

# THE CHANGING SHAPE OF GEOMETRY

Celebrating a Century of Geometry and  
Geometry Teaching

*Edited on behalf of The Mathematical Association by*

CHRIS PRITCHARD

MATHEMATICAL ASSOCIATION



supporting mathematics in education



THE MATHEMATICAL ASSOCIATION OF AMERICA



**CAMBRIDGE**  
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom  
MATHEMATICAL ASSOCIATION OF AMERICA  
1529 Eighteenth Street, NW Washington DC 20036

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK  
40 West 20th Street, New York, NY 10011-4211, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
Ruiz de Alarcón 13, 28014 Madrid, Spain  
Dock House, The Waterfront, Cape Town 8001, South Africa  
<http://www.cambridge.org>

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First published 2003

Printed in the United Kingdom at the University Press, Cambridge

*Typeface* Times 11/14 pt    *System* L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> [TB]

*A catalogue record for this book is available from the British Library*

*Library of Congress Cataloguing in Publication data*

ISBN 0 521 82451 6 hardback  
ISBN 0 521 53162 4 paperback

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# General Introduction: Simplicity, Economy, Elegance

CHRIS PRITCHARD

As children we build sandcastles and snowmen, construct buildings with LEGO and play computer games that create the impression of rapid movement through three-dimensional space. In later life, we hang wallpaper, negotiate narrow spaces in our cars and tease furniture through doorways. A surprising number of us use specially-honed spatial skills to earn a living, such as barbers and sculptors, interior designers and footballers, bus-drivers and architects, supermarket shelf-stackers, couturiers and civil engineers. Occasionally a Pelé, an Yves Saint Laurent or a Barbara Hepworth comes along. But regardless of the extent to which our spatial talents are developed, from the cradle to the grave, we are all geometers.

This book celebrates the best of geometry in all its simplicity, economy and elegance. Such a wonderful tray of attributes might have formed a ‘splendid title’ for the book as a whole, in the view of H. S. M. Coxeter, but instead it serves as a theme for the Desert Island Theorems and as the title of this general introduction. The source is a lovely anecdote about Peter Frazer, related by H. E. Huntley in his book *The Divine Proportion* [1, p. 5]:

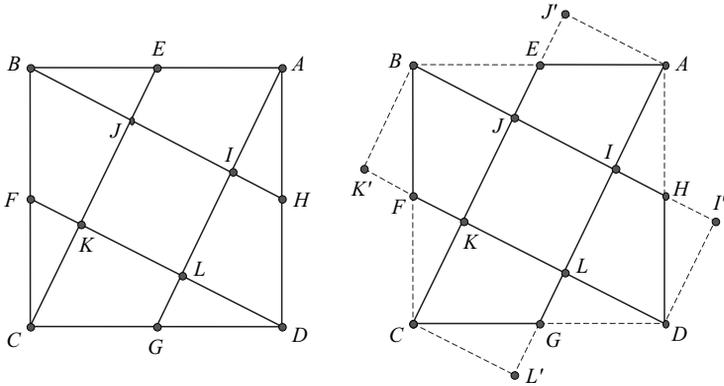
Peter Frazer . . . a lovable man and a brilliant teacher, was discussing cross ratios with a mathematics set. Swiftly, he chalked on the blackboard a fan of four straight lines, crossed them with a transversal and wrote a short equation; he stepped down from the dais and surveyed the figure. . . . Striding rapidly up and down between the class and the blackboard, waving his arms about excitedly, with his tattered gown, green with age, billowing out behind him, he spoke in staccato phrases: ‘Och, a truly beautiful theorem! Beautiful! . . . Beautiful! . . . Look at it! *Look at it!* What simplicity! What economy! Just four lines and one transversal . . . What elegance!’

It appears that Frazer saw simplicity, economy and elegance as complementary essences of beauty in geometry. If I were to add a fourth attribute, it would be ‘surprise’. To elaborate on this theme consider two results from elementary Euclidean geometry. Firstly, in the figure,  $ABCD$  is a square and  $E, F, G, H$  are

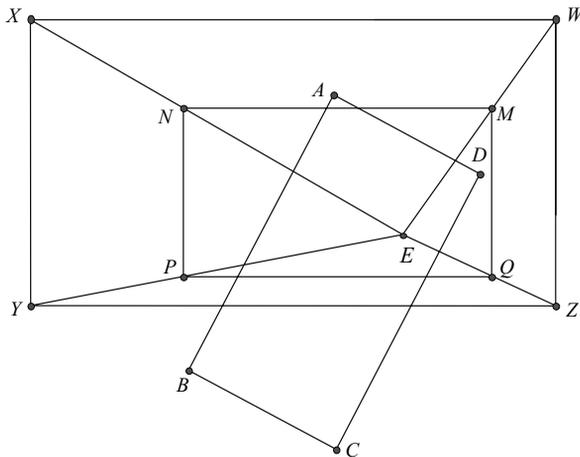
the midpoints of  $AB$ ,  $BC$ ,  $CD$ ,  $DA$  respectively. Then:

$$\text{Area } IJKL = \frac{1}{5} \text{Area } ABCD.$$

A simple dissection proof consists of rearranging the figure into a pentomino in the form of a cross by rotating  $\triangle EJB$  through 180 degrees about  $E$ , and rotating the other three small triangles in like manner.



Secondly, take two maps of the same area, one a pocket map and the other a comprehensive map on a larger scale. (Alternatively, take two differently-sized prints of the same photograph.) Place the pocket map on the detailed map at any angle. Then there is one and only one point on the pocket map lying immediately on top of the point corresponding to it on the map beneath. This can be demonstrated using simple transformation geometry. First rotate the pocket map  $ABCD$  about the common point,  $E$ , to give  $MNPQ$  with the same orientation as the larger map



$WXYZ$ . The common point then acts as the centre of a dilatation (or enlargement) of the pocket map. Now let us reverse the argument. Given the two maps with the smaller cast carelessly onto the larger, how would we go about finding the common point? (A solution using cyclic quadrilaterals is shown at the end of the introduction.)

The Mathematical Association came into existence in 1871 at a time when geometry teaching was in something of a tumult. The school and university geometry curricula and Euclid's *Elements* were as two peas in a pod. So completely was the curriculum determined by the standard text that Sylvester sarcastically referred to the *Elements* as 'one of the advanced outposts of the British Constitution'. Calls for reform in the universities met with the antipathy of De Morgan, Cayley and Kelland and with positions entrenched, it fell to schoolteachers to advance the case for 'loosening the shackles' of Euclid [2, ch. 2].

The sort of issues that challenged teachers at the time, concerned the choice and order of the Euclidean propositions to be taught, to whom and at what age? The role of riders in geometry education was also considered. Geometry continued to be viewed as the ideal vehicle for developing an understanding of formal proof. It is only in recent decades, with the advent of modern mathematics and its emphasis on transformation geometry, that the status of formal proof has fallen away and its 'natural habitat' relocated to algebra. We stand on the threshold of a possible reversal of this trend, though as ever it will be difficult to reach a consensus.

In its first thirty years, the movement to improve geometry teaching had only limited success. But by the turn of the century there was a more general acknowledgement of the problems by both educationalists and administrators and the time was ripe for significant change. A growing understanding of the need to broaden the scope and methods of geometry in schools saw the first use of measuring instruments such as protractors and the adoption of four-figure tables.

To digress a little, at high school in the late 1960s, my classmates and I made use of such four-figure tables, compiled according to the frontispiece by 'the late C. Godfrey and the late A. W. Siddons'. We had the great good fortune to be taught geometry by Wil Williams, a teacher of unusual clarity and not a little humour – "they died calculating the entries", was his quip. At the time, little did he or we know of the enormous roles Godfrey and Siddons played in The Mathematical Association. Twenty-three of its early members formed the first Teaching Committee in 1902 and Godfrey and Siddons were prominent among them. This book celebrates the centenary of Teaching Committee, a body which in its early years helped to subjugate the geometry text to the requirements of the designed curriculum and which has continued ever since to provide advice and resources for mathematics teachers and to seek to influence national policies on mathematics education.

At the Annual Meeting of the National Council of Teachers of Mathematics in Florida in April 2001, I sought the views of one of its senior figures on The Mathematical Association. It was no great surprise that the first thing he said in reply was that the association has very good journals. And indeed, the journals attract contributions from all parts of the world. Since it is from two of these journals, the *Mathematical Gazette* and *Mathematics in School*, that a goodly proportion of the material for this book is taken, a little more information about them is in order.

The *Mathematical Gazette*, the premier journal of The Mathematical Association, was established in April 1894 under the editorship of Edward Mann Langley. The opening editorial made it plain that it would enable mathematics teachers to share successful approaches to their art, other than those to be found in the texts of the day [2, p. 40]. In this role it has met with fluctuating success during its 108-year history, whilst maintaining greater consistency as a minor mathematical serial. It was Langley, incidentally, who as a teacher at Bedford Modern School converted E. T. Bell to mathematics and quite likely helped shape that accomplished number theorist and somewhat unreliable mathematical biographer [3, ch. 2]. Bell duly contributed an article to the *Mathematical Gazette*'s 250<sup>th</sup> number in July 1938 and, with its reputation soaring both in Britain and internationally, the 500<sup>th</sup> number was published in July 2000. The launch of *Mathematics in School* in November 1971 provided members with a journal designed to support the teaching of mathematics to younger children. Its style remains to this day rather more informal than that of the *Mathematical Gazette*, with a larger proportion of articles focussing on classroom practice, often with a hands-on flavour. Elementary geometry and its teaching have formed the subject of several thousand articles which have appeared in these two journals over the last century; many of the best get a second airing in this volume.

Following an initial trawl some twenty geometrical themes were identified for possible inclusion in this book. Each perhaps merited a separate part but six were finally selected for inclusion. The first two parts, on the nature and history of geometry, and the last part, on its teaching, are somewhat weightier than the other three on Pythagoras' Theorem, the golden section and recreational geometry. They tend to set the scene, to help the reader to understand the contexts within which geometry and its teaching developed. A number of articles in these sections refer to Euclidean definitions and theorems simply by taking the book number and the proposition number from the *Elements*. Thus, Pythagoras' Theorem is Euclid I, 47 because it is the forty-seventh proposition of the first book of the *Elements*. Note that Euclid and his book are synonymous. Since a large proportion of a modern audience will be unfamiliar with these tags, each of the propositions referred to in this way has been stated in full in the second appendix.

Two of the most illustrious mathematicians of this century, G. H. Hardy and Michael Atiyah offer views on what constitutes geometry. Hardy confesses from the outset that ‘I do not claim to know any geometry, but I do claim to understand quite clearly what geometry is’ and comes to a pessimistic conclusion about ever reaching a consensus on the geometry curriculum. Atiyah takes an historical approach to tease out the nature of the subject.

Following a general overview of the history of geometry, the history articles open with an essay on Greek mathematics by Thomas Heath. A notable feature of this section is a run of items highlighting non-European contributions to the development of geometry, especially one by the author of *The Crest of the Peacock*, George Gheverghese Joseph. It is rounded off with a detailed review of nineteenth century geometry by Gaston Darboux, penned at the turn of the century without the benefit that a time lapse often affords. The reader may judge whether Hardy is harsh in suggesting in his article that Darboux was a great geometer with little feel for the nature of his discipline. The section on the teaching of geometry is introduced by a new and comprehensive essay from The Mathematical Association’s historian, Michael Price. It is followed by what Price describes as a ‘concise and stylish attack on Euclid from an advanced pure mathematical standpoint’ [2, p. 55]. Its author is Bertrand Russell, the year 1902, the very year that Teaching Committee came into existence.

There is a greater focus on elucidating elementary geometry in the other three parts of the book. Articles on Pythagoras’ Theorem are introduced by Janet Jagger, an educationalist and former Chair of Teaching Committee. Ron Knott casts his expert eye over the articles on the golden section. Incidentally, his peerless Fibonacci website is a treasury of information on the sequence and the associated ratio. Articles on recreational geometry are prefaced by the thoughts of Brian Bolt, whose numerous books on recreational mathematics are much in the vein of Martin Gardner. It is a real pleasure to see reproduced here an extract of an early *Mathematical Gazette* article on ‘rep-tiles’ (replicating figures in the plane) by another hugely influential recreational mathematician, Solomon Golomb. His enthusiastic followers will also be pleased to find among the Desert Island Theorems previously unpublished proofs of a tromino theorem.

Since 1942, the BBC has broadcast a popular weekly radio programme called *Desert Island Discs*. These days its guests are invited to select eight pieces of music that they would take with them to a desert island. They also nominate a favourite book though, presumably to guarantee variety week on week, the key works of the great religions and the works of Shakespeare are excluded. Thirdly, a luxury inanimate object of no practical use is selected. The choices made by guests are little more than devices to prompt them to discuss their lives.

For this book, the ‘desert island theme’ has been developed in a different direction and for a different reason. Imagine you are a mathematician, cast away

on a desert island. You have no access to the modern world and little prospect of being reunited with the rest of humanity. Which geometrical theorem would you least wish to be without, in some sense; which perhaps are you truly glad was discovered. The focus is more on the theorem but might well have something of the personal too. No claim is made to novelty in this shift from desert island music to desert island mathematics. Those who have previously written or spoken on desert island mathematics include David Burghes, Tony Crilly and Colin Fletcher and it is appropriate that their theorems should be included in this book.

Prompted by an early success in persuading Coxeter to offer his Desert Island Theorem – he actually took no persuading at all, such is his love of geometry – a number of eminent mathematicians and physicists, teachers and educationalists were invited to follow suit. They were asked to nominate an elementary, geometrical theorem, to try to keep to a maximum of about 500 words plus a diagram and to adopt where possible what might be termed a ‘popular science’ style. As you will see, some found the brief restrictive but the brief was always intended to guide rather than stifle. Only in a very small number of cases was the same theorem chosen by two contributors.

The response from those approached was overwhelmingly positive and enthusiastic. The Mathematical Association notes with gratitude the affection shown towards it by the community of mathematicians, especially in Britain and North America during the preparation of this book. Among those in higher education and research there would appear to be fulsome recognition of the valuable support and advice given to mathematics teachers by The Mathematical Association and similar organizations worldwide.

The informality and brevity of the contributor’s designations has raised the eyebrows of some who have previewed material in this book. Leaders in their fields usually need little introduction and certainly no puffing up. Indeed, what needs to be added to an author’s name tends to vary inversely with eminence. In the minds of thousands of students or former students, Tom Apostol is the author of the two best calculus texts of all time. It is fitting therefore to refer to him as a teacher of the calculus, especially in a book put together to celebrate a centenary of relevance to mathematics teachers. Further, in most cases, academic positions (though not institutions) have been omitted as have titles and other honours. This is an attempt – possibly a vain attempt, human nature being what it is – to focus on the Desert Island Theorems themselves rather than the eminence of the writer. Several of them are beautifully crafted. Where a contributor is or has been a servant of The Mathematical Association, this is noted especially for the benefit of members. Each castaway has either suggested the designation provided or else expressed contentment with it.

There are thirty Desert Island Theorems in total, arranged very loosely into five groups:

- A. ancient Greek geometry
- B. elementary Euclidean geometry (of the last four centuries)
- C. advanced Euclidean
- D. spherical geometry and topology
- E. geometrical physics.

As a rule of thumb, the difficulty level of the theorems tends to increase as the book proceeds. Arguably, there is a move further and further from pure geometry at the same time. The vast majority of the theorems should be accessible to those who have studied mathematics to the age of eighteen and possess a willingness to wrestle with some of the geometrical arguments. Readers with an interest in the development of geometry will find much to savour.

If it can ever be said that a single person has pushed back the boundaries of a branch of mathematics, particularly in recent times, then it might justifiably be said of Coxeter in relation to geometry. Coxeter's sixty-six year association with the University of Toronto may have obfuscated the fact that he was born and raised in England and received his university education at Cambridge. The first recorded evidence of his prodigious grasp of geometry is to be found in the *Mathematical Gazette*, where in the issue of October 1926, the young 'Donald' Coxeter asked the readership via Alan Robson, his teacher at Marlborough School, if it knew of an 'elementary verification' of  $\int_0^{\frac{\pi}{2}} \sec^{-1}(\sec x + 2) dx = \frac{5\pi^2}{24}$  and two similar results 'suggested by a geometrical consideration and verified graphically' [4]. At the time, Robson was already prominent in The Mathematical Association. He would go on to hold high office as Chair of Teaching Committee and as President, either side of the Second World War [2, pp. 151, 161]. The direct influence of Robson on Coxeter is undoubted, the indirect influence of the Mathematical Association and its Teaching Association rather more speculative.

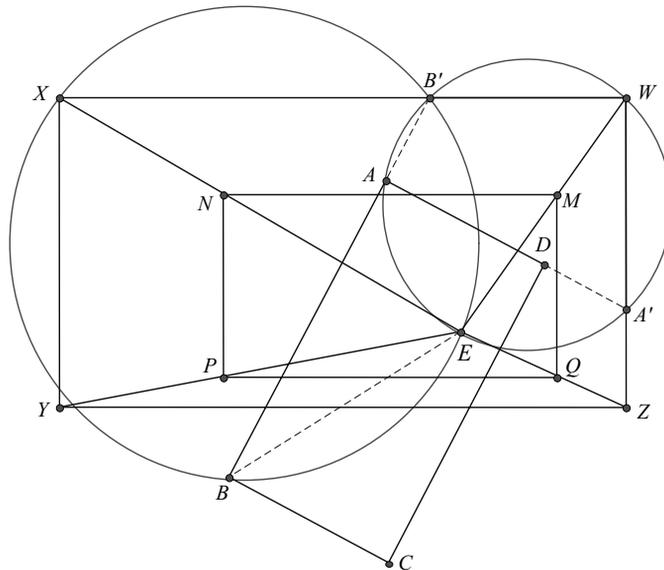
Finally, let me return to my previous theme to consider the simple, economical, elegant and surprising geometry of the cauliflower. Almost any elementary treatment of the Fibonacci sequence and the golden section draws attention to the structure of the spiralling segments of the pineapple. Yet the cauliflower appears to have exactly the same structure and it seems a shame that this humble vegetable should have been so thoroughly eclipsed by an exotic fruit. If you look carefully enough the geometrical structure is certainly evident in the common white variety, though it is striking in the romanesco variety of Fibonacci's homeland of northern Italy. A few years ago I was delighted to find that the cauliflower was not without its champion. Touring the Cité des Sciences et de l'Industrie at La Villette in Paris, I came across an exhibit which highlighted both the cauliflower's spirals and



The spirals and fractals of the romanesco cauliflower

its self-replicating fractal form. But whether we invoke the pineapple, the nautilus shell, or Fibonacci's romanesco cauliflower one thing seems clear – the attributes of the most attractive and pleasing geometry remain the same irrespective of whether the geometry is of our invention or of our discovery, crafted by man or designed by nature. In the words of John Keats: *A thing of beauty is a joy for ever.*

***Solution to the Maps Question (by Doug French)***



First, let  $BA$  produced meet  $XW$  at  $B'$ .

$$\angle BEX = \angle BB'X = \text{angle of rotation}$$

These angles are subtended by the same chord,  $XB$ . So  $XBEB'$  is a cyclic quadrilateral.

Similarly, let  $AD$  produced meet  $WZ$  at  $A'$ .

$$\angle AEW = \angle AA'W.$$

These angles are subtended by the same chord,  $AW$ . So  $WAEA'$  is a cyclic quadrilateral.

Since  $E$  lies on the circumference of both circles, it is located at one of the points of intersection.

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3. Constance Reid, *The Search for E. T. Bell, Also Known as John Taine* (Washington DC: Mathematical Association of America, 1993).
4. D. Coxeter (per A. R.), Note 853, *Mathematical Gazette* **13** (October 1926), 205. This was Coxeter's first publication. [See Coxeter, H. S. M. *Twelve Geometry Essays* (Southern Illinois University Press, 1968) (Preface, p. vii).]

