactical, models (Pielou, 1981). I am interested in simple mechanistic models that generate interesting hypotheses or explanations rather than in detailed and complex models that provide detailed forecasts. You will also find many equations, but few formal theorems and proofs. Applied scientists and pure mathematicians both have reason to be offended! Because of the interdisciplinary nature of my class and because of my own preference for solving problems over proving theorems, I have tried to hold to a middle course that should appear natural to applied mathematicians and to theoretical biologists. I hope that this middle course will appeal to a broad range of (present and future) scientists. Failing that, I hope that you, gentle reader, can use this book as a springboard for more detailed applied and theoretic investigations.

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Knoxville, Tennessee

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