

Contents

<i>Preface</i>	<i>page</i> ix
<i>Acknowledgements</i>	xiii
1 Sets and structures	1
1.1 Sets and logic	2
1.2 Subsets, unions and intersections of sets	5
1.3 Cartesian products and relations	7
1.4 Mappings	10
1.5 Infinite sets	13
1.6 Structures	17
1.7 Category theory	23
2 Groups	27
2.1 Elements of group theory	27
2.2 Transformation and permutation groups	30
2.3 Matrix groups	35
2.4 Homomorphisms and isomorphisms	40
2.5 Normal subgroups and factor groups	45
2.6 Group actions	49
2.7 Symmetry groups	52
3 Vector spaces	59
3.1 Rings and fields	59
3.2 Vector spaces	60
3.3 Vector space homomorphisms	63
3.4 Vector subspaces and quotient spaces	66
3.5 Bases of a vector space	72
3.6 Summation convention and transformation of bases	81
3.7 Dual spaces	88
4 Linear operators and matrices	98
4.1 Eigenspaces and characteristic equations	99
4.2 Jordan canonical form	107

Contents

4.3	Linear ordinary differential equations	116
4.4	Introduction to group representation theory	120
5	Inner product spaces	126
5.1	Real inner product spaces	126
5.2	Complex inner product spaces	133
5.3	Representations of finite groups	141
6	Algebras	149
6.1	Algebras and ideals	149
6.2	Complex numbers and complex structures	152
6.3	Quaternions and Clifford algebras	157
6.4	Grassmann algebras	160
6.5	Lie algebras and Lie groups	166
7	Tensors	178
7.1	Free vector spaces and tensor spaces	178
7.2	Multilinear maps and tensors	186
7.3	Basis representation of tensors	193
7.4	Operations on tensors	198
8	Exterior algebra	204
8.1	r -Vectors and r -forms	204
8.2	Basis representation of r -vectors	206
8.3	Exterior product	208
8.4	Interior product	213
8.5	Oriented vector spaces	215
8.6	The Hodge dual	220
9	Special relativity	228
9.1	Minkowski space-time	228
9.2	Relativistic kinematics	235
9.3	Particle dynamics	239
9.4	Electrodynamics	244
9.5	Conservation laws and energy–stress tensors	251
10	Topology	255
10.1	Euclidean topology	255
10.2	General topological spaces	257
10.3	Metric spaces	264
10.4	Induced topologies	265
10.5	Hausdorff spaces	269
10.6	Compact spaces	271

Contents

10.7	Connected spaces	273
10.8	Topological groups	276
10.9	Topological vector spaces	279
11	Measure theory and integration	287
11.1	Measurable spaces and functions	287
11.2	Measure spaces	292
11.3	Lebesgue integration	301
12	Distributions	308
12.1	Test functions and distributions	309
12.2	Operations on distributions	314
12.3	Fourier transforms	320
12.4	Green's functions	323
13	Hilbert spaces	330
13.1	Definitions and examples	330
13.2	Expansion theorems	335
13.3	Linear functionals	341
13.4	Bounded linear operators	344
13.5	Spectral theory	351
13.6	Unbounded operators	357
14	Quantum mechanics	366
14.1	Basic concepts	366
14.2	Quantum dynamics	379
14.3	Symmetry transformations	387
14.4	Quantum statistical mechanics	397
15	Differential geometry	410
15.1	Differentiable manifolds	411
15.2	Differentiable maps and curves	415
15.3	Tangent, cotangent and tensor spaces	417
15.4	Tangent map and submanifolds	426
15.5	Commutators, flows and Lie derivatives	432
15.6	Distributions and Frobenius theorem	440
16	Differentiable forms	447
16.1	Differential forms and exterior derivative	447
16.2	Properties of exterior derivative	451
16.3	Frobenius theorem: dual form	454
16.4	Thermodynamics	457
16.5	Classical mechanics	464

Contents

17	Integration on manifolds	481
17.1	Partitions of unity	482
17.2	Integration of n -forms	484
17.3	Stokes' theorem	486
17.4	Homology and cohomology	493
17.5	The Poincaré lemma	500
18	Connections and curvature	506
18.1	Linear connections and geodesics	506
18.2	Covariant derivative of tensor fields	510
18.3	Curvature and torsion	512
18.4	Pseudo-Riemannian manifolds	516
18.5	Equation of geodesic deviation	522
18.6	The Riemann tensor and its symmetries	524
18.7	Cartan formalism	527
18.8	General relativity	534
18.9	Cosmology	548
18.10	Variation principles in space-time	553
19	Lie groups and Lie algebras	559
19.1	Lie groups	559
19.2	The exponential map	564
19.3	Lie subgroups	569
19.4	Lie groups of transformations	572
19.5	Groups of isometries	578
	<i>Bibliography</i>	587
	<i>Index</i>	589